QUANTWAY CORE

Carnegie Math Pathways
WestEd

Introductory Quantitative Reasoning

Quantway Core

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Detailed Licensing

Detailed Licensing

Licensing

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CHAPTER OVERVIEW

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1.1: Setting Course Expectations and Productive Classroom Norms

Quantway Cohort Contract

By signing below, I agree to fulfill the following requirements for participation in Quantway, and acknowledge that I understand the requirements for continued enrollment.

Specifically:

- I commit to successfully completing Quantway with the members of my cohort.
- I commit to helping all of my cohort members understand mathematics and complete Quantway.
- I will come to class everyday prepared to participate in all classroom activities.
- I will contribute to creating a productive classroom atmosphere that supports everyone learning.
- I will keep an open mind and a positive attitude, and will be willing to try out new learning strategies and study skills.

Signature: ______________________________ Date: ________________

Witness: _______________________________ Date: _______________

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1.2: Introduction to Quantitative Reasoning

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- quantitative reasoning is the ability to understand and use quantitative information. It is a powerful tool in making sense of the world.
- relatively simple math can help make sense of complex situations.
- 1 billion = 1,000 × 1,000 × 1,000.
- the representations 1 billion, 1,000,000,000, and 10^9 have the same meaning.

By the end of this collaboration, you should be able to

- identify quantitative information.
- convert units from feet to miles.
- round numbers (based on homework).
- name large numbers (based on homework).
- work in groups and participate in discussion using the group norms for the class.

PROBLEM SITUATION 1: DOES THIS INFORMATION MAKE SENSE?

During this course, you will be presented with a number of problem situations. These problem situations will help you learn how to evaluate the types of quantitative information you may encounter in everyday life.

Problem Situation 1 asks you to use quantitative information to figure out if a statement makes sense.

Imagine that you just received a flier in the mail with the following statement:[1]

> Every year since 1980, the number of American children gunned down has doubled.

(1) What groups might have wanted to mail a flier like this? What are some social issues or political ideas that this statement might support?

Is The Information Reasonable?

The flier in Problem Situation 1 includes **quantitative information**. Quantitative information uses concepts about quantity or number (this can be specific numbers or a pattern based on numerical relationships such as doubling).

You hear and see statements that include quantitative information every day. People use these statements as evidence to convince you to do things like

- vote a certain way;
- donate or give money to a cause; or
- understand a health risk.

You often do not know whether these statements are true. You may not be able to locate the information that supports these statements, but you can start by asking if the statements are **reasonable**. This means asking if the statements make sense. You will be asked if information is *reasonable* throughout this course. This lesson will help you to understand what is meant by this question.

(2) Do you think the statement, “Every year since 1980, the number of children gunned down has doubled” is a reasonable statement? Discuss with your group.

(3) Using only the information that was included in the flier, how can you decide if the statement was reasonable? Talk with your group about the different ways in which you might answer this question. Write some strategies below.

(4) In Question 3, you thought about how to decide if the statement was reasonable. For this problem, choose a starting number for gun deaths in 1980. Put this number into the table below in cell (a). Work in your group to complete the other values in the second

column of the table (b)-(f). Write the numbers in words in (g)-(l). For example, if “1000” is in column 2, “one thousand” would be written in the corresponding space in column 3.

Year	Number of Children Gunned Down	Number in words (rounded)
1980	*(a)*	*(g)*
1990	*(b)*	*(h)*
2000	*(c)*	*(i)*
2010	*(d)*	*(j)*
2015	*(e)*	*(k)*
2020	*(f)*	*(l)*

(5) Refer to the table you created in Question 4. Does the number of children gunned down in the year 2020 seem reasonable? What kind of information might help you decide?

About This Course

This course is called a *quantitative reasoning* course. This means you will learn to use and understand quantitative information. It will be different from many other math classes you have taken. You will learn and use mathematical skills connected to situations like the one you just discussed in this lesson. You will talk, read, and write about quantitative information.

The material in this course will focus on three themes.

- **Citizenship**: You will learn how to understand information about your society, government, and world that can impact many decisions you make.
- **Personal Finance**: You will study how to understand financial information and how to use it to make decisions in your life.
- **Medical Literacy**: You will learn how to understand information about health issues and medical treatments.

This collaboration (1.1) is part of the “Citizenship” theme. In this collaboration, you learn about ways to decide if information is reasonable. This can help you form an opinion about an issue.

Today, the goal is to introduce you to the idea of quantitative reasoning. This will help you understand what to expect from the class. Do not worry if you did not understand all of the math concepts. You will have more time to work with these ideas throughout the course.

In this course you will learn to do the following things:

- Understand and interpret quantitative information.
- Evaluate quantitative information. Today you did this when you answered if the statement was reasonable.
- Use quantitative information to make decisions.

PROBLEM SITUATION 2: HOW BIG IS A BILLION?

Scientists have worried about human population growth for nearly 200 years. The population of Earth has grown over time and is still growing. We do not know how many people Earth can support. (In Lesson 1.2, you will get a sense of how many people there are and how that number has changed over time.)

The world population is estimated to be about 7 billion people. That is seven times as many people as there were 200 years ago.

It is difficult to understand just how big a billion is. Here is a way to help you think about it:

$$1 \text{ billion} = 1{,}000 \times 1{,}000 \times 1{,}000 = 1{,}000{,}000{,}000 = 10^9$$

The following questions will also help you think about how big 1 billion is.

(6) Imagine a line of 1,000 people standing shoulder to shoulder. How long is the line? Complete the following steps to answer this question. For each step, write your calculations clearly so that someone else can understand your work.

(a) Estimate the shoulder width of an “average” person. Write your estimate in the space below.

(b) Use that estimate in the following calculations. Calculate how far a line of 1,000 people, standing shoulder to shoulder, would measure in miles (5,280 feet = 1 mile).

(c) Now imagine 1,000 lines of 1,000 people.

(i) How many people would be in the line?

(ii) How long would the line be measured in miles?

(d) Imagine 1,000 lines like the one in Question 6(c).

(i) How many people would be in line?

(ii) How long is the line measured in miles? Round to the nearest whole mile.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[1] Adapted from Joel Best, *Damned Lies and Statistics: Untangling Numbers from the Media, Politicians, and Activists* (Berkeley: University of California Press, 2001).

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1.2.1: Exercise 1.2

INTRODUCTION

Since this is your first assignment, the authors will explain the structure of your daily assignments. These assignments are referred to as **Exercises**. Each Exercise is comprised of the same three sections:

- Making Connections to the Collaboration
- Developing Skills and Understanding
- Making Connections Across the Course

Note: the online version of an Exercise has a fourth section called Questions.

MAKING CONNECTIONS TO THE COLLABORATION

The purpose of this section is to help make sure you understand the most important ideas of the collaboration. Sometimes it is hard to know what to focus on when you are in class. The authors have designed this curriculum to help you identify and remember important ideas through the following steps:

- Every collaboration session ends with a discussion that asks you to think about the important mathematical ideas of the collaboration.
- The first section of the exercise is “Making Connections to the Collaboration.” This section always starts with a question that asks you to identify a main mathematical idea from the collaboration. You are given four statements to choose from.
- In future exercises, you will describe how mathematical ideas connect across collaborations.

A **main mathematical idea** means the idea is an important concept that helps explain how to do many different types of problems and helps connect different problems together. It may take you a while to be able to identify the main mathematical ideas of collaborations. Your instructor will help you at first by making sure these ideas are highlighted in their summary feedback of the collaboration sessions.

(1) Which of the following statements correctly illustrates one of the *main mathematical ideas* of the collaboration?

(i) Asking good questions about quantitative information is important in quantitative reasoning.

(ii) Doubling means to multiply by 2.

(iii) Gun violence is a problem in the United States.

(iv) You should not use estimation.

Since this is your first time with this type of question, the authors are going to explain the answer to Question 1. The answer is (i) because asking questions about quantitative information is important in many different problem situations. The other answers may or may not be true, but they are not the main mathematical ideas for this collaboration. Specifically,

- (ii) is true, but it only applies to one type of procedure: doubling.
- (iii) is an opinion. You cannot say if it is true or false, and it is not a mathematical idea.
- (iv) is not true.

As you saw in the collaboration, estimation is a valuable skill.

DEVELOPING SKILLS AND UNDERSTANDING

The purpose of this Exercise section is for you to practice the skills and concepts you learned from the lesson. You will see questions directly related to the lesson. You will also see questions that apply the skills and concepts to different situations. This section will sometimes have reading material that helps explain the topics from the lesson. Later in the course, you can look back at this information as you review what you have learned.

Questions 2 and 3 below highlight important quantitative reasoning skills that you will learn in this course.

Quantitative Reasoning Skill: Reading and Interpreting Quantitative Information

Collaboration 1.1 focused on a statement about children “gunned down”. How was such an inaccurate statement published? It was based on another statement published at an earlier time.[2] Both statements are shown below. Read them carefully and decide what each means mathematically

Original Statement: The number of American children killed each year by guns has doubled since 1980.	**Reworded Statement**: Every year since 1980, the number of American children gunned down has doubled.

(2) Based on the original statement and the reworded statement, which of the following comments are valid?

(i) Assume that the original statement is true. If approximately 100 children were killed by guns in 1980, the number of children killed by guns in the present year was about 200.

(ii) Assume that the reworded statement is true. If approximately 100 children were killed by guns in 1980, the number of children killed by guns in the present year was about 200.

Quantitative Reasoning Skill: Identifying Information that Can Be Verified (Checked to See If It Is True)

One characteristic of quantitative information is that it contains numerical information. Another is that it has information that can be checked or evaluated. Being able to evaluate a claim based on quantitative information is an important quantitative reasoning skill.

(3) Which of the following statements contain quantitative information? There may be more than one correct answer.

(i) Many Americans have diabetes.

(ii) ABC News reported that the number of Americans that have diabetes could triple in the next 40 years.[3]

(iii) About a fourth of Americans with diabetes are over 65 years old according to the American Diabetes Association.[4]

(iv) Diabetes is a terrible disease.

The statement “Many Americans have diabetes” sounds quantitative. *Many* implies a number, but it is also a judgment. How much is *many*? There is no way to verify this statement because you could have different opinions about the meaning of “many.” “Diabetes is a terrible disease” is also a judgment. You can offer quantitative information to support the statement, but you cannot verify that this is true or false.

Quantitative Reasoning Skill: Naming and Estimating Large Numbers

Large numbers often occur in real-life situations, but it is hard to make sense of them. It can be difficult to imagine the distinction between 1 million and 1 billion. Collaborations in Module 1 will help you understand the difference in size between these two numbers and other large numbers. You will work on recognizing and naming large numbers as part of the collaboration.

(4) The following place-value chart is partially labeled.

Place-Value Chart

Place-value chart. Starting on the right: Ones = 10 to the power of 0, Tens = 10 to the power of one, Hundreds = 10 to the power of two. Thousands, Ten thousands, Ten millions, Hundred millions, Ten billions, and Hundred billions are blank.

(a) Fill in the missing name labels on the Place-Value Chart.

(b) Fill in the power of 10 that corresponds to each position on the Place-Value Chart. (Entries for 10^0 in the ones place, 10^1 in the tens place, and 10^2 in the hundreds place have already been entered.)

(5) Which of the following represents the number: “Six billion, nine hundred ten million, one hundred fifty-two thousand, eight hundred twenty-four”?

(i) 6,910,152,824

(ii) 600,910,152,824

(iii) 6,000,910,152,824

(iv) 6,910,001,052,824

(6) Select the number/word combination that best estimates each number.

(a) 87,300,000

(i) 8.73 million

(ii) 8.73 billion

    - (iii) 87.3 million
    - (iv) 87.3 billion
- (b) 2,670,000,000,000
    - (i) 2.7 trillion
    - (ii) 2.7 billion
    - (iii) 2700 million
    - (iv) 2.7 million
- (c) 234,700,000,000
    - (i) 235 trillion
    - (ii) 235 billion
    - (iii) 235 million
    - (iv) 235 thousand

(7) The following table provides some data about diabetes in the United States from the American Diabetes Association.[5] Complete the table either by writing the quantitative information as a number or as a combination of words and numbers.

	Number	**Word/Number Combination**
Number of children and adults with diabetes in 2019	(i)	37.3 million
Number of children under age 20 with diabetes in 2019	283,000	(ii)
Cost due to diagnosed diabetes cases in 2017—includes medical costs, disability payments, loss of work, and premature death	$327,000,000,000	(iii)

Quantitative Reasoning Skill: Rounding Numbers

Another important skill you used in this collaboration is rounding. People often round numbers when they are trying to make sense out of them or make comparisons and do not need exact numbers. In Problem Situation 1 of this collaboration, you found that the statement on the flier claimed trillions of children were gunned down in the present year. This was enough to know that the statement is not reasonable because that was more than the entire population of the United States (and in fact, the world). You did not need to have exact numbers.

MAKING CONNECTIONS ACROSS THE COURSE

This section of the Exercises will help you make connections between concepts across the course. In “Making Connections,” you will be using concepts, skills, and situations from previous assignments and previewing topics you will use in later assignments.

The first five collaborations (1.1–1.5) will help you develop some very important skills that you will use throughout the course. These include the following:

- Reading quantitative information
- Writing statements using quantitative information
- Understanding large numbers
    - Place value
    - Reading and writing large numbers in both words and digits
    - The size of numbers
    - Comparing the relative size of numbers
- Estimation

- Understanding, estimating, and calculating percentages
- Fundamentals of calculations
    - Order of operation
    - Different ways to write and perform calculations

By the end of this module, you should also understand some important points about this course:

- The meaning of quantitative reasoning
- Your responsibility for
    - creating and contributing to the classroom learning environment
    - being prepared for class
    - completing your work
    - planning and monitoring your own learning and course progress
- How to be an effective member of a work group
- Strategies for working on difficult problems

Exercises are also available online. The online versions contain the following sections:

- Making Connections to the Collaboration
- Developing Skills and Understanding
- Making Connections Across the Course
- Questions

The first three sections of online Exercises match your workbook and no score is given for these sections. The **Questions** section of online Exercises contains similar questions, but the numbers and figures vary from your workbook, so they are good for extra practice. Completing the **Questions** section will generate a score.

Tip: You can use the print version or the **Learning** sections online for practice. Then you can do the **Questions** section for additional practice and a score.

After Exercise 1.1 (survey)

The following questions will help you prepare for this course:

What are your goals for this class?

What strengths or supports do you have that will help you achieve your goals? Examples: You are hard working, caring, organized, and work well with others. You have time to work in the tutoring center, support from classmates, support at home, and confidence in yourself based on your past experiences either in school or in other aspects of your life.

What questions or concerns do you have for your instructor?

---

[2] Joel Best, Damned Lies and Statistics: Untangling Numbers from the Media, Politicians, and Activists (Berkeley: University of California Press, 2001).

[3] http://abcnews.go.com/WN/diabetes-rise-america-slow-growth-world-news-question/story?id=11945648

[4] http://www.diabetes.org/diabetes-basics/statistics/?loc=db-slabnav

[5] http://www.diabetes.org/diabetes-basics/statistics/?loc=db-slabnav

1.3: Seven Billion and Counting

INTRODUCTION

Discuss the following questions with your group members:

- Consider the following sentence, “In the Metamorphoses, Ovid (43 BCE to 17 CE) tells the story of King Lycaeon who was changed into a werewolf for displeasing the gods.”
- What do BCE and CE refer to?
- Do you know what the acronyms BCE and CE stand for?

Quick Check-in:

Please discuss the following questions with your group:

- What specific math concept from Unit 1.1 do you feel fairly confident about?
- Are there any concepts that you find challenging?

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- population growth can be measured in terms of doubling time.
- doubling times can be used to compare population growth during different periods.

By the end of this collaboration, you should be able to

- calculate quantities in the billions.
- use data to estimate a doubling time.
- compare and contrast population growth via population doubling times.

PROBLEM SITUATION: MEASURING POPULATION GROWTH

In this collaboration, you will look at doubling times to determine how the human population of the earth has changed over time. The **doubling time** of a population is the amount of time it takes a population to double in size. Calculating doubling time helps you understand how fast a population is growing. Comparing doubling times helps you understand how growth is changing over time.

Table 1 gives historical estimates of the human population. The population in 8,000 BCE was estimated to be 5 million people. Two thousand years later, in 6,000 BCE, the population had doubled to 10 million people. Therefore, the population doubling time for 8,000 BCE is about 2,000 years.

Table 1
Population Estimates Throughout History[6]

Year	World Population (lower bound, in millions)		Year	World Population (lower bound, in millions)
10,000 BCE	1		1850	1,262
9,000 BCE	3		1900	1,650
8,000 BCE	5		1950	2,519
7,000 BCE	7		1960	2,982
6,000 BCE	10		1970	3,692
5,000 BCE	15		1980	4,435
4,000 BCE	20		1985	4,831
3,000 BCE	25		1990	5,263

2,000 BCE	35		1995	5,674
1,000 BCE	50		2000	6,070
500 BCE	100		2005	6,454
1 CE	200		2010	6,922
1000 CE	310		2015	7,339
1800 CE	978		2020	7,753

(1) Using the information in Table 1, estimate the doubling times of Earth's human population. Start with the year given below and estimate how long it took for the population in that year to double. The first entry is done for you.

Let's take a moment to explore why the doubling time for the population in 8,000 BCE was 2,000 years. The population in 8,000 BCE was estimated to be 5 million people. Two thousand years later, in 6,000 BCE, the population had doubled to 10 million people. Therefore, we can say that the population doubling time for 8,000 BCE is about 2,000 years.

Think about (a) - (g) in the following table on your own for a minute before sharing your ideas with your group. Be prepared to explain how you arrived at your answers.

Year	Doubling Time
8,000 BCE	2,000 years
6,000 BCE	(a)
3,000 BCE	(b)
1 CE	(c)
1800 CE	(d)
1850 CE	(e)
1900 CE	(f)
1970 CE	(g)

(2) Discuss the results from Question 1 with your group. What do you notice about the doubling times? What does this tell you about how the human population has changed over time? Be sure to describe how you estimated.

A Writing Tip

One of the skills you will learn in this course is how to write quantitative information. A writing principle that you will use throughout the course is given below and followed by Question 3, which offers examples of how to use this principle.

Writing Principle: Use specific and complete information. Readers should understand what you are trying to say even if they have not read the question or writing prompt. This includes

- information about context, and
- quantitative information.

(3) With reference to the Writing Principle, which of the following statements best describes the change in doubling times before 1800 CE?

(i) The doubling times decreased.

(ii) Before 1800 CE, estimated population doubling times decreased from 2,000 to 1,000.

(iii) The doubling times decreased from 2,000 to 1,000.

(4) With reference to the Writing Principle, write a statement that describes the change in doubling times after 1800 CE.

FURTHER APPLICATIONS

(5) Imagine that you are explaining the relationship of million, billion, and trillion to someone else. Your explanation should follow the Writing Principle. Think about this on your own first before sharing with your group. Write an example below.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

---

[6] https://www.worldometers.info/world-population/world-population-by-year/

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1.3.1: Preparation 1.3

Your instructor expects you to be prepared for the next collaboration. This section explains what you need to know and be able to do to be prepared. You will be asked to rate how confident you are in your ability to do certain things. Be honest when you rate yourself. You will not be graded on the rating. If you do not feel confident, get help on the topic before the collaboration. Talk to your instructor about ways you can get help on campus.

Reread the information from Unit 1.1 that describes this course:

This course is called a **quantitative reasoning** course. This means you will learn to use and understand quantitative information. It will be different from many other math courses you have taken. You will learn and use mathematical skills connected to real-life situations. You will talk, read, and write about quantitative information.

The lessons in this course will focus on three themes.

- **Citizenship**: You will learn how to understand information about your society, government, and world that can impact many decisions you make.
- **Personal Finance**: You will study how to understand financial information and how to use it to make decisions in your life.
- **Medical Literacy**: You will learn how to understand information about health issues and medical treatments.

The purpose of Collaboration 1.1 was to introduce you to the idea of quantitative reasoning and give you a picture of what the course will be like. Do not worry if you did not understand all of the math concepts. You will have more time to work with these ideas throughout the course. Some other skills you will learn are

- how to understand and make sense of quantitative information.
- how to evaluate quantitative information (like you did in Problem Situation 1 of Collaboration 1.1 when you were asked if the statement was reasonable).
- how to use quantitative information to make decisions.

Self-Regulating Your Learning: An Introduction

One goal of this course is to increase your ability to learn efficiently and effectively. This means learning faster and learning smarter—what scientists call being a “self-regulated learner.” The following section explains what this means.

Self-regulating your learning means you *plan* your work, monitor your *work* and progress, and then *reflect* on your planning and strategies and what you could do to be more effective. These are the three phases of **Self-Regulated Learning (SRL)**. They are introduced below, and will be followed up on later in the course.

> **Plan**: Before doing a problem or assignment, self-regulated learners *plan*. They think about what they already know or do not know, decide what strategies to use to finish the problem, and plan how much time it will take. Research has shown that math experts often spend much more time planning how they will do a problem than they do actually completing it. Novices, the people who are just starting out, often do the opposite.
>
> **Work**: Self-regulated learners use effective strategies as they *work* to solve problems. They actively monitor what study strategies are working and make changes when they are not working. When they do not know which strategy would be better, they ask for help. Self-regulated learners also keep themselves focused while they are working and pay attention to their feelings to avoid getting frustrated.
>
> **Reflect**: Usually after an assignment or problem is done, self-regulated learners take time to reflect about what worked well and what did not. Based on that reflection, they think about how to change their approach in their future. The *reflect* phase helps self-regulated learners understand more about how they learn so they can become more efficient and more effective the next time. Reflecting is important for doing a better job the next time one plans for a new problem or assignment.

You can think of these three phases as a cycle. You incorporate what you learned during the reflect stage into your next plan phase, making you a more effective learner as you repeat this process many times. The most effective students get in the habit of working this way:

Diagram showing the Self-Regulated Learning (SRL) phases, indicating the following phases, each leading to the next: - Plan - Work - Reflect

For most people, self-regulating takes time, practice, and hard work, but it is always possible. People can improve even if, in the beginning, they did not self-regulate their learning very well. The more you practice something and the more you train your brain to think in certain ways, the easier it becomes.

Since thinking this way takes practice, you will have opportunities to practice some of these skills as you progress through this course. As you read through the course material, you will encounter activities that are designed to help you incorporate the *Plan*, *Work*, and *Reflect* phases in specific ways. Take the time to thoughtfully complete these exercises. The payoff will be worth it!

Self-Regulated Learning: Plan

Part of effectively planning for what could be new material is figuring out how much you already know. In Collaboration 1.2, you will need to be able to do the following things:

- Double values in contextual situations
- Identify place value to the trillions
- Read a table of numbers
- Add and subtract numbers

Upon completing Preparation 1.2, be sure to engage with the After Preparation 1.2 Survey (below) in which you can reflect on your confidence with the concepts to be engaged with in Collaboration 1.2.

After Preparation 1.2 (survey)

To effectively plan and use your time wisely, it helps to think about what you know and do not know. For each of the following, rate how confident you are that you can successfully do each task. Use the following descriptions to rate yourself:

5—I am extremely confident I can do this task.

4—I am somewhat confident I can do this task.

3—I am not sure how confident I am.

2—I am not very confident I can do this task.

1—I am definitely not confident I can do this task.

Before beginning Collaboration 1.2, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can…	**Rating from 1 to 5**
double values in contextual situations.	
identify place value to the trillions.	
read a table of numbers.	
add and subtract numbers.	

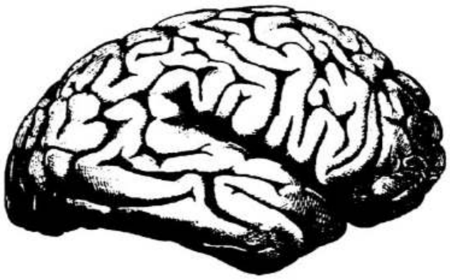
1.3.2: Growth Mindset Activity

YOU CAN GROW YOUR BRAIN

New Research Shows the Brain Can Be Developed Like a Muscle

Many people think of the brain as a mystery. We don't often think about what intelligence is or how it works. And when you do think about what intelligence is, you might think that a person is born either smart, average, or dumb—either a "math person" or not—and stays that way for life.

But new research shows that the brain is more like a muscle—it changes and gets stronger when you use it. Scientists have been able to show just how the brain grows and gets stronger when you learn.



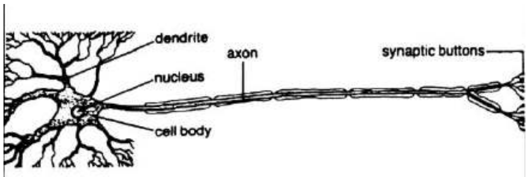
Everyone knows that when you lift weights, your muscles get bigger and you get stronger. A person who can't lift 20 pounds when they start exercising can get strong enough to lift 100 pounds after working out for a long time. That's because muscles become larger and stronger with exercise. And when you stop exercising, the muscles shrink and you get weaker. That's why people say "Use it or lose it!"

But most people don't know that when they practice and learn new things, parts of their brain change and get larger, a lot like the muscles do. This is true even for adults or older teenagers. So it's not true that some people are stuck being "not smart" or "not math people." You can improve your abilities a lot if you practice and use good strategies.

Inside the outside layer of the brain—called the cortex—are billions of tiny nerve cells, called neurons. The nerve cells have branches connecting them to other cells in a complicated network. Communication between these brain cells is what allows us to think and solve problems.

nerve fibers (white matter)

A Section of the Cerebrum



A Typical Nerve cell

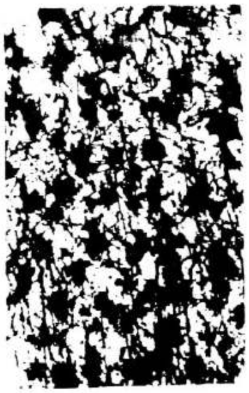
When you learn new things, you actually grow more of these tiny connections and they get stronger. The more you challenge your mind to learn, the more your brain cells grow. Then, things that you once found very hard or even impossible to do—like understanding history, doing algebra, or completing complicated paperwork—become easier. The result is a stronger, smarter brain.

How Do We Know That The Brain Can Grow Stronger?

Scientists started thinking the human brain could develop and change when they studied adult animals' brains. They found that animals who lived in a challenging environment, with other animals and toys to play with, were different from animals who lived alone in bare cages.

The animals who lived alone just ate and slept all the time, but the ones who lived with different toys and other animals were always active. They spent a lot of time figuring out how to use the toys and how to get along with other animals.

Nerves in brain of animal living in bare cage.



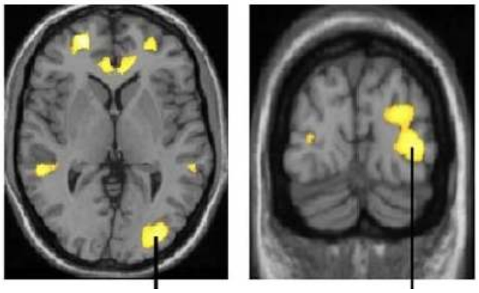
Brain of animal living with other animals and toys.

These animals had more connections between the nerve cells in their brains. The connections were bigger and stronger, too. In fact, their whole brains were about 10% heavier than the brains of the animals who lived alone without toys.

The adult animals who were exercising their brains by playing with toys and each other were also "smarter" –they were better at solving problems and learning new things.

Can Adults Grow Their Brains?

Scientists have recently shown that adults can grow the parts of their brains that control their abilities—like the ability to do math or even to juggle.



In Yellow: Parts of the brain that grew when adults learned to juggle

doi:10.1371/journal.pone.0002669.g001

In one study, scientists found a group of adults who were not jugglers. They taught half how to practice juggling in the right way. These people practiced for a long time and got much better at juggling. The other half didn't practice, and didn't get better.

Next, the scientists used a brain scanner to compare the brains of the two groups of people. They found that the people who learned how to juggle actually grew the parts of their brains that control juggling skills. Their brains had changed, so they actually had more ability.

This was surprising because these people said before the study that they couldn't juggle—just like some people say they're "not good at math" or "not smart at school." But when they kept trying and learned good strategies for practicing, they actually learned and grew their brains.

Two Parts of Your Brain That Need to Grow: The "Knowing" Part and the "Know How" Part

The brain doesn't just learn things; it also learns how to do things. Scientists have shown that people can grow two types of intelligence: the "knowing" type and the "know how" type. The "knowing" type of intelligence is what most people think about when they are learning new things in science or English or history. But you also need to learn skills that let you use your brain in a smarter way.

How do scientists know that you can grow the "know how" part of your brain? One example comes from the brains of taxi drivers in London, England. The streets of London are like a terrible maze. They are very hard to get around in. Brain scientists found that the longer the taxi drivers spent practicing driving in the streets of London, the more their brains grew. The "hippocampus" area of the brain—which controls memory for how to do everyday things—was actually larger! This means the more they practiced driving the confusing streets of London, the more the "know how" part of their brain grew. Although you might not realize it, we all grow the "know how" part of our brains whenever we learn to do new, hard things.

People often forget that we need to grow the "know how" part of our brains. If you study a long time but use a bad strategy, you may not learn—even if you try very hard. A few students study by doing the same set of easy problems and skipping the hard ones. Or they just re-read the textbook but don't try to challenge the ideas. After all, it's easier that way. But when it comes time to do the test, they don't do well because they didn't do things that grew their brains and taught them new things. When this happens, they may even say "I must not be smart at this."

But brain scientists have shown that a stronger brain doesn't just happen to you because you put in the time. You have to practice in the right way. If a weightlifter watched other people exercise all day long, he wouldn't get any stronger. You actually have to practice the right way—and usually that means the hard way—to get better at something. In fact, scientists have found that the brain grows more when you use new and different strategies. The brain grows less when you stick to things you already know.

How do you grow the "know how" part of your brain? You do it by seeking out help from others, like teachers or older students who do well. Or by going to the academic support center or getting tutoring. And then you practice successful strategies. At first it can be hard—sometimes people feel dumb for not knowing simple things like how to take notes, how to study, or even how to sign up for their classes. Or they feel not smart if they go to a tutor. But it's actually the opposite. Practicing better strategies makes your brain smarter.

The Truth About “Smart” and “Dumb”

People aren’t “smart” or “dumb” in school. At first, no one can read or solve equations. But with practice, they can learn to do it. And the more a person learns, the easier it gets to learn new things. This is because the “knowing” and “know how” brain muscles have gotten stronger.

This is true for everyone, even adults or college students. Dr. Wittenberg, the scientist from Wake Forest University who did the research on juggling, said “We used to think adults can’t form new brain connections, but now we know that isn’t true… The adult brain is like a muscle, and we need to exercise it.”

People who don’t know this can miss out on the chance to grow a stronger brain. It does take work to learn, just like lifting more weights or becoming a better juggler does. Sometimes it even hurts! But when you feel yourself get better and stronger, you realize that all the work is worth it.

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Questions: You Can Grow Your Brain

Questions

(1) Think about the article that you just read. What are all the reasons why scientists say that people's math ability can grow and get better with effort and practice?

Please summarize them briefly below.

(2) In the article, you learned three things:

- When you work hard and learn new things, your brain grows new connections and you get smarter.
- The more you challenge yourself, the smarter you will become.
- Smart kids are the kids who have practiced more—they have built up their reading and math "muscles."

Think about an example from your own life. What is something you weren’t good at first but then practiced using a good strategy and became really good at it? Write about it and explain how you became good at it.

(3) Not all math students know that the brain can get smarter, even though it may help them have success. And we want to get your help so that we can learn more about how to explain it to them. We're hoping you can explain—in your own words—that the brain gets smarter when people use good strategies and try hard.

Imagine a friend who is struggling in school. This friend used to do pretty well in school but now is having a hard time and is starting to feel dumb. Write a letter to your friend to encourage him or her—tell them about what you just learned about the brain and why they shouldn’t be discouraged. For example, you can tell them:

“Dear friend,

I realize that you feel discouraged in your math class because it is becoming a bit challenging for you, but after learning that our brain grows after practicing something hard I strongly encourage you to continue. I have learned that changing our studying strategies, asking for help, using resources, and practicing will help us improve what we do not understand. Life is a challenge and if everything was easy then the whole world would be boring. We have to grow our "know how" parts of our brain which requires a *lot of effort. Our brain can become stronger and smarter by practicing what we do not know. So don't give up on your math class; challenge yourself."*

Here are some other general ideas you can share with them:

1. How they can get smarter if they work hard and use a good strategy.
2. How they should work hard to build their reading and math muscles.
3. How they are not dumb, they just need to practice using a good strategy.
4. How they can ask the teacher or other students to help them learn better ways to study.
5. Any other tips you have for learning in school and getting smarter.

(Don't worry about writing a perfect final draft. We just want to know how you would say this to another student in your own words.)

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1.3.3: Exercise 1.3

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) The human population is quickly growing.

(ii) It is important to take time to make sense of large numbers because they occur in many important situations.

(iii) The world population in the year 1 CE was approximately 200 million people.

(iv) There is not much difference between a million and a billion.

DEVELOPING SKILLS AND UNDERSTANDING

(2) Which of the following statements is true?

(i) A trillion is 10^{10}.

(ii) A trillion is 10 billion.

(iii) A trillion is 100 billion.

(iv) A trillion is 1,000 billion.

(3) Refer back to the table in Question 1 of Collaboration 1.2. One of your classmates estimates the doubling time to be 500 years in 1000 CE. Does that answer seem reasonable? Meaning, does that number fit in with the numbers you see in your table? Write one or two sentences supporting your statement. Use the Writing Principle.

(4) There are many types of **investments**. Putting money into a savings account or an individual retirement account are two examples of investments.

Some types of investments earn **interest** based on a percentage rate. People often estimate the doubling time of investments to predict how much money the investment will be worth in the future. An investment that earns 5% interest will double in value about every 14 years. Use this information to complete the missing values in the table below for $7,500 invested at 5% interest. Fill in the missing values in the table.

Year	Value of Investment
2020	$7,500
(i)	$15,000
(ii)	$30,000
2062	(iii)

(5) Which of the following is the best estimate for the amount of time it would take the investment in Question 4 to reach a hundred thousand dollars?

(i) Less than 40 years

(ii) Between 40 and 60 years

(iii) Between 60 and 80 years

(iv) More than 80 years

MAKING CONNECTIONS ACROSS THE COURSE

Exercise 1.1 explained the purpose of each exercise section. Refer back to that information to answer the following questions.

(6) The first section of every assignment is called “Making Connections to the Collaboration.” The purpose of this section is to

(i) help you identify and remember the important mathematical ideas of the collaboration.

(ii) help you make a personal connection to the material in the collaboration.

(iii) help you review all the work you did in the collaboration.

(7) Which of the following are ways to use the “Developing Skills and Understanding” section to support your learning? There may be more than one correct answer.

(i) To earn points to improve your grade.

(ii) This section is not important unless you did not understand the work in the collaboration.

(iii) To assess how well you understand the new material from the collaboration.

(iv) To review information from previous collaborations.

(8) Why do you rate yourself in the ”After Preparation” surveys? There may be more than one correct answer.

(i) So you can show the instructor how much you know.

(ii) To honestly assess if you are ready for the next collaboration.

(iii) To get the best rating in the collaboration.

(iv) So you know what is expected in the next collaboration.

After Exercise 1.2 (survey)

Self-Regulated Learning: Reflect

It is not enough to complete the rating in the “After Preparation” surveys. First, you should use the rating to get ready for your next lesson. If your rating is a 3 or below, you should get help with the material before class. Remember, your instructor is going to assume that you are confident with the material and will not take class time to answer questions about it. If you need help, you should see your instructor or a tutor. You might also consider setting up a study group with classmates so you can help each other.

Second, you should use this rating to help you get better at self-assessment. Just like any other skill, being good at self-assessment takes practice. If you rate yourself as confident but then find that you are not prepared for class, you are not doing a good job of self-assessment. In this case, it is a good idea to talk to your instructor or a tutor about how you can do a better job of assessing yourself and preparing for class.

Self-regulating your learning includes looking back and reflecting on what you understand.

Use the following descriptions to rate yourself:

5—I am extremely confident I can do this task.

4—I am somewhat confident I can do this task.

3—I am not sure how confident I am.

2—I am not very confident I can do this task.

1—I am definitely not confident I can do this task.

Skill or Concept: I can …	**Rating from 1 to 5**
double values in contextual situations.	
identify place value to the trillions.	
read a table of numbers.	
add and subtract numbers.	

After Preparation 1.2, you rated your confidence in applying these same mathematical skills. (Please review your ratings from Preparation 1.2). After applying these skills in this assignment, has your confidence in your ability to successfully apply these skills changed? If so, why?

1.4: Percentages in Many Forms

INTRODUCTION

Recall from Preparation 1.3 the following:

- **Educated guess**: One type of estimation might be called an educated guess about something that has not been measured exactly. In Collaboration 1.2, you used estimations of the world population. This quantity cannot be measured exactly—it would be impossible to count how many people live on the earth at any given time. Scientists can use good data and mathematical techniques to estimate the population, but it will always be an estimate.
- **Convenient estimation**: Sometimes estimations are used when it is inconvenient or not worthwhile to make an exact count. Imagine that you need to know how much paint to buy to paint the baseboard trim in your house. (The baseboard trim is the piece of wood that follows along the bottom of the walls.) You need to know the length of the baseboard. You could measure the length of each wall to the nearest 1/8 inch and carefully subtract the width of halls and doors. It would be much quicker and just as effective to measure to the nearest foot or half foot. If you were cutting a piece of baseboard to go along the floor, however, you would want an exact measurement.
- **Estimated calculation**: This usually involves rounding numbers to make calculations simpler. This collaboration (1.3) focuses on estimating and calculating percentages. You will find in this course that percentages are used in many contexts. One of the most important skills you will develop is understanding and being comfortable working with percentages in a variety of situations.

Quick Check-in

Let’s take a minute to consider why mistakes, challenges, and struggles can be key ingredients of success.

Discuss the following questions in your group. Make a note of any interesting or illuminating points from your discussion.

- How do you feel when you make a mistake? Why?
- How do you think other people see you when you make a mistake?
- Have you ever discovered something new from making a mistake?
- Have you ever felt proud of making a mistake? Has a mistake ever made you think more deeply about a problem? (You can start by talking about a non-academic setting, and then talk about academics.)

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- estimation is a valuable skill.
- standard benchmarks can be used in estimation.
- there are many strategies for estimating.
- percentages are an important quantitative concept.

By the end of this collaboration, you should be able to

- use a few standard benchmarks to estimate percentages.
- estimate the percent one number is of another.
- estimate the percent of a number, including situations involving percentages less than one.
- calculate the percent one number is of another.
- calculate the percent of a number, including situations involving percentages less than one.

PROBLEM SITUATION 1: ESTIMATIONS WITH PERCENTAGES

This collaboration will help you improve your estimation skills. Strong estimation skills allow you to make quick calculations when it is inconvenient or unnecessary to calculate exact results. You can also use estimation to check the results of a calculation. If the answer you calculated is not close to your estimate, you know that you need to check your work in your calculation.

In this course, you will make a lot of estimations and be asked to explain the strategies you used to generate your estimations. There is not one best strategy. It is important that you develop strategies that make sense to you. An estimation strategy is wrong only if it is mathematically incorrect (like saying that 25% is 1/2). In the following section, you will practice your use of estimation strategies to answer questions and calculate percentages.

Building Your Estimation Skills

Use estimation to answer the following questions. Try to make your estimation calculations mentally. Write down your work if you need to, but do not use a calculator. First, complete the problem yourself. When you complete the problem, discuss your estimation strategy with your group. Your group should discuss at least two different strategies for each problem.

(1) Imagine you are shopping for a coat and find one that is on sale. The coat’s regular price is $87.99.

(a) What is your estimate of the sale price based on each of the following discounts? Try to do the estimate mentally (you may write some notes down, but avoid using a calculator) if you can. Do this on your own first before sharing in your group.

(i) 20% off

(ii) 25% off

(iii) 35% off

(iv) 70% off

(b) Explain to your group your sale price estimates for (i) - (iv) above and your estimation strategies. Make note of any strategies that are different from your own. **Note**: It is okay if people in your group use different strategies and for your estimates to be different. Describe two of the strategies you discussed in the spaces below.

Estimations help you make calculations quickly in daily situations. This problem shows how estimates of percentages can be used to make comparisons among groups of different sizes.

A law enforcement officer reviews the following data from two precincts.

Precinct	Number of Violent Incidents	Number of Violent Incidents Involving a Weapon
1	25	5
2	122	18

(2) She makes a quick estimate to answer the following question: “If a violent incident occurs, in which precinct is it more likely to involve a weapon?” Think about how to answer this question on your own for one minute before sharing your thoughts and estimation strategy with your group. Write one of your group's answers, and the strategy, below.

(3) You have a credit card that awards you a “cash back bonus.” This means that every time you use your credit card to make a purchase, you earn back a percentage of the money you spend. Your card gives you a bonus of 0.5%. If you charge $462 on your credit card, what would your bonus be?

PROBLEM SITUATION 2: FROM ESTIMATION TO EXACT CALCULATION

Being able to calculate with percentages is also very important. In the situation in Question 1, an estimate of the sale price would help you to decide whether to buy the coat. However, the store owner needs to make an exact calculation to know how much to charge. In Question 2, an estimate helps the officer get a sense of the situation, but if she is writing a report, she will want exact figures.

In the problems below, you will find **exact calculations** instead of estimates for the same situations you encountered in Questions 1–3. You may use a calculator. Show your work.

(4) You are shopping for a coat and find one that is on sale. The coat’s regular price is $87.99.

(a) What is the exact sale price based on each of the following discounts? Remember, we are trying to find the sale price here, so we need to figure out the discount first, then subtract to find the sale price. Do this on your own first before sharing in your group.

(i) 20% off

(ii) 25% off

(iii) 35% off

(iv) 70% off

(b) Explain to your group your sale price calculations for (i) - (iv) above. Enter the sale price for each of the discounts (i) - (iv). Round your calculations to 2 decimal places.

(5) For each precinct, what is the exact percentage of violent incidents that involve a weapon? Round your calculation to the nearest whole percent.

Precinct	Number of Violent Incidents	Number of Violent Incidents Involving a Weapon
1	25	5
2	122	18

Precinct 1:

Precinct 2:

(6) You have a credit card that awards you a “cash back bonus.” This means that every time you use your credit card to make a purchase, you earn back a percentage of the money you spend. Your card gives you a bonus of 0.5%. **Calculate** your exact bonus if you charge $462 on your credit card. Round to the nearest cent.

FURTHER APPLICATIONS

(7) Estimate an answer to each of the following. Explain your estimation strategy. Record an estimate and strategy for (i) - (iv).

(i) 62% of 87

(ii) 22% of 203

(iii) 37 is what percent of 125

(iv) 2 is what percent of 310

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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1.4.1: Preparation 1.4

In this course, you will talk about different types of **estimation**.

- **Educated guess**: One type of estimation might be called an educated guess about something that has not been measured exactly. In Unit 1.2, you used estimations of the world population. This quantity cannot be measured exactly—it would be impossible to count how many people live on the earth at any given time. Scientists can use good data and mathematical techniques to estimate the population, but it will always be an estimate.
- **Convenient estimation**: Sometimes estimations are used when it is inconvenient or not worthwhile to make an exact count. Imagine that you need to know how much paint to buy to paint the baseboard trim in your house. (The baseboard trim is the piece of wood that follows along the bottom of the walls.) You need to know the length of the baseboard. You could measure the length of each wall to the nearest 1/8 inch and carefully subtract the width of halls and doors. It would be much quicker and just as effective to measure to the nearest foot or half foot. If you were cutting a piece of baseboard to go along the floor, however, you would want an exact measurement.
- **Estimated calculation**: This usually involves rounding numbers to make calculations simpler. Lesson 1.3 focuses on estimating and calculating percentages. You will find in this course that percentages are used in many contexts. One of the most important skills you will develop is understanding and being comfortable working with percentages in a variety of situations.

The following questions will help you prepare for Collaboration 1.3 by reviewing some concepts about percentages.

(1) Each large square below represents 100%.

(a) Select the square with 35% shaded.

i) Image showing a square composed of ten columns and ten rows, and 100 smaller, equal sized squares. Seven rows are completely shaded. In the eighth row, only the first two squares are shaded. ii) Image showing a square composed of ten columns and ten rows, and 100 smaller, equal sized squares. Six rows are completely shaded. In the seventh row, only the first five squares are shaded.

iii) Image showing a square composed of ten columns and ten rows, and 100 smaller, equal sized squares. Three rows are completely shaded. In the fourth row, only the first five squares are shaded. iv) Image showing a square composed of ten columns and ten rows, and 100 smaller, equal sized squares. The first, third, fifth, seventh and ninth rows are completely shaded.

(b) (i) What percentage of the large square below is shaded? (ii) What percentage is not shaded?

Image showing a square composed of ten columns and ten rows, and 100 smaller, equal sized squares. Seven rows are completely shaded. In the eighth row, only the first two squares are shaded, the last eight squares are not shaded.

(c) (i) What percentage of the large square below is shaded? (ii) What percentage is not shaded?

Image showing a square composed of ten columns and ten rows, and 100 smaller, equal sized squares. The first square and half of the second square , in the first row, are shaded.

(d) (i) What percentage of the large square below is shaded? (ii) What percentage is not shaded?

Image showing a square composed of ten columns and ten rows, and 100 smaller, equal sized squares. Half of the first square , in the first row, is shaded.

(2) **Equivalent** means that numerical expressions are equal to each other. Complete the following table of equivalent percentages, fractions, and decimals. The first row in the table is done for you. Recall that the first numeral to the right of the decimal point is referred to as the "tenths" place while the second numeral to the right of the decimal point is the "hundredths" place. Round repeating numbers to two decimal places.

Simplified Fraction	Decimal	Percent
$\frac{1}{100}$	0.01	1%
$\frac{1}{10}$		
	0.2	
		25%
$\frac{1}{3}$	round to nearest hundredth	round to the nearest one percent
	0.5	
$\frac{2}{3}$	round to nearest hundredth	round to the nearest one percent

	0.75	

(3) There are similarities between writing the value of a coin as a dollar amount and writing decimals. To help you understand the similarities, write the value of each coin as a dollar amount. **Example**: Dime = $0.10

(a) Penny =

(b) Nickel =

(c) Quarter =

(d) Half dollar =

(e) Dollar =

(4) You probably know that 100 pennies equals one dollar. The value of 100 pennies written as a dollar amount is $1.00. Similarly, 100% written as a decimal is 1.00. Since you probably write dollar amounts all the time in your real life, it may help you to think about dollar amounts when you are converting percents to decimals.

For each of the following, convert between percent and decimal forms.

(a) Convert 45% to a decimal. (**Hint**: Think about how to write 45 cents as a dollar amount.)

(b) Convert 0.12 to a percent. (**Hint**: Think about how many cents you would have if you had 0.12) dollars.

(c) Convert 50% to a decimal. (**Hint**: Think about two quarters.)

Preparations are also available online. The online versions contain one or more **Learning** sections and a **Questions** section. The **Learning** sections of online Preparations match what is in this workbook and no score is given for these sections. The **Questions** section of online Preparations contains similar questions, but the numbers and figures vary from your workbook, so they are good for extra practice. Completing the **Questions** section will generate a score.

Tip: You can use the print version or the **Learning** sections online for practice. Then you can do the **Questions** section for additional practice and a score.

After Preparation 1.3 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 1.3, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can...	**Rating from 1 to 5**
understand the meaning of percent.	
convert between fractions, decimals, and percentages.	
round numbers to a given place value.	

1.4.2: Exercise 1.4

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main mathematical ideas of the collaboration?

(i) Percentages are used to calculate sale prices.

(ii) To calculate 35% of a number, multiply the number by 0.35.

(iii) Percentages are a ratio of a number out of 100. For example, 16% means 16 out of 100.

(iv) You should always calculate percentages exactly.

(2) Refer back to Collaboration 1.1 and 1.2. Which statement below is a good description of how the important *mathematical* ideas from these collaborations connect to this collaboration (1.3)?

(i) Many people worry that the world population is growing too rapidly. The rate of growth has been increasing throughout history.

(ii) Estimation is used in quantitative reasoning for many things, including estimating measurements, understanding large numbers, and making quick mental calculations.

(iii) Large numbers are hard to understand.

(iv) Calculating percentages is an important skill in quantitative reasoning because percentages are used in many situations.

DEVELOPING SKILLS AND UNDERSTANDING

Language of Percentages and Fractions

There are several important vocabulary words you should know and use.

- A **ratio** is a comparison of two numbers by division. You will see many different types of ratios in this course. In this lesson, you worked with a special type of ratio called a percentage. A percentage is a ratio because it is a number compared to 100.
- **Percentages** are a relationship between two values: the **comparison value** (the numerator of a fraction or percentage) and the **reference value** (the denominator of a fraction or percentage). A percentage is shown with a percent sign (%). This indicates that the rate is out of 100.

Example: 10 is 20% of 50.

10 is the comparison value.

50 is the reference value.

20% is the percentage rate; it can be written as a decimal by using the relationship to 100: $\frac{20}{100}$ = 0.2

- Fractions have two parts: $\frac{numerator}{denominator}$
- Every fraction can be written in *equivalent* forms. **Equivalent** means that the expressions mean the same thing (e.g., $\frac{2}{3}=\frac{4}{6}=\frac{6}{9}$). It is often useful to write the fraction in the form with the smallest numbers. This is called **simplified** or **reduced**. In the example, $\frac{2}{3}$ is in simplest form.

The Language of Estimation

Certain words or phrases are often used to indicate that a number is an estimate rather than an exact figure. Read the following statement: “Almost 30% of the patients had less pain.” The word *almost* indicates that the percentage was a little less than 30. Some words and phrases that are commonly used to signal estimates are shown below.

almost	about	approximately
more than	less than	close to
just over	just under	nearly

(3) At Gillway Community College, 43 out of 381 students earned honors. At Montessa Valley Community College, 17 out of 108 students earned honors.

(a) **Estimate** the percentage of students at Gillway CC who earned honors.

(b) **Estimate** the percentage of students at Montessa Valley CC who earned honors.

(c) Which school had a higher percentage of students who earned honors?

- (i) Gillway Community College
- (ii) Montessa Valley Community College

(d) Write a statement about the estimated percentage of students who earned honors at Gillway CC. (Use a word or phrase from the list above.) You may want to refer to the Writing Principle in the Module 1 Introduction.

(4) Select all of the options that are either exactly equal to the given ratio or a good estimate of the ratio. There may be more than one correct answer.

(a) 60%

- (i) 1 out of 6
- (ii) 1 out of 60
- (iii) 6 out of 10
- (iv) close to 2/3
- (v) 6 out of 100

(b) 8 out of 1,000

- (i) less than 1%
- (ii) about 8%
- (iii) about 1/8
- (iv) more than 8%
- (v) 0.8%

(c) $\frac{8}{100}$

- (i) less than 1%
- (ii) almost 10%
- (iii) 8 out of 10
- (iv) 2 out of 25
- (v) 80%

Estimate or Calculate

(5) Your bill at a restaurant is $23.17. You want to leave about 20% for a tip. **Estimate** how much you should leave for the tip.

(6) You are completing a tax form. The tax is 15.3% of $47,000. **Calculate** how much you have to pay.

(7) During an election for city council, you hear a candidate say that 68% of children in the city live in poverty. You know that your child's school has about 1,200 students. Based on the candidate's statement, **estimate** how many children in the school live in poverty.

(8) You are a teacher and are grading a test. A student earned 42 out 58 points. **Calculate** the student's grade as a percentage. (Round to the nearest whole percent)

(9) Some checking accounts pay a small amount of interest on the money in the account. In this case, **interest** is money that is paid to the account holder by the financial institution issuing the checking account. The interest is a percentage of the amount of money in the account. The percentage is called the **annual interest rate**. Compare the following two offers.

- Bank of Avalon pays 0.8% with no annual fee.
- Cypress Savings pays 1.5%, but charges a $10 annual fee.

Which bank would be the better offer if you have $1,000 in an account for one year?

(i) Bank of Avalon

(ii) Cypress Savings

MAKING CONNECTIONS ACROSS THE COURSE

(10) There are about 330 million people in the United States. A 2022 article claimed that the richest 1% of Americans controlled 32% of the nation's wealth.[7] About how many people is this?

(i) 300,000

(ii) 1,000,000

(iii) 3,000,000

(iv) 126,000,000

(11) The same article claims that the poorest 90% of Americans controlled only 30% of the nation's wealth. About how many people is this?

(i) 3,000,000

(ii) 21,000,000

(iii) 27,000,000

(iv) 300,000,000

(12) The nation's wealth in 2021 was about 143 trillion dollars. About how much money did the richest 1% of Americans control? (Recall that they controlled 32% of the nation's wealth.)

(i) $720,000,000

(ii) $5,000,000,000

(iiii) $56,000,000,000

(iv) $45,000,000,000,000

Working with Large Numbers

Large numbers such as those used in Question 14 can be hard to read when written out as a number. In Lesson 1.1, you used exponents to write powers of 10. For example, $100{,}000{,}000{,}000 = 10^{11}$. You can use this idea to write other large numbers in the form of a number multiplied by a power of 10. For example, the number 124,000 can be written as 1.24×10^5. You can check that this is true by multiplying this expression out:

$$1.24 \times 10^5$$

$$\rightarrow 1.24 \times 100{,}000$$

$$\rightarrow 124{,}000$$

There are other ways that 124,000 could be written as a power of 10. For example, 12.4×10^4 is also equal to 124,000.

(13) Which of these are equal to 135,230,000,000? There may be more than one correct answer.

(i) 1.3523×10^7

(ii) 13.523×10^{10}

(iii) 1.3523×10^{11}

(iv) One hundred thirty-five million, two hundred thirty thousand

(v) One hundred thirty-five billion, two hundred thirty million

(14) Write 68,000,000 in equivalent forms as instructed below.

(a) As a number times 10^5 : _______________ x 10^5

(b) As a number times 10^7 : _______________ x 10^7

(c) In words:

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[7] https://www.cnbc.com/2022/04/01/richest-one-percent-gained-trillions-in-wealth-2021.html

1.5: The Flexible Quantitative Thinker

INTRODUCTION

Recall the Consumer Expenditures Survey, which you first saw in Preparation 1.4 (chart pasted below).

- Are the expense categories realistic?
- Do the relative sizes of expenses match your personal experience?
- What do you spend the most (or the least) on?
- Where would you cut from this budget if you had to save some money?

Diagram showing the 2020 Consumer Expenditure Survey, indicating the following: Circular chart showing how average household income of 84352 is spent. Average annual income = 84352. Average annual expenditures = 61334. Food at home = 4942. Food away from home = 2375. Housing (Shelter) = 12603. Housing (utilities) = 4158. Housing (operations) = 4648. Apparel = 1434. Transport (vehicle purchase) = 4523. Transport (gas etc) = 5039. Transport (public) = 263. Healthcare = 5177. Entertainment = 2912. Personal care = 646. Reading and education = 1385. Insurance and pension = 7426. Other = 3983.

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- flexibility with calculations is an important quantitative skill.
- different methods of calculation are often possible and helpful.

By the end of this collaboration, you should be able to

- write a calculation in at least two different ways based on

- equivalent forms of fractions/decimals/percentages.
- relation of multiplication and division.

- the *Commutative Property* (knowing when the order of numbers can be reversed, such as 3 + 4 = 4 + 3, but 3 – 4 ≠ 4 – 3).

- order of operations.
- the *Distributive Property* (5(3 + 4) = 5 × 3 + 5 × 4).

PROBLEM SITUATION: PERFORMING CALCULATIONS IN MULTIPLE WAYS

Today’s collaboration asks you to brainstorm different ways to calculate the exact answer to a problem. This flexibility is important because different strategies are often useful in different situations. Sometimes changing the order of operations or grouping operations in other ways can make it easier to solve a problem. It is important to know when you can make changes such as these and still make correct calculations.

You will use information from the 2020 Consumer Expenditure Survey for today’s Collaboration. This survey provides detailed information about how American consumers spend money. It contains information about individuals and what they purchase. The survey also has information about a typical family’s income and what that family uses its money to buy. The survey refers to each family as an “average household.”

The 2020 Consumer Expenditure Survey studied how Americans spend their income. The survey found that the

- average household had an income before taxes of $84,352.
- average household spent about one-fourth (1/4) of its income on housing (either rent, if the family rented a home, or mortgage payments, if the family owned its home).
- average household’s annual expenditures were $61,334. An expenditure is something you spend money on.

You will use the above information to answer the following questions.

For the next question, complete (1a) on your own. When everyone in your group is ready, complete (1b) as a group.

(1) (a) Estimate how much the average household spent on housing. Try to do the estimate mentally (without writing it down or using a calculator) if you can.

(b) Explain your strategy for your estimation. Make note of the strategies that are different from your own. **Note**: It's okay if people in your group use different strategies and for your estimates to be different.

(c) Explain one of the strategies you discussed or might consider below.

(2) How would you write a mathematical expression to find how much the average household spends on housing? A **mathematical expression** is when numbers, symbols, and operators are grouped together to show the value of something. An example of a mathematical expression is 2 + 4. Try to find as many different mathematical expressions as possible. Write your answer below.

Look back at the graphic from the Introduction section.

(3) What fraction of ***expenditures*** went toward housing?

(4) What fraction of ***expenditures*** went toward **all other** expenses?

(5) How could you calculate the amount spent (as a dollar amount) on expenses other than housing? As a group, think of as many different ways as you can.

(6) Which of the methods from Question 5 makes the most sense to you? Explain why. (**Note**: Your answer does not have to agree with your group.)

(7) The Hudson family's housing expenses include rent and utilities (electricity, gas, water and sewer). They have the following average housing expenses per month.

Hudson Family Housing Expenses

Rent	$1,600
Electricity	$115
Gas	$140
Water and sewer	$92
Other household costs	$120
Total Housing Per Month	

(a) Calculate how much they spend on housing each month.

(b) Calculate how much the Hudson family spends on housing in one year.

(8) Look at your answer in Question 7(b). Does it seem like a reasonable amount to spend on housing in a year? Explain your answer.

FURTHER APPLICATIONS

(9) The **pie chart** below represents the amount of spending money a college student needs, or a *budget* of an average college student.[9] Write three questions about these data that require calculations or estimation.

You may refer to Questions 1–4 in the Collaboration for examples. Include the answers to your questions. (**Note**: You may need to make up amounts to represent a student budget, as that information is not given.)

Student budget pie chart. Food = 41%. Book and supplies = 7%. Travel = 4%. Entertainment = 15%. Clothing = 6%. Gas and insurance = 12%. Other = 15%.

Write your three questions below.

1)

2)

3)

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[9] http://college.lovetoknow.com/Amount_of_Spending_Money_a_College_Student_Needs

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1.5.1: Preparation 1.5

(1) Match the fractions to their equivalent percent form (rounded to the nearest percent).

(a) $\frac{1}{5}$	(i) 10%
(b) $\frac{2}{3}$	(ii) 20%
(c) $\frac{3}{4}$	(iii) 25%
(d) $\frac{1}{2}$	(iv) 33%
(e) $\frac{1}{3}$	(v) 50%
(f) $\frac{1}{4}$	(vi) 67%
(g) $\frac{1}{10}$	(vii) 75%

(2) In Exercise 1.1, you used a place-value chart for place values greater than one. Place value also extends to the right of the decimal to represent numbers less than one.

(a) Complete the missing names in the chart below.

Hundred thousands
Ten thousands
Thousands
Hundreds
Tens
Ones
Tenths
(i)
(ii)
(iii)
Hundred thousandths

(b) What is the name in words for the number 0.035?

(c) What is the name in words for the percent 0.02%?

(3) Round 54,927.2382 to the specified place value.

(a) Hundredths

(b) Hundreds

(c) Tens

(d) Thousands

(e) Tenths

(f) Thousandths

(4) Convert each fraction to a decimal. Round to the nearest thousandth.

(a) $\frac{2}{5}$

(b) $\frac{12}{7}$

(c) $\frac{25}{3}$

(d) $\frac{25}{6}$

(e) $\frac{254}{100}$

(f) $\frac{6}{25}$

(5) Collaboration 1.4 focuses on skills needed to be a “flexible quantitative thinker.” One of these skills is recognizing when calculations can be done in different ways. In the following question, you will be asked if two number expressions are equivalent. A number expression is a statement that uses numbers and operations, such as 3 + 4. An equivalent number expression is indicated by an equal sign (=), such as 3 + 4 = 7. Also note 3 + 4 is equivalent to 4 + 3 because it does not matter in what order you add the numbers.

Complete the table by indicating whether the first and second expressions are equivalent.

First Expression	Second Expression	Equivalent? Yes/No
5 × 7	7 × 5	(a)
8 – 4	4 – 8	(b)
10 ÷ 2	2 ÷ 10	(c)
20 ÷ 2	$20 \times \frac{1}{2}$	(d)

Multiplying Fractions

You can think of multiplying fractions in terms of area. Look at the squares below.

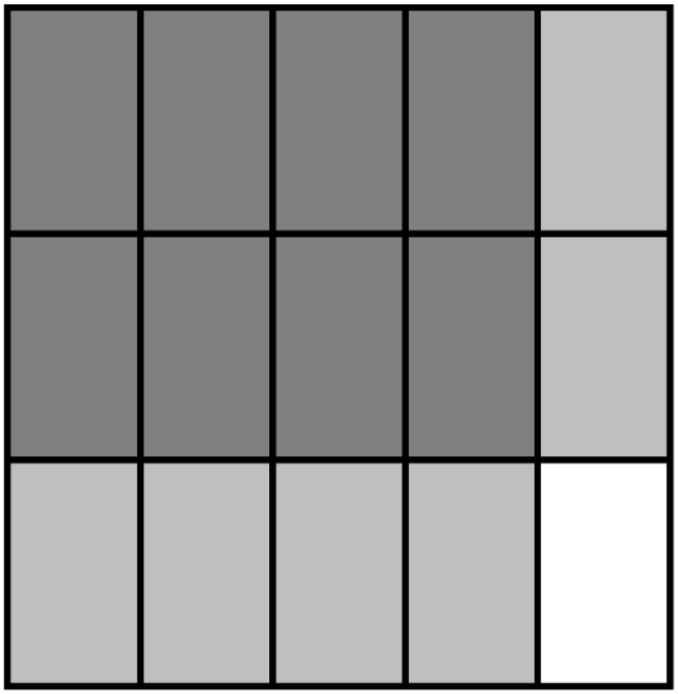
Image showing a square composed of three horizontal rectangles of equal size. The first two rectangles are shaded.

Image showing a square composed of five vertical rectangles of equal size. The first four rectangles are shaded.

The square on the left is divided into thirds. Two of the thirds are shaded gray, that is, the shaded part is $\frac{2}{3}$ of the whole square.

The square on the right is divided into fifths. Four of the fifths are shaded gray, that is, the shaded part is $\frac{4}{5}$ of the whole square.

Now, see what happens when we overlap the two squares.



The region that is shaded twice is darker than the rest. The dark gray area represents the product $\frac{2}{3} \times \frac{4}{5}$

Notice that the square is now divided into 15 regions (15 = 3 × 5), and the number of those regions that are dark gray is 8 (8 = 2 × 4). So 8 out of 15 pieces of the square are dark gray, or $\frac{8}{15}$.

Recall the rule for multiplying fractions: Multiply the numerators (2 × 4); multiply the denominators

(3 × 5); then simplify if possible. Therefore, $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$.

(6) Multiply the following fractions. Write your answer in simplified form.

(a) $\frac{3}{4} \times \frac{2}{5} =$

(b) $\frac{1}{8} \times \frac{3}{4} =$

(7) The Bureau of Labor Statistics conducts an annual survey to study how Americans spend their income. This is called the Consumer Expenditure Survey. (An expenditure is something you spend money on.)

The results from the 2020 Consumer Expenditure Survey[8] are summarized in the diagram below. You will use this information in Collaboration 1.4.

Diagram showing the 2020 Consumer Expenditure Survey, indicating the following: Circular chart showing how average household income of 84352 is spent. Average annual income = 84352. Average annual expenditures = 61334. Food at home = 4942. Food away from home = 2375. Housing (Shelter) = 12603. Housing (utilities) = 4158. Housing (operations) = 4648. Apparel = 1434. Transport (vehicle purchase) = 4523. Transport (gas etc) = 5039. Transport (public) = 263. Healthcare = 5177. Entertainment = 2912. Personal care = 646. Reading and education = 1385. Insurance and pension = 7426. Other = 3983.

Use the information in the diagram above to answer the following questions:

(a) What is the average annual household income?

(b) What percentage of a household's annual income is used to pay for housing? Round to the nearest hundredth of a percent.

(c) Which fraction would you use to approximate the percentage of a household's income that is used to pay for housing?

- (i) $\frac{1}{5}$
- (ii) $\frac{1}{4}$
- (iii) $\frac{1}{3}$

(iv) $\frac{2}{5}$

(v) $\frac{1}{2}$

After Preparation 1.4 (survey)

You should be able to do the following things for the next class. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 1.4, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
recognize common fraction benchmarks and equivalent percent forms.	
round a whole number to a given place value.	
perform calculations using a calculator.	
understand the relationship of multiplication and division (dividing by 3 is the same as multiplying by 1/3).	
convert between a fraction and its decimal form.	
multiply fractions.	

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[8] https://www.bls.gov/cex/csxreport.htm

1.5.2: Exercise 1.5

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main mathematical ideas of the collaboration?

(i) You can change a calculation in any way that you think will make it easier to do.

(ii) Calculations can often be performed in different ways based on mathematical rules.

(iii) Multiplying by 2/3 is the same as multiplying by 2 and then dividing by 3.

(iv) About 33% of the average family’s expenditures goes towards housing.

DEVELOPING SKILLS AND UNDERSTANDING

In Collaboration 1.4, you used several important mathematical rules and relationships to perform calculations in different ways. Those rules are summarized for you here so you can refer back to them. The rules also have formal names. You do not have to memorize these names for this course, but you may use them in other math classes. If you want more help with any of the rules, use their formal names to find resources on the Internet.

Mathematical rules are defined in terms of **variables**. The variables are symbols, usually letters that represent numbers. You use variables to show that the rule can apply to multiple numbers. This is called **generalizing** because it shows that a rule can be used in *general* and not just in specific cases. The mathematical rules below are shown using both variables and numbers.

While mathematical rules are very important, in this course the authors emphasize *reasoning* over memorizing rules. As you review the rules, try to make sense of the rules so that they will become a part of your thinking.

Commutative Property

The **Commutative Property** states that the order of addition and multiplication can be changed.

General Rule	Example
$a + b = b + a$	$8 + 3 = 3 + 8$
$a \times b = b \times a$	$5 \times 6 = 6 \times 5$

Note: It is important to remember that the Commutative Property does not apply to subtraction and division.

Order of Operations

The **order of operations** defines the order in which operations are performed.

General Rule	Example
1. Operations within grouping symbols, innermost first. Grouping symbols include: • Parentheses ( ) • Brackets [ ] • Fraction Bar – the line between the top number and the bottom number in a fraction as shown to the right	15 + [12 – (3 + 2)] – 2 × 32 ÷ 6 15 + [12 – (5)] – 2 × 32 ÷ 6 15 + [7] – 2 × 32 ÷ 6 $\frac{a}{b}$
2. Exponents	15 + [7] – 2 × 9 ÷ 6
3. Multiplication and division, left to right=	15 + [7] – 18 ÷ 6 15 + [7] − 3
4. Addition and subtraction, left to right	22 – 3 19

Distributive Property

The **Distributive Property** general rule can be written as:

$$a\,(b + c) = a \times b + a \times c$$

Note about subtraction: Subtraction is related to addition. The Distributive Property is shown using addition, but it also works with subtraction, such as $8\,(5 - 1) = 8 \times 5 - 8 \times 1$.

Notation: The operation of multiplication is shown in many ways. You have already seen the use of the multiplication symbol (×). Another way to indicate multiplication is a number or variable in front of parentheses with no other symbol. For example $6(2) = 6 \times 2$, or $a(b) = a \times b$. You will learn other symbols for multiplication later in the course.

The Distributive Property is easiest to understand by looking at examples.

Example: $4\,(3 + 1) = 4 \times 3 + 4 \times 1$.

To demonstrate that these two calculations are equivalent, each side is done separately.

Left side: Using order of operations, the operation inside the parentheses is done first.

$$4\,(3 + 1)$$

$$4\,(4)$$

$$16$$

Right side: Using the Distributive Property, the multiplication is *distributed* over the addition.

$$4\,(3 + 1)$$

$$4 \times 3 + 4 \times 1$$

Order of operations tells you to multiply first.

$$12 + 4$$

$$16$$

Division

Division is the same as multiplication by the **reciprocal**. You get the reciprocal of a number when you write the number as a fraction and reverse the numerator (the top number) and the denominator (bottom number).

General Rule	Example
$\frac{a}{b} = a \times \frac{1}{b}$	$15 \div 5 = 15 \times \frac{1}{5}$
$a \div \frac{b}{c} = a \times \frac{c}{b}$	$10 \div \frac{3}{5} = 10 \times \frac{5}{3}$

(2) In Collaboration 1.4, you saw that there was a relationship between multiplication and division. Refer back to this work to complete the following statement. (Fill in the blank with a fraction.)

65,596 ÷ 4 is the same as 65,596 × ____________.

(3) Using the concept from the previous question, fill in the blanks in the table below to create equivalent statements.

Multiplication	Division
$85 \times \frac{1}{5}$	85 ÷ ___
1.23 × ___	1.23 ÷ 7
1.23 × ___	$1.23 \div \frac{2}{3}$

(4) Which expressions are equivalent to $16 \times \frac{3}{4}$? There may be more than one correct answer.

(i) 16 × 3 ÷ 4

(ii) 16 ÷ 0.75

(iii) 3 × 16 ÷ 4

(iv) 3 ÷ 4 × 16

(v) 16 × 0.75

(vi) 16 ÷ 4 × 3

(vii) 0.75 × 16

(viii) 16 × 4 ÷ 3

(5) According to the Consumer Expenditure Survey, the average American household spent $7,316 on food in 2020. About one-third of that was spent on eating out at restaurants. Calculate one-third of $7,316.

Introduction to Spreadsheets

A spreadsheet is a computer program used to organize and analyze data. In the example here, Lisa has created a spreadsheet for her monthly budget. Data is entered into **cells**, like the boxes in a table. The cells are named by the letter of the column along the top and numbered rows down the side. (**Note**: The cell that contains the word *income* is labeled as A2, not 2A.)

Screen capture showing the Monthly Budget spreadsheet, indicating the following: A1 = Monthly Budget A2 = Income B2 = $1,677 A3 = Rent B3 = $750 A4 = Utilities B4 = $230 A5 = Food B5 = $160 A6 = Insurance B6 = $89 A7 = Gas for Car B7 = $80

Use this spreadsheet to answer Questions 6–8.

(6) What is in Cell B4?

(7) What does the number in Cell B4 represent in Lisa's budget?

(i) The money she plans to spend on rent each month.

(ii) The money she plans to spend on utilities each month.

(iii) The money she plans to spend on food each month.

(iv) The money she plans to spend on insurance each month.

(v) The money she plans to spend on gas for her car each month.

Formulas and Spreadsheets

Formulas are algebraic expressions that show important and non-changing relationships. They can be used to perform calculations in spreadsheets. The formulas use the cell name as a *variable* that represents the value in that cell. For example, in the spreadsheet above, the formula "=B3+B4" would result in the calculation $750 + $230, and "$980" would be displayed. Spreadsheets are a valuable tool because once a formula is written, the result will change when the values change. So if Lisa's rent increases, she can change the number in Cell B3 and the formula will calculate the new result(s) automatically.

(8) Lisa put the following formula in her spreadsheet:

=B2-B3-B4-B5-B6-B7

(a) Calculate the result of this formula.

(b) What does this value represent for Lisa?

(i) The amount of money she expects to lose each month.

(ii) The amount of money she expects to have left after paying bills each month.

(iii) The percentage of her income that she will be able to save each month.

(iv) The value has no meaning for Lisa.

(c) Which of the following expressions would give the same result as Lisa's formula?

(i) =B2-B3+B4+B5+B6+B7

(ii) =B2-(B3+B4+B5+B6+B7)

(iii) =(B3+B4+B5+B6+B7)-B2

(iv) =B3+B4+B5+B6+B7-B2

MAKING CONNECTIONS ACROSS THE COURSE

(9) Which of these expressions shows how to calculate 25% of 2,310? There may be more than one correct answer.

(i) $2{,}310 \div 4$

(ii) $2{,}310 \times 4$

(iii) $2{,}310 \div 25$

(iv) $2{,}310 \times 25$

(v) $2{,}310 \times 0.25$

(vi) $2{,}310 \div 0.25$

(vii) $\frac{1}{4} \times 2{,}310$

(viii) $\frac{1}{4} \div 2{,}310$

(ix) $0.25 \times 2{,}310$

(x) $0.25 \div 2{,}310$

(10) Which expression is the same as 20% of a billion? There may be more than one correct answer.

(i) $0.2 \times 1{,}000{,}000{,}000$

(ii) $0.2 \times 1{,}000{,}000$

(iii) $10^9 \div 5$

(iv) $10^9 \div 20$

(v) $10^6 \div 5$

(vi) One-fifth of 1,000 million

(vii) 20,000,000

(viii) $20 \div 100 \times 1{,}000{,}000{,}000$

Scientific Notation

In Exercise 1.3, you saw that a large number can be written as a number times a power of 10 in many different ways. For example, the number 124,000 can be written as 1.24×10^5 or 12.4×10^4. These different forms are all equivalent.

Scientific notation is a very specific way to write a large number as a power of 10. The purpose of scientific notation is to make it easier for people to use and communicate with large numbers. It would be confusing if two people working together on one project wrote the same number in two different ways. To avoid this, people decided that numbers in scientific notation would always be written in the same way: a number between 1 and 10 times a power of 10.

From the previous example:

- 1.24×10^5 is in scientific notation because 1.24 is a number between 1 and 10.
- 12.4×10^4 is not in scientific notation because 12.4 is larger than 10.

(11) Write 16,900,000 in scientific notation.

(12) Write 4,275,000,000 in scientific notation.

Self-Regulating Your Learning: The Plan Phase

At the start of this module, the authors briefly described what it means to be a “self-regulated learner.” As you already learned, being a self-regulated learner involves going through three phases when you are working on a problem or an assignment. The phases are:

1. Plan
2. Work
3. Reflect

Let’s look at what you should be doing during the ***plan*** phase. As you might imagine, the planning phase involves thinking about all the things you need to do to successfully complete a problem or assignment before you begin working on it. As was said previously, researchers who study how people learn found that experts often spend a lot more time planning how they are going to finish a task than they spend actually doing the task.

The planning phase involves several important aspects. The following are some that will be explored in this course:

- How much *confidence* you have that you can successfully complete the problem.
- The amount of *time and effort* you think it will take to understand and work on the problem.
- The *strategies* you might use to solve the problem.
- The *goals* you have as you try to work on the problem.

The authors will now describe each aspect in a little more detail. You will also continue to revisit them throughout the rest of the course.

Confidence: People who study how we learn have found that our beliefs regarding our ability to do a given task, like work on a particular math problem, often predicts how well we will actually do. Here is one way to think about it: If you really believe you can succeed at a problem, you are more likely to keep trying and keep working on that problem even if you get stuck. Because you invest more effort, you are more likely to be successful. On the other hand, if you look at a problem and immediately think “I cannot do this,” then when you do get stuck or confused, you might be more likely to give up and not be successful. Researchers call your beliefs about your abilities your **self-efficacy**.

In this course, you will be asked to rate your self-efficacy on certain problems. If you rate yourself low, then you might want to allow more time to do that problem, plan to go get help, or try being more patient than you might normally be. Thinking about your confidence can help you plan your time and effort when you work on a problem or task.

Time and Effort: Obviously, some problems or assignments take more time than others. Some assignments require more effort than others. It can be frustrating to jump into an assignment thinking you can finish it easily or quickly only to discover it is harder or takes more time than you thought it would. You can avoid some or all of that frustration if you have a realistic idea of how hard the assignment will be. Also, having a good idea of how much time and effort will be needed helps you manage your time. For example, you might need to allocate time to discuss the assignment with your instructor, classmates, or tutors. For these reasons, approximating the time and effort needed before starting work on an assignment is a good planning tool.

Strategies: When you start working on a problem or assignment, you often have to try several different strategies before you find an approach that will help you complete it successfully. Sometimes, it is the first strategy you think of, but often it is not. If you think about possible strategies before you begin working, you immediately have another one to try if your first one does not work. Self-regulated learners think about many different possible strategies before trying to solve a problem.

Goals: Education researchers have shown that students who have **learning goals** are more likely to succeed than students who have what are called **performance goals**. If you have *learning goals*, you are trying to understand what you are learning and trying to make connections between ideas and concepts. If you have *performance goals*, you care most about finishing an assignment to get points or have it done; you are not focused on understanding the material. Self-regulated learners try to have learning goals more than performance goals. This helps them stay focused and motivated to learn when the problems are challenging. Good planning means making an effort to change your thinking so you have learning goals as often as possible.

In future collaborations and assignments, you will have opportunities to practice the planning ideas presented here. Before then, start incorporating the planning phase whenever you start an assignment. If you do, you will be better prepared and more likely to succeed.

1.6: The Credit Crunch

INTRODUCTION

Discuss the following questions in your group:

- Credit cards are just one way that people have to pay for things. Which method(s) do you prefer to use (e.g. cash, debit, credit, checks). Why?
- Are the choices the same for people of different ages? Discuss.

Quick Check-In:

Please discuss the following questions with your group:

- What specific concept from Preparation 1.5 do you feel fairly confident about?
- Are there any concepts from Preparation 1.5 that you find challenging?

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- quantitative reasoning and math skills can be applied in various contexts.
- creditworthiness affects credit card interest rates and the amount paid by the consumer.
- reading quantitative information requires filtering out unimportant information.

By the end of this collaboration, you should be able to

- recognize common mathematical concepts used in different contexts.
- apply skills and concepts from previous lessons in new contexts.
- identify a complete response to a prompt that asks for connections between mathematical concepts and a context.
- write a formula in a spreadsheet.

PROBLEM SITUATION: UNDERSTANDING CREDIT CARDS

When you use a credit card, you can pay off the amount you charge each month. If you do not pay the full amount, you are borrowing money from the credit card company. This is called **credit card debt**. Many people in the United States are concerned about the amount of credit card debt both for individuals and for society in general. In this collaboration, you will use skills and ideas from previous collaborations to think about some issues related to credit cards. You may want to refer back to the previous material.

(1) The statement below came from a website that reported statistics for credit card debt in 2022:

- "American household debt hit a record $14.6 trillion in the spring of 2021, according to the Federal Reserve."[10]

Does the statement "US households had $14,600 billion in debt in 2021," express the same amount of debt? Justify your answer with an explanation.

Understanding the Fine Print

When you apply for a new credit card, you usually receive a *disclosure* in the mail along with the new credit card. The information below is excerpted from the disclosure that came with a new credit card. Use this information to answer Questions 2 and 3.

Annual Percentage Rate (APR) for Purchases	**0.00%** introductory APR for 6 months from the date of account opening. After that, your APR will be **10.99%** to **23.99%** based on your creditworthiness. This APR will vary with the market based on the Prime Rate.

A **credit score** is a number assigned to a person that indicates to lenders his or her ability to repay a loan. A high credit score indicates good credit. In the following questions, you will explore how your credit score can affect how much you have to pay in interest for the purchases you make with your credit card. Juanita and Brian both have a credit card with the terms in the above disclosure form. They have both had their credit cards for more than six months.

(2) (a) Juanita has good credit and gets the lowest interest rate possible for this card. She is not able to pay off her balance each month, so she pays interest. **Estimate** how much interest Juanita would pay in a year if she maintained an average balance of $5,000 each month on her card. Explain your estimation strategy.

(b) Brian has a very low credit score and has to pay the highest interest rate. He is not able to pay off his balance each month, so he pays interest. **Calculate** how much interest he would pay in a year if he maintained an average balance of $5,000 each month. Show your calculation and round to two decimal places.

(c) What are some things that might affect your credit score?

(3) The **Annual Percentage Rate (APR)** of a credit card is the interest rate for a full year. The **periodic rate** of a credit card is the interest rate for a month. You can find the periodic rate by dividing the APR by the number of months in a year.

(a) What is the periodic rate for Juanita's card? Round to two decimal places.

(b) Juanita has a balance of $1,082 on her January credit card statement. The statement says the minimum payment required in January is $100. Juanita makes only the minimum payment, so the rest of the balance carries over to February. Which of the following is the best estimate of how much interest she will owe in February?

Less than a dollar	$5–$10	$10–$20	More than $20

Credit Cards and Cash Advances

You will use the following information from the credit card disclosure for Question 4. A **cash advance** is when you use your credit card to get cash instead of using it to make a purchase.

Annual Percentage Rate (APR) for Purchases	After that, your APR will be **10.99%** to **23.99%** based on your creditworthiness. This APR will vary with the market based on the Prime Rate.
APR for Cash Advances	**28.99%**. This APR will vary with the market based on the Prime Rate.

(4) Discuss each statement (a) and (b) below and decide if it is reasonable.

(a) Jeff pays the highest interest rate for purchases he makes with his credit card. For a cash advance, he would pay $0.05 more for each dollar he receives in cash than he pays for purchases he makes with his credit card.

(b) The APR for cash advances is about two-and-a-half times as much as the lowest APR for purchases.

(5) Brian used a spreadsheet to record his credit card charges for a month.

Screen capture showing Brian's spreadsheet, indicating the following: A1 = Credit Card Charges for the Month A2 = Gas B2 = $56.08 A3 = Groceries B3 = $36.72 A4 = Restaurant B4 = $12.82 A5 = Movie tickets B5 = $16.00 A7 = Interest paid

Brian used the following expression to calculate his interest for that month's charges.

$$\frac{0.2399}{12}(B2+B3+B4+B5)$$

Which of the following statements best explains what the expression means in terms of the context?

(i) Brian added his individual charges. Then he divided 0.2399 by 12. Then he multiplied the two numbers.

(ii) Brian found the interest charge for the month by dividing 0.2399 by 12 and multiplying it by the sum of Column B.

(iii) Brian added the individual charges to get the total amount charged to the credit card. He found the periodic rate by dividing the APR by 12 months and multiplied the rate by the total charges. This gave the interest charge for the month.

FURTHER APPLICATIONS

(6) Refer to Question above. **Estimate** Brian's interest charge for that month. Explain your estimation strategy.

(7) You will encounter this expression in Exercise 1.5:

$$\frac{0.2399*B2+0.2399*B3+0.2399*B4+0.2399*B5}{12}$$

Why would you add the terms in the numerator before dividing by 12? (**Hint**: the numerator is: 0.2399 * B2 + 0.2399 * B3 + 0.2399 * B4 + 0.2399 * B5.)

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[10] https://www.debt.org/faqs/americans-in-debt/demographics/

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1.6.1: Preparation 1.6

(1) Which expressions are ways to write the number 5,200,000? There may be more than one correct answer.

(i) 52 million

(ii) 5.2 billion

(iii) 5.2 million

(iv) Five million, two hundred thousand

(v) Fifty-two million

(vi) 5,200

(2) **Estimate** the following percentages without using a calculator.

(a) 10.1% of 7,800

(b) 0.99% of 83,583

(c) 20% of 5,008,340

(3) Which expressions are equivalent to $\frac{32}{9}(53-31+48)$? There may be more than one correct answer.

(i) $\frac{32}{9}\cdot 53-\frac{32}{9}\cdot 31+\frac{32}{9}\cdot 48$

(ii) $\frac{32\cdot 53-32\cdot 31+32\cdot 48}{9}$

(iii) $\frac{32}{9}(53-31-48)$

(iv) $32\left(\frac{53}{9}-\frac{31}{9}+\frac{48}{9}\right)$

(v) $\frac{32}{9}\cdot 53-31+48$

An Introduction to Credit Cards

The following information will be used in Collaboration 1.5. You will be given some information about how much you have to pay to borrow money on a credit card. The credit card company charges interest on the amount you do not pay off each month. The amount that you do not pay off each month is called your balance. Interest is based on a percentage of the amount you have borrowed. The annual interest rate is the **APR** (also known as the **annual percentage rate**).

Creditworthiness is how likely it is that you will pay your bills on time. It is measured by a credit score that can range from 300 to 850. Someone with a high credit score has good credit and will get a lower interest rate than someone with a low credit score.

Credit cards are very complicated. Because it is important for people to understand how much credit cards charge, the U.S. government has a law called the Credit Card Accountability, Responsibility, and Disclosure Act of 2009, which requires companies to publish information about rates and fees in a standard format. This is called a *disclosure* or the *pricing and terms*.

The disclosure begins with a summary like the one shown below. Scan this form. (This means to read it quickly to get a general sense of the information without focusing on every detail.) This information will be discussed in more detail in Collaboration 1.5.

Interest Rates and Interest Charges

Annual Percentage Rate (APR) for Purchases	**0.00%** introductory APR for 6 months from the date of account opening. After that, your APR will be **10.99%** to **23.99%** based on your creditworthiness. This APR will vary with the market based on the Prime Rate.

CC BY NC

APR for Balance Transfers	**0.00%** introductory APR for 24 months after the first transaction posts to your account under this offer. After that, your APR will be **10.99%** to **23.99%** based on your creditworthiness. This APR will vary with the market based on the Prime Rate.
APR for Cash Advances	**28.99%**. This APR will vary with the market based on the Prime Rate.
Penalty APRs and When It Applies	Between up to **16.99%** and up to **26.99%** based on your creditworthiness and other factors. This APR will vary with the market based on the Prime Rate. This APR may be applied to new purchases and balance transfers on your account if you make a late payment. How long will the penalty APR apply?: If your APRs for new purchases and balance transfers are increased for a late payment, the Penalty APR will apply indefinitely.
How to Avoid Paying Interest on Purchases	Your due date is at least 25 days after the close of each billing period (at least 23 days for billing periods that begin in February). We will not charge you any interest on purchases if you pay your entire balance by the due date each month.
Minimum Interest Charge	If you are charged interest, the charge will be no less than $0.50.

After Preparation 1.5 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 1.5, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	Rating from 1 to 5
name and understand large numbers written in different forms.	
use benchmarks to estimate percentages including percentages less than 1%.	
use order of operations and the Distributive Property to write expressions in different forms.	

Self Regulated Learning (SRL): Plan

The next collaboration (1.5) will be a review of concepts in the course so far. After reading the information above about the plan phase, think about what you might do to be well prepared for your next session. Try to incorporate the ideas of confidence (self-efficacy), time and effort, strategies, and your goals. Write out your planning ideas.

1.6.2: Exercise 1.6

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) The number \$14.6 trillion can also be written as \$14,600 billion.

(ii) Credit cards are expensive to use if you do not pay off your balance each month. You pay more interest for cash advances than for the balance on purchases. Credit card debt is a problem in the United States.

(iii) A percentage is always a number greater than one.

(iv) Understanding numbers includes knowing how numbers compare in size, knowing what numbers represent in situations, and using estimation to answer questions about numbers.

DEVELOPING SKILLS AND UNDERSTANDING

(2) Below is the information from Question 5 in Collaboration 1.5, which you will use to answer (a) and (b) below.

Screen capture showing Brian's spreadsheet, indicating the following: A1 = Credit Card Charges for the Month A2 = Gas B2 = \$56.08 A3 = Groceries B3 = \$36.72 A4 = Restaurant B4 = \$12.82 A5 = Movie tickets B5 = \$16.00 A7 = Interest paid

Brian used the following expression to calculate his interest for that month's charges.

= (0.2399 / 12) * (B2 + B3 + B4 + B5)

(a) A student used a different expression (shown below) to calculate Brian's monthly interest. Choose the sentence that best explains what the expression means **in terms of the context** and the order in which the calculations were done. (**Note**: Spreadsheets use an asterisk (*) to indicate multiplication. For example, 3 * 4 means 3 × 4.)

= (0.2399 * B2 + 0.2399 * B3 + 0.2399 * B4 + 0.2399 * B5) / 12

(i) The student found the annual interest for each individual credit card charge, added to find the total annual interest, and then divided by 12 months to find the interest for 1 month.

(ii) The student distributed 0.2399 to the sum of the numbers and then divided by 12.

(iii) The student divided the annual interest rate by 12 to find the monthly interest rate, then multiplied each of the charges by that amount to find the monthly interest for each charge. The student then added the monthly interest for each charge together to find the total monthly interest.

(iv) The student multiplied each entry in the B column by 0.2399, added the results, and divided by 12 to find the final answer.

(b) Open a spreadsheet program like Excel or create a spreadsheet in Google Drive. Enter the information shown above into a new spreadsheet. Enter the formula given above:

= (0.2399 / 12) * (B2 + B3 + B4 + B5)

into cell B7. To do this, click on the cell. First type the equals sign (=). Note that a formula in a spreadsheet always starts with "=". Type the formula. Notice as you type, your formula appears in the cell and also in the formula bar above the spreadsheet cells. Press enter and record the result (what appears in the cell) when you are done.

(3) Below is information from Question 1 in Collaboration 1.5, which you will use to answer (a) and (b) below.

- "American household debt hit a record \$14.6 trillion in the spring of 2021, according to the Federal Reserve."

(a) Write the debt in standard form (written as a number like 374,000).

(b) What is the projected debt in scientific notation?

(i) 1.46×10^{12}

(ii) $14,600 \times 10^{12}$

(iii) 1.46×10^{13}

(iv) 14.6×10^{12}

(4) The Consumer Financial Protection Bureau has useful consumer information about credit cards. Go to the website www.consumerfinance.gov/credit-cards/knowbeforeyouowe. This is an interactive site in which you can get information by clicking on parts of the offer form. Use the information to answer the following questions.

(a) Which of the following can trigger a penalty annual percentage rate (APR)?

(i) You are late in paying your bill.

(ii) You pay your bill too early.

(iii) You do not use the credit card for six consecutive months.

(iv) You use your credit card every day.

(b) How can you avoid paying interest on purchases?

(i) Always make the minimum payment on time.

(ii) Avoid late fees.

(iii) Pay the entire balance by the due date.

(iv) Pay the minimum interest charge.

(5) A college student is talking to her family about a news story she read at InsideHigherEd.com.[11] It states:

> Faculty members and students at Connecticut community colleges are concerned about a recent vote by the state Board of Regents to raise tuition at the institutions by 5 percent.

The student attends Gateway Community College in New Haven, Connecticut and paid about $4,700 in tuition in 2021. Which of the following statements is a good *quantitative* description of how her tuition will change for 2022 based on the news story? There may be more than one correct answer.

(i) Her tuition is going to increase by almost a half.

(ii) Her tuition will go up by more than $200.

(iii) Her tuition is going to double!

(iv) Her tuition will be around $5,000.

MAKING CONNECTIONS ACROSS THE COURSE

Big budget movies are tracked by investors and consumers. The following table gives data on the eight movies with the largest budgets that had been released as of August 22, 2022.[12] The data includes an estimate of the U.S. gross earnings and gross earnings in other countries. Gross earnings is the amount of money that a movie takes in.

Release Date	Movie	Budget	U.S. Gross Earnings	Gross Earnings Outside U.S.
5/18/2011	Pirates of the Caribbean: On Stranger Tides	$379,000,000	$241,071,802	$804,642,000
4/22/2015	Avengers: Age of Ultron	$365,000,000	$459,005,868	$943,803,672
4/24/2019	Avengers: Endgame	$356,000,000	$858,373,000	$1,939,128,328
4/25/2018	Avengers: Infinity War	$325,000,000	$678,815,482	$1,369,544,272
5/25/2007	Pirates of the Caribbean: At World's End	$300,000,000	$309,420,425	$651,576,067
10/17/2017	Justice League	$300,000,000	$229,024,295	$428,902,692
5/23/2018	Solo: A Star Wars Story	$275,000,000	$213,767,512	$179,157,295
12/18/2019	Star Wars: The Rise of Skywalker	$275,000,000	$515,202,542	$558,946,737

(6) Write the number in words for the gross earnings outside the United States for Justice League.

(7) Which of the following calculations shows a correct method to estimate the net earnings for *Pirates of the Caribbean: At World's End*? Net earnings is the total amount the movie makes after expenses (the budget) are taken out. There may be more than one correct answer.

(i) ($310,000,000 + $650,000,000) – $300,000,000 = $660,000,000

(ii) $650,000,000 – ($310,000,000 + $300,000,000) = $40,000,000

(iii) The budget and the U.S. gross earnings are about the same and cancel each other out. The net earnings would be about the same as the gross earnings outside the United States, or about 650 million dollars.

(iv) The gross earnings are about $650 million plus $300 million, or $950 million. The expenses are about $300 million. So, the net earnings are about 650 million dollars.

(8) Refer to the data for *Star Wars: The Rise of Skywalker*.

(a) Estimate the net earnings.

(b) Write a numerical statement to explain a way to estimate the net earnings using the budget and total gross earnings for *Star Wars: The Rise of Skywalker*, as in Question 7(i). This should look like the following, which was calculated for *Pirates of the Caribbean: At World's End*: $960,000,000 - $300,000,000 = $660,000,000. Note that this example rounds figures to the nearest ten million so we can estimate easier. Use figures rounded to the nearest hundred million.

(9) The **return on investment** is the net earnings as a percentage of the budget. Which of the following statements best estimates the return on investment for *Star Wars: The Rise of Skywalker*? There may be more than one correct answer.

(i) The net earnings for *Star Wars: The Rise of Skywalker* were almost triple the investment.

(ii) The net earnings for *Star Wars: The Rise of Skywalker* were more than triple the investment.

(iii) The return on investment for *Star Wars: The Rise of Skywalker* was more than 300%.

(iv) The return on investment for *Star Wars: The Rise of Skywalker* was more than 200%.

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[11] https://www.insidehighered.com/news/2022/03/25/several-community-colleges-raise-tuition-fall

[12] http://en.Wikipedia.org/w/index.php?title=List_of_most_expensive_films, http://www.imdb.com/ ,and http://boxofficemojo.com/

1.7: Whose Footprint Is Bigger?

INTRODUCTION

Throughout this course, we will be often using the term “rational reasoning.” In mathematics, by rational reasoning, we mean making statements that involve division. There are many variables that are defined using division. Here are some examples:

Variable	Numerator	Denominator	Ratio
Velocity (speed)	89 miles	2 hours	$\frac{89\ miles}{2\ hours}$ = 44.5 miles per hour
Fuel consumption	150 miles	4 gallons	$\frac{150\ miles}{4\ gallons}$ = 37.5 miles per gallon
Population density	20,000 people	80 square miles	$\frac{20,000\ people}{80\ square\ miles}$ = 250 people per square mile
Cost per pound	$25	16 pounds	$\frac{\$25}{16\ pounds}$ = $1.5625 per pound
Download speed	858 MB	3 min	$\frac{858\ MB}{3\ min}$ = 286 MB per minute

As a group, discuss other examples of variables that are defined using division. Make sure to clearly state their units.

There are two other terms that you will become very familiar with throughout this course. Those terms are "integers" and "scientific notation."

- An integer is a positive or negative whole number.
- We use scientific notation to write very large or very small numbers in a format that is easier for us to work with them. You will see an expression where $a \times 10^n$ is being discussed in Problem Situation 1. "a" can be replaced with a number between 1 up to 10, and "n" can be replaced with an integer when we are writing a number in scientific notation.

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- the magnitude of large numbers is seen in place value and in scientific notation.
- proportions are one way to compare numbers of varying magnitudes.
- different comparisons may be needed to accurately compare two or more quantities.

By the end of this collaboration, you should be able to

- express numbers in scientific notation.
- estimate ratios of large numbers.
- calculate ratios of large numbers.
- use multiple computations to compare quantities.
- compare and rank numbers including those of different magnitudes (millions, billions).

PROBLEM SITUATION 1: COMPARING POPULATIONS

In this problem situation, you are going to compare the populations of China and the United States, and you will compare the population of each country to the population of the world. Since the population of each of these countries is very large, you will use scientific notation to write your answers. Recall that **scientific notation** is a special way of writing very large or very small numbers. A number in scientific notation is written in the form: $a \times 10^n$ where $1 \leq a < 10$; and n is an *integer*. An **integer**, commonly known as a whole number, is a number that can be written without a fractional component. Below are two examples of very large numbers written in scientific notation.

Examples:

- In 2020, the total population of Asia was approximately 4.7 billion people.[15] You can write this as 4,700,000,000 or you can use scientific notation to write it as 4.7×10^9 people.
- 28,930,000 can be written in scientific notation as 2.893×10^7.

(1) In late-2022, the population of the United States was 333,000,000.[16] In late-2022, Earth's population was about 7.9 billion. Write these numbers in scientific notation. How does writing the two quantities in scientific notation help you compare them?

(a) U.S. population in scientific notation:

(b) World population in scientific notation:

(2) What are some other ways you could compare the population of the United States to the population of the earth *besides* writing the two numbers in scientific notation? Think about forms of comparisons using both estimation and calculation.

(3) A **ratio** is an expression that compares how much there is of one thing to another. Ratios are often written as fractions. In mid-2022, the population of China was 1.412 billion.[17] Use a ratio to compare the population of China in 2022 to the population of Earth in 2022 (7.9 billion). Write your answer as a fraction and as a percent.

PROBLEM SITUATION 2: COMPARING WATER FOOTPRINTS

While there are countries with larger populations than that of the United States, the people who live in the U.S. consume (or use up) a larger percentage of some natural resources, such as water. This means that the United States has a large *water footprint*.

According to the website, waterfootprint.org, "People use lots of water for drinking, cooking, and washing, but even more for producing things such as food, paper, cotton clothes, etc. The water footprint is an indicator of water use that looks at both direct and indirect water use of a consumer or producer. The water footprint of an individual, community, or business is defined as the total volume of freshwater that is used to produce the goods and services consumed by the individual or community or produced by the business."

You will go on to look at the water footprint of three countries. You will examine each nation's water footprint as a whole and per capita (per person) to make some comparisons and to consider what this information might mean for the planet's sustainability—that is, Earth's ability to continue to support human life. While there is no one definition of **sustainability**, most agree that "sustainability is improving the quality of human life while living within the carrying capacity of supporting ecosystems." **Carrying capacity** refers to how many living plants, animals, and people Earth can support into the future.

The table below gives the population and water footprints of China, India, and the United States between 1996–2005 (the most recent data available).[18]

Country	Population (in thousands)	Total Water Footprint (in 109 cubic meters per year)
China	1,277,208	1,368
India	1,051,290	1,145
United States	288,958	821

(4) Compare China's population in 2022 (1.412 billion) with the population of the United States (333 million) using a ratio (with the U.S. population as the reference value). Discuss in your group. Write a sentence that interprets this ratio in the given context.

(5) Notice that in the table above, the countries are listed from highest to lowest population during that period. Using the "Total Water Footprint" data, rank the countries from highest (top) to lowest (bottom) according to their total water footprint. Rank the countries according to the highest total water footprint.

(6) (a) Calculate the water footprint **per capita** for China, India, and the United States. Round your answers to the nearest whole number. Include units.

(b) Rank the countries in order of water footprint per capita from highest (top) to lowest (bottom).

(7) How many times larger was the population of China than the population of the United States? Take a minute to think about this individually before sharing your ideas in your group. (**Note**: You calculated the ratio of China's population with the population of the United States in 2022 in Question 4. The table shows populations of these countries between 1996-2005 when the water footprint data was collected. Compare this to your answer in Question 4. Are the ratios the same or different? Why?

(8) Calculate how many times more water the average person in the United States used (from 1996-2005) compared to the average person in China. Round to two decimal places.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[15] https://www.prb.org/international/geography/asia/

[16] http://www.census.gov/popclock/

[17] https://www.prb.org/international/geography/china/

[18] http://www.waterfootprint.org/Reports/Report50-NationalWaterFootprints-Vol1.pdf

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1.7.1: Preparation 1.7

(1) Which of the following can be used to represent 5 out of 20? There may be more than one correct answer.

(i) $\frac{1}{2}$

(ii) 0.25

(iii) 25%

(iv) 0.25%

(v) 25

(2) What is another way to represent $\frac{3}{5}$? There may be more than one correct answer.

(i) 1.66

(ii) 166%

(iii) 60%

(iv) 0.6

(v) $\frac{6}{10}$

(vi) 3 out of 5

(3) Which of the following is the standard form of 1.23×10^{11}?

(i) 1,230,000,000,000

(ii) 123,000,000,000

(iii) 12,300,000,000

(iv) 1,230,000,000

Scientific Notation and Calculators

Scientific notation is useful because it is easy to make mistakes when working with numbers that contain a lot of zeros. You can use scientific notation with a calculator that has an exponent feature. Instructions for using exponents with two different types of calculators are given as follows.

Scientific Calculator: These calculators have a key that looks like one of the following:

y^x.png

or

x^y

The keystrokes for entering 10^8 are

keys 1 keys 0 x^y keys 8

Graphing Calculators and Computers: Graphing calculators and computers have a key that looks like an upside down “v” (see picture below). This is called a *caret* symbol. It is also used for exponents in computer programs, including spreadsheets.

keys ^

The keystrokes for entering 10^8 are

keys 1 keys 0 keys ^ keys 8

Calculators automatically display results to calculations in scientific notation when the numbers have too many digits to be displayed on the screen. The way that these are displayed varies slightly with different calculators. One common display is 2.34 E9, which represents 2.34×10^9.

(4) As of 2021, the world population is estimated to be about 7.9×10^9. (Recall the discussion of scientific notation from Exercise 1.4.) About 4.2% of the world's population lives in the United States. Approximately how many people live in the United States?

(i) 1.63×10^7

(ii) 3.32×10^7

(iii) 1.63×10^8

(iv) 3.32×10^8

(v) 1.63×10^9

(vi) 3.32×10^9

(5) Water usage varies greatly in different countries, from as little as 20 liters a day per capita (per person) in some third world countries, to several hundred liters a day in some industrialized countries. How much water would be needed for one day if every person in the world used 50 liters of water a day? Use the world population from the previous problem.

(a) Write your answer in scientific notation.

(b) Write your answer in standard notation (as a number).

(c) Write your answer in words.

Background Information for the Upcoming Collaboration

The following information will be used in Collaboration 1.6. You will again examine the situation of the Earth's population. Recall in Collaboration 1.2 that you looked at how the population has grown and is currently growing. Numerous scientists have conjectured about how long we can sustain ourselves. One of the most important natural resources that humans need for survival is water.

An article published by UNICEF in 2022 predicts that 700 million people worldwide could be displaced by intense water scarcity by 2030. The article also said that roughly 1 in 4 children worldwide will be living in areas of extremely high water stress by 2040.[13] You need clean water not just for drinking, but also for necessary tasks such as sanitation, growing food, and producing goods.

You will use a measure of water consumption called a "water footprint" that includes all of the ways that people use fresh water. According to WaterFootprint.org, "The water footprint measures the amount of water used to produce each of the goods and services we use."[14] Goods are physical products such as food, clothes, books, or cars. Services are types of work done by other people. Examples of services are having your hair cut, having a mechanic fix your car, or having someone provide daycare for your children. Fresh water is often used to make goods and to provide you with services.

To prepare for your class, make sure you understand this information and understand the term *water footprint*. For more information, you can do an Internet search for "definition of water footprint."

After Preparation 1.6 (survey)

Did you read and understand the information to be used in class?

You should be able to do the following things for the next class. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 1.6, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
calculate a quotient (one number divided by another).	
use a calculator to divide numbers.	
use scientific notation.	
convert between fractions, percentages, and decimals.	

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[13] https://www.unicef.org/wash/water-scarcity

[14] https://waterfootprint.org/en/water-footprint/what-is-water-footprint/

1.7.2: Exercise 1.7

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Ratios are a way to compare measurements in different situations.

(ii) A water footprint measures the amount of water used by a person or group. This includes water used for cooking, drinking, cleaning, and to produce all the goods and services used by the person.

(iii) The number 333,000,000 can be written in scientific notation as 3.33×10^8.

(iv) A nation's water footprint can be calculated by dividing the nation's population by the amount of water used in that nation.

DEVELOPING SKILLS AND UNDERSTANDING

(2) The website for the nonprofit organization Charity: Water discusses the need for clean water around the world.

(a) The website states that worldwide "Access to clean water and basic sanitation can save around 16,000 lives per week… 43% of those deaths are children under the age of five years old."[19] The following statements are all correct interpretations of this statistic. Which gives the most complete information?

(i) 6,880 children die every week from unsafe water and unhygienic living conditions.

(ii) 6,880 out of the 16,000 deaths that occur every week from unsafe water and unhygienic living conditions are of children under five years old.

(iii) 43% of deaths from unsafe water and unhygienic living conditions are of children under five years old.

(iv) 43 out of 100 deaths that occur every week from unhealthy living conditions are of children under five years old.

(b) On their website, the organization Action Against Hunger claims: "Nearly two billion people worldwide drink unsafe water."[20] If there are 7.9×10^9 people in the world, then approximately what percent of them live without clean drinking water? Round to the nearest tenth of a percent.

(3) According to data on the website for the Centers for Disease Control and Prevention (CDC), 41.9% of American adults were obese in 2020.[21] In this report, adults were considered to be 20 years of age or older.

(a) According to the U.S. Census Bureau, about 75% of Americans are adults (20 and over). Use the U.S. population estimate of 332 million people to calculate the number of American adults age 20 and older. Write your answer as a number in standard form.

(b) According to the CDC, about how many of the American adults (20 and over) were obese? Round to the nearest hundred thousand adults. Write your answer as a number in standard form.

(c) How do you write the number found in (b) in scientific notation?

(i) 0.104331×10^8 adults

(ii) 0.104331×10^9 adults

(iii) 1.04331×10^7 adults

(iv) 1.04331×10^8 adults

(v) 10.4331×10^7 adults

(4) From 2007 to 2020, there was a substantial increase in obesity rates in the United States. The following information comes from an earlier CDC report on obesity:[22]

> "Results from the 2007–2008 National Health and Nutrition Examination Survey (NHANES), using measured heights and weights, indicate that an estimated 34.2% of U.S. adults aged 20 years and over are overweight, 33.8% are obese, and 5.7% are extremely obese."

Use the information in the paragraph above to answer the following questions:

(a) In 2007-2008, approximately ________ out of 100 adults aged 20 years and older in the United States were obese. Round to the nearest adult.

(b) Use your answer from part (a) to calculate how many of the 220 million American adults aged 20 years and older in 2007-2008 were obese. Round to the nearest million adults. Write your answer as a number in standard form.

(c) About how many more American adults (20 and over) were obese in 2020 than in 2007-2008? (**Hint**: Use your answers from 3(b) and 4(b).)

(i) 2.93×10^6 adults

(ii) 2.93×10^7 adults

(iii) 4.4×10^6 adults

(iv) 4.4×10^8 adults

(v) 7.5×10^7 adults

MAKING CONNECTIONS ACROSS THE COURSE

(5) Tannika has a health insurance plan that will reimburse her for 60% of her family’s health care expenses after she pays a $2,000 deductible. A **deductible** is the amount a person pays (to a hospital, for example) before an insurance company will begin to pay for a percentage of the remaining expenses. Tannika has to pay the deductible and the percentage not covered by the insurance company. These are called “**out-of-pocket expenses**” because they are paid for by the person who owns the policy.

Tannika records the total of her health care expenses in the spreadsheet below.

Screen capture showing Tanika's spreadsheet, indicating the following: A1 = Service B1 = Cost D1 = Amount Paid by Insurance A2 = Doctors visits B2 = $1,235 D2 = Out-of-pocket expenses A3 = Prescriptions B3 = $1,044 A4 = Hospital bills B4 = $8,270 A5 = Lab tests B5 = $600

(a) Which of the following formulas could Tannika use in Cell E1 to calculate the amount paid by her insurance?

(i) = 0.6(B2 + B3 + B4 + B5) – 2000

(ii) = 0.6 * B2 + B3 + B4 + B5 – 2000

(iii) = 2000 − 0.6(B2 + B3 + B4 + B5)

(iv) = 0.6(B2 + B3 + B4 + B5 – 2000)

(b) Which of the following formulas could Tannika use in Cell E2 to calculate her out-of-pocket expenses? There may be more than one correct answer.

(i) = 0.4(B2 + B3 + B4 + B5 – 2000) + 2000

(ii) = 0.4 * B2 + B3 + B4 + B5 + 2000

(iii) = 2000 + 0.4(B2 + B3 + B4 + B5)

(iv) = (B2 + B3 + B4 + B5) – E1

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[19] http://www.charitywater.org/whywater/

[20] http://www.actionagainsthunger.org/impact/water-sanitation-hygiene

[21] https://www.cdc.gov/obesity/data/adult.html

[22] http://www.cdc.gov/nchs/data/hestat/obesity_adult_07_08/obesity_adult_07_08.htm

1.8: A Taxing Set of Problems

INTRODUCTION

Let's review some of the terms defined in Preparation 1.7. Please make sure you understand these terms as they will be referred to throughout this collaboration.

Revenue: This is the amount of money that a business receives when it sells a product or service.

Net profit: The net profit is the actual amount of money a business makes after expenses. Expenses are something a person or people spend money on. The expression for this is:

$$\text{Net profit} = \text{Revenue} - \text{Expenses}$$

Example: A restaurant might charge a customer \$10 for a meal, but it cost the restaurant \$4 for the food, \$1 for the waiter's paycheck, and \$1 for the building. You need to add up all the restaurant's expenses (\$4 + \$1 + \$1 = \$6). Then you subtract it from the total amount they make (\$10) to figure out the net profit. The expression would be 10 – (4 + 1 + 1) = 4. The restaurant's net profit is \$4.

Net loss: A net loss is similar to net profit, but a business has a net loss if the net profit is a negative number.

Example: If the restaurant's expenses were higher than the revenue, they would have a net loss. They could pay the waiter more (\$5) and the building could cost more (\$3). The expression would be 10 – (4 + 5 + 3) = −2. The restaurant's net loss is \$2.

FICA: This is an acronym for "Federal Insurance Contributions Act." FICA tax is the money that is taken out of workers' paychecks to pay older Americans their Social Security retirement and Medicare (Hospital Insurance) benefits. It is a mandatory payroll deduction.

Quick Check-in:

Please discuss the following questions with your group:

- What specific concept from the last unit do you feel fairly confident about?
- Are there any concepts from the last unit that you find challenging?

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- the order of operations is needed to communicate mathematical expressions to others.

By the end of this collaboration, you should be able to

- perform multistep calculations using information from a real-world source.
- rewrite multistep calculations as a single expression.
- explain the meaning of a calculation within a context.

PROBLEM SITUATION: FICA TAXES

The United States government requires that businesses pay into two national insurance programs—Social Security and Medicare—which help senior citizens pay for their personal **expenses** (things that a person or people spend money on) and health care. Businesses take money out of their employees' paychecks in order to pay the government. In other words, if you work for a business, your employer deducts Social Security and Medicare taxes from your paycheck. Also, the business pays a portion of the taxes for you. These taxes are called Federal Insurance Contributions Act (FICA) taxes.

People who own their own businesses are self-employed. They have to pay their own taxes. This is called the self-employment tax. In this problem situation, you will calculate how much three self-employed individuals owe in self-employment tax. You will do this by using a tax worksheet called the Schedule SE. This is an **Internal Revenue Service (IRS)** tax form. The IRS is the part of the government that collects taxes. It has many different types of forms that help individuals figure out how much they owe in taxes.

(1) Below is a copy of a blank Schedule SE.[23] Recall that you saw this in Preparation 1.7, which also included some important vocabulary words and examples that can help you answer this question.

Marianne Lopez has a part-time job as a math tutor. She offers tutoring services to students both in person and online. Last year, she earned $11,385 in **revenue**, or the amount of money she received from selling her service. Her expenses totaled $3,862. You are going to calculate how much self-employment tax Marianne owes. That is, you are going to determine what value should be put into the cell on line 5. Take a moment to do this on your own before sharing your calculations in your group.

Note: Marianne does not have a farm, and she does not receive social security retirement or disability benefits. She is also not a minister or a member of a religious order and does not have other income. You do not need to know which schedule or form Marianne's **net profit** (the actual amount of money made after expenses) comes from. These cells have been zeroed out to avoid confusion. You also do not need to calculate line 13.

Screenshot of Self-Employment tax form. At the top, it notes: If your only income subject to self-employment tax is church employee income, see instructions for how to report your income and the definition of church employee income. Part A: If you are a minister, member of a religious order, or Christian science practitioner and you filed Form 4361, but you had $400 or more other net earnings from self-employment, check here and continue with Part I. Skip lines 1a and 1b if you use the farm optional method in part II. See instructions. 1a: Net farm profit or (loss) from Schedule F, line 34, and farm partnerships, Schedule K-1 (Form 1065), box 14, code A. b: If you received social security retirement or disability benefits. enter the mount of conservation Reserve program payment included on Schedule F, line 4b, or listed on Schedule K-1 (Form 1065), box 20, code AH. Skip line 2 if you use the non farm optional method in Part II. See instructions. 2: Net profit or (loss) from Schedule C, line 31, and Schedule K (Form 1065), box 14, code A (other than farming). See instructions for other income to report or if you are a minister or member of a religious order. 3: Combine lines 1a a, 1b, and 2. 4a: If line 3 is more than zero, multiply line 3 by 92.35% (0.9235). Otherwise, enter amount from line 3. Note: If line 4a is less than $400 due to Conservation Reserve Program payments on line 1b, see instructions. b: If you elect one or both of the optional methods, enter the total of line 15 and 17 here. c: Combine lines 4a and 4b. If less than $400, stop. You don;t ow self-employment tax. Exception: if less than $400 and you had church employee income, enter 0 and continue. 5a: Enter your church employee income from Form W-2. See instructions. 5b: Multiply line 5a by 92.35% (0.9235). If less than $100, enter 0. 6: Add line 4c and 4b. 7: Max amount of combined wages and self-employment earnings subject to social security tax or the 6.2% portion of the 7.56% railroad retirement (tier 1) for 2021. 8a: Total social security wages and tips (total of boxes 3 and 7 on Form W-2) and railroad retirement (tier 1) compensation. If $142,800 or more, skip lines 8b through 10, and go to line 11. b: Unreported tips subject to social security tax from Form 4137, line 10. c: Wages subject to social security tax from Form 89191, line 10. d: Add lines 8a, 8b, and 8c. 9: Subtract line 8d from line 7. If zero or less, enter 0 here and on line 10 and go to line 11.10: Multiply the smaller number of line 6 or line 9 by 12.4%. 11: multiply line 6 by 2.9%. 12: Self-employment tax. Add lines 10 and 11. Enter here and Schedule 2 (Form 1040), line 4. 13: Deduction for one-half of self-employment tax. Multiply line 12 by 50% (0.50). Enter here and on Schedule 1 (Form 1040), line 15.

(a) What value should be put into the cell on line 2?

(b) What value should be put into the cell on line 3?

(c) What value should be put into the cell on line 4a? Round your answer to two decimal places.

(d) What value should be put into the cell on line 6? Write answer to two decimal places.

(e) What value should be put into the cell on line 9?

(f) What value should be put into the cell on line 10? Round your answer to two decimal places.

(g) What value should be put into the cell on line 11? Round your answer to two decimal places.

(h) What value should be put into the cell on line 12? Round your answer to two decimal places.

(2) Leigh Olson started a small bakery last year. She has no income to report on line 1a or line 1b. She earned $1,050 in revenue selling cookies and cupcakes. Her expenses totaled $630. In your group, work through the questions below to figure out how much self-employment tax Leigh Olson owes.

Note: Leigh does not have a farm, and she does not receive social security retirement or disability benefits. She is also not a minister or a member of a religious order. You do not need to know which schedule or form Leigh's net profit comes from. These cells have been zeroed out to avoid confusion. You also do not need to calculate line 13.

Screenshot of Self-Employment tax form. At the top, it notes: If your only income subject to self-employment tax is church employee income, see instructions for how to report your income and the definition of church employee income. Part A: If you are a minister, member of a religious order, or Christian science practitioner and you filed Form 4361, but you had $400 or more other net earnings from self-employment, check here and continue with Part I. Skip lines 1a and 1b if you use the farm optional method in part II. See instructions. 1a: Net farm profit or (loss) from Schedule F, line 34, and farm partnerships, Schedule K-1 (Form 1065), box 14, code A. b: If you received social security retirement or disability benefits. enter the mount of conservation Reserve program payment included on Schedule F, line 4b, or listed on Schedule K-1 (Form 1065), box 20, code AH. Skip line 2 if you use the non farm optional method in Part II. See instructions. 2: Net profit or (loss) from Schedule C, line 31, and Schedule K (Form 1065), box 14, code A (other than farming). See instructions for other income to report or if you are a minister or member of a religious order. 3: Combine lines 1a a, 1b, and 2. 4a: If line 3 is more than zero, multiply line 3 by 92.35% (0.9235). Otherwise, enter amount from line 3. Note: If line 4a is less than $400 due to Conservation Reserve Program payments on line 1b, see instructions. b: If you elect one or both of the optional methods, enter the total of line 15 and 17 here. c: Combine lines 4a and 4b. If less than $400, stop. You don;t ow self-employment tax. Exception: if less than $400 and you had church employee income, enter 0 and continue. 5a: Enter your church employee income from Form W-2. See instructions. 5b: Multiply line 5a by 92.35% (0.9235). If less than $100, enter 0. 6: Add line 4c and 4b. 7: Max amount of combined wages and self-employment earnings subject to social security tax or the 6.2% portion of the 7.56% railroad retirement (tier 1) for 2021. 8a: Total social security wages and tips (total of boxes 3 and 7 on Form W-2) and railroad retirement (tier 1) compensation. If $142,800 or more, skip lines 8b through 10, and go to line 11. b: Unreported tips subject to social security tax from Form 4137, line 10. c: Wages subject to social security tax from Form 89191, line 10. d: Add lines 8a, 8b, and 8c. 9: Subtract line 8d from line 7. If zero or less, enter 0 here and on line 10 and go to line 11.10: Multiply the smaller number of line 6 or line 9 by 12.4%. 11: multiply line 6 by 2.9%. 12: Self-employment tax. Add lines 10 and 11. Enter here and Schedule 2 (Form 1040), line 4. 13: Deduction for one-half of self-employment tax. Multiply line 12 by 50% (0.50). Enter here and on Schedule 1 (Form 1040), line 15.

(a) What value should be put into the cell on line 2?

(b) What value should be put into the cell on line 3?

(c) What value should be put into the cell on line 4a? Round your answer to two decimal places.

(d) How much tax does Leigh owe? Be careful here!

(3) In Question 1, you learned about Marianne Lopez. You used the Schedule SE form to calculate how much self-employment tax Marianne owes. Now, write the calculation you completed to answer Question 1 as a single expression. An example of a single expression is shown below.

Example: The following calculations, 19 - 4 = 15

5.75*15 = 86.25

0.3*86.25 = 25.875

Can be written as a single expression: 0.3(5.75)(19 - 4) = 25.875

(4) Look back at your single expression from Question 3. Imagine you have to explain the expression to Marianne to help her understand how you calculated the amount of self-employment tax that she owes. Answer these questions about the expression:

(a) What does the operation $11,385 − $3,862 represent for Marianne?

(b) What does the operation of multiplying by 0.9235 represent for Marianne?

(c) What does the operation of multiplying by (0.124+0.029) represent for Marianne?

Self-Employment and the Medicare Tax

The Affordable Care Act was passed by Congress and signed into law in 2010. The law reforms the health care system in the United States by protecting consumers from losing their health insurance, bringing down insurance costs, expanding access to health care services, strengthening Medicare with added benefits, and improving the quality of health care for Medicare patients. **Medicare** is a federal (government-run) health insurance program for seniors, people who are 65 and older, in the United States. All taxpayers pay into the Medicare program.

In order to raise revenue to support the expansion of care under the Affordable Care Act, the U.S. government created an additional Medicare tax for individuals with higher incomes. People who are self-employed use Part II of Form 8959 to determine how much additional Medicare tax they have to pay (if any).[24] Only individuals and couples with incomes over certain amounts have to pay this tax. These amounts appear in line 9 of Form 8959.

Screen capture showing Part II - Additional Medicare Tax on Self-Employment Income, indicating the following values: 8 = $215,500 9 = $200,000 10 = $0 11 = $200,000

(5) Andie Henson is a single, self-employed psychiatrist. Her self-employment income from Schedule SE (Form 1040) Section A was $215,500. She did not earn wages from another job during this year. This information has already been entered in lines 8–11 of Form 8959. Use mental math to find the exact value of line 12 and add it to the form. Then, *estimate* the value of line 13 and record your estimate in the space below. Do not use a pencil and paper or a calculator. Briefly explain your estimation strategy.

(6) Now, *calculate* the Additional Medicare Tax that Andie owes (line 13). How does the calculation compare to your estimate in Question 5?

FURTHER APPLICATIONS

(7) Workers in the United States pay several types of taxes on income. This collaboration discussed the FICA tax. You also have to pay federal income tax. Your federal income tax rate is based on the amount of money you make. Income is broken into levels called tax brackets. The table below shows the tax brackets for 2021.[25]

Taxable Income	Tax
$0–$9,950	10% of taxable income
$9,951–$40,525	$995.00 plus 12% of excess over $9,950
$40,526–$86,375	$4,664.00 plus 22% of excess over $40,525
$86,376–$164,925	$14,751.00 plus 24% of excess over $86,375
$164,926–$209,425	$33,603.00 plus 32% of excess over $164,925
$209,426–$523,600	$47,843.00 plus 35% of excess over $209,425
$523,601 and up	$157,804.25 plus 37% of excess over $523,600

(a) Write a single expression that you can use to calculate the income tax for a person with taxable income of $63,500.

(b) The $4,664.00 in the third line of the table is based on information from the previous two lines. Explain how the $4,664.00 is calculated. Hint: Start by thinking about where the $995.00 in line 2 came from.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[23] https://www.irs.gov/pub/irs-prior/f1040sse--2021.pdf

[24] https://www.irs.gov/forms-pubs/about-form-8959

[25] https://taxfoundation.org/2021-tax-brackets/

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1.8.1: Preparation 1.8

(1) Which of the following calculations solves this problem according to the order of operations?

$$8 + 6 \times (3 + 6) \div 2 - 4$$

(i) 8 + 6 × (3 + 6) ÷ 2 – 4
→ 8 + 6 × (3 + 6) ÷ 2
→ 8 + 6 × (3 + 3)
→ 8 + (18 + 3)
→ 8 + 21
→ 29

(ii) 8 + 6 × (3 + 6) ÷ 2 – 4
→ 8 + 6 × 9 ÷ 2 – 4
→ 8 + 54 ÷ 2 – 4
→ 8 + 27 – 4
→ 35 – 4
→ 31

(iii) 8 + 6 × (3 + 6) ÷ 2 – 4
→ 8 + (18 + 6) ÷ 2 – 4
→ 8 + (18 + 3) – 4
→ 8 + 21 – 4
→ 29 – 4
→ 25

(iv) 8 + 6 × (3 + 6) ÷ 2 – 4
→ 8 + 6 × (3 + 3) – 4
→ 8 + (18 + 3) – 4
→ 8 + 21 – 4
→ 29 – 4
→ 25

(v) 8 + 6 × (3 + 6) ÷ 2 – 4
→ 14 × (9) ÷ 2 – 4
→ 14 × (9) ÷ 2
→ (126) ÷ 2
→ (63)
→ 63

(2) Miguel has a coupon for 20% off any purchase in a furniture store. He decides to purchase a desk originally priced at $80. Excluding tax, how much does Miguel save on his purchase?

(i) $2

(ii) $4

(iii) $16

(iv) $40

(3) Sylvia is charged 8% tax for her $2 cheeseburger. How much does Sylvia owe the cashier?

(i) $1.84

(ii) $2.08

(iii) $2.16

(iv) $2.80

Profit, Loss, and Revenue

The following terms will be used in the next class. Make sure you understand what they mean.

- **Revenue**: This is the amount of money that a business receives when it sells a product or service.
- **Net profit**: The net profit is the actual amount of money a business makes after **expenses**. Expenses are something a person or people spend money on. The expression for this is:

$$\text{Net profit} = \text{Revenue} - \text{Expenses}$$

Example: A restaurant might charge a customer $10 for a meal, but it cost the restaurant $4 for the food, $1 for the waiter’s paycheck, and $1 for the building. You need to add up all the restaurant’s expenses ($4 + $1 + $1 = $6). Then you subtract it from the total amount they make ($10) to figure out the net profit. The expression would be 10 – (4 + 1 + 1) = 4. The restaurant’s *net profit* is $4.

- **Net loss**: A net loss is similar to net profit, but a business has a net loss if the net profit is a negative number.

Example: If the restaurant’s expenses were higher than the revenue, they would have a net loss. They could pay the waiter more ($5) and the building could cost more ($3). The expression would be 10 – (4 + 5 + 3) = −2. The restaurant’s *net loss* is $2.

In Collaboration 1.7, you are going to be working through a tax form like the one shown below. This is a Schedule SE tax form. People who are self employed use this form to calculate how much tax they owe. Please take a few minutes to read it in preparation for Collaboration 1.7.

Screenshot of Self-Employment tax form. At the top, it notes: If your only income subject to self-employment tax is church employee income, see instructions for how to report your income and the definition of church employee income. Part A: If you are a minister, member of a religious order, or Christian science practitioner and you filed Form 4361, but you had $400 or more other net earnings from self-employment, check here and continue with Part I. Skip lines 1a and 1b if you use the farm optional method in part II. See instructions. 1a: Net farm profit or (loss) from Schedule F, line 34, and farm partnerships, Schedule K-1 (Form 1065), box 14, code A. b: If you received social security retirement or disability benefits. enter the mount of conservation Reserve program payment included on Schedule F, line 4b, or listed on Schedule K-1 (Form 1065), box 20, code AH. Skip line 2 if you use the non farm optional method in Part II. See instructions. 2: Net profit or (loss) from Schedule C, line 31, and Schedule K (Form 1065), box 14, code A (other than farming). See instructions for other income to report or if you are a minister or member of a religious order. 3: Combine lines 1a a, 1b, and 2. 4a: If line 3 is more than zero, multiply line 3 by 92.35% (0.9235). Otherwise, enter amount from line 3. Note: If line 4a is less than $400 due to Conservation Reserve Program payments on line 1b, see instructions. b: If you elect one or both of the optional methods, enter the total of line 15 and 17 here. c: Combine lines 4a and 4b. If less than $400, stop. You don;t ow self-employment tax. Exception: if less than $400 and you had church employee income, enter 0 and continue. 5a: Enter your church employee income from Form W-2. See instructions. 5b: Multiply line 5a by 92.35% (0.9235). If less than $100, enter 0. 6: Add line 4c and 4b. 7: Max amount of combined wages and self-employment earnings subject to social security tax or the 6.2% portion of the 7.56% railroad retirement (tier 1) for 2021. 8a: Total social security wages and tips (total of boxes 3 and 7 on Form W-2) and railroad retirement (tier 1) compensation. If $142,800 or more, skip lines 8b through 10, and go to line 11. b: Unreported tips subject to social security tax from Form 4137, line 10. c: Wages subject to social security tax from Form 89191, line 10. d: Add lines 8a, 8b, and 8c. 9: Subtract line 8d from line 7. If zero or less, enter 0 here and on line 10 and go to line 11.10: Multiply the smaller number of line 6 or line 9 by 12.4%. 11: multiply line 6 by 2.9%. 12: Self-employment tax. Add lines 10 and 11. Enter here and Schedule 2 (Form 1040), line 4. 13: Deduction for one-half of self-employment tax. Multiply line 12 by 50% (0.50). Enter here and on Schedule 1 (Form 1040), line 15.

After Preparation 1.7 (survey)

You should be able to do the following things for the next class. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 1.7, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	Rating from 1 to 5
follow the order of operations.	
find a percent of a number.	
estimate 1% of a number.	

If your confidence ratings are below 3 for any of these skills/concepts, what are three things you might do to increase your confidence in these areas? If your confidence ratings are all 3 or above for these skills/concepts, what are three things your classmates might do to increase their confidence in these areas?

1.8.2: Exercise 1.8

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) The order of operations follows this order: 1) Within parentheses; 2) Exponents; 3) Multiplication; 4) Division; 5) Addition; 6) Subtraction.

(ii) Taxes are very complicated, and tax forms are hard to complete.

(iii) Part of quantitative reasoning is being able to read, interpret, and use quantitative information to perform a task.

(iv) It does not matter how you write your calculations as long as you get the correct answer.

(2) Refer back to Question 5 in Collaboration 1.5 and Question 4 in this collaboration (1.7). What important quantitative reasoning skill was used in both of these questions? Choose the best answer from the following.

(i) Both questions related to making sense of credit card fees.

(ii) Both questions related to making sense of numbers and calculations.

(iii) Both questions related to making sense of personal finance.

(iv) Both questions related to making sense of taxes.

DEVELOPING SKILLS AND UNDERSTANDING

(3) Martin Binford is an author. He has no income he would report on line 1a or line 1b of his Schedule SE.[26] He earned $163,380 last year from his books. He had $3,563 in expenses. (**Note**: He records his net profit in line 2, where it says “Net profit or (loss).”) You do not need to know which schedule or form it comes from, and he is not a minister or member of a religious order. He also does not get a deduction for one-half of self employment tax. How much self-employment tax does he owe (line 12)? Round to the nearest dollar.

Screenshot of Self-Employment tax form. At the top, it notes: If your only income subject to self-employment tax is church employee income, see instructions for how to report your income and the definition of church employee income. Part A: If you are a minister, member of a religious order, or Christian science practitioner and you filed Form 4361, but you had $400 or more other net earnings from self-employment, check here and continue with Part I. Skip lines 1a and 1b if you use the farm optional method in part II. See instructions. 1a: Net farm profit or (loss) from Schedule F, line 34, and farm partnerships, Schedule K-1 (Form 1065), box 14, code A. b: If you received social security retirement or disability benefits. enter the mount of conservation Reserve program payment included on Schedule F, line 4b, or listed on Schedule K-1 (Form 1065), box 20, code AH. Skip line 2 if you use the non farm optional method in Part II. See instructions. 2: Net profit or (loss) from Schedule C, line 31, and Schedule K (Form 1065), box 14, code A (other than farming). See instructions for other income to report or if you are a minister or member of a religious order. 3: Combine lines 1a a, 1b, and 2. 4a: If line 3 is more than zero, multiply line 3 by 92.35% (0.9235). Otherwise, enter amount from line 3. Note: If line 4a is less than $400 due to Conservation Reserve Program payments on line 1b, see instructions. b: If you elect one or both of the optional methods, enter the total of line 15 and 17 here. c: Combine lines 4a and 4b. If less than $400, stop. You don;t ow self-employment tax. Exception: if less than $400 and you had church employee income, enter 0 and continue. 5a: Enter your church employee income from Form W-2. See instructions. 5b: Multiply line 5a by 92.35% (0.9235). If less than $100, enter 0. 6: Add line 4c and 4b. 7: Max amount of combined wages and self-employment earnings subject to social security tax or the 6.2% portion of the 7.56% railroad retirement (tier 1) for 2021. 8a: Total social security wages and tips (total of boxes 3 and 7 on Form W-2) and railroad retirement (tier 1) compensation. If $142,800 or more, skip lines 8b through 10, and go to line 11. b: Unreported tips subject to social security tax from Form 4137, line 10. c: Wages subject to social security tax from Form 89191, line 10. d: Add lines 8a, 8b, and 8c. 9: Subtract line 8d from line 7. If zero or less, enter 0 here and on line 10 and go to line 11.10: Multiply the smaller number of line 6 or line 9 by 12.4%. 11: multiply line 6 by 2.9%. 12: Self-employment tax. Add lines 10 and 11. Enter here and Schedule 2 (Form 1040), line 4. 13: Deduction for one-half of self-employment tax. Multiply line 12 by 50% (0.50). Enter here and on Schedule 1 (Form 1040), line 15.

(4) Which of the following expressions can be used to compute how much self-employment tax Martin Binford owes?

(i) 0.029(0.9235)($163,380 – $3,563)

(ii) 0.124($142,800) + 0.029(0.9235)($163,380 – $3,563)

(iii) 0.124(0.9235)($163,380 – $3,563) + 0.029(0.9235)($163,380 – $3,563)

(iv) 0.029(0.9235)($163,380 – $3,563 + $17,707.20)

(5) The expression below shows another way to calculate Martin’s tax.

$$0.124(142{,}800) + 0.029(150{,}881.43 - 142{,}800)$$

Based on this expression, select the statement that describes how Martin’s income is taxed.

(i) Martin pays 15.59% tax on his income.

(ii) Martin pays 44.3% tax on his income.

(iii) Martin pays 12.4% in tax on the first $142,800 of his income. He pays 29% on his income over $142,800.

(iv) Martin pays 12.4% in tax on the first $142,800 of his income. He pays 2.9% on his income over $142,800.

(6) Miguel is moving and wants to estimate what his electricity bill will be in his new apartment. He looks at old bills and sees that he uses around 700 kWh of electricity each month. The utility company charges \$6 each month plus 6.726 cents per kWh for the first 500 kWhs and 8.136 cents for each kilowatt-hour above 500.

(a) How much will Miguel pay for 700 kWh of electricity?

(b) People often make a common error in situations like the one in (a). The purpose of the next two questions is to help you recognize this error and correct your work in (a) if necessary.

If someone bought three items for \$1.50, 37 cents and 5 cents, how much did they spend?

(c) Which of the following is most likely the common error in (b)?

(i) Making an addition error such as 37 + 5 = 45 cents

(ii) Forgetting to change the cents to dollars: 1.50 + 37 + 5 = \$43.50

(iii) Leaving off the decimal: 1.50 + 37 + 5 = \$4,350

(7) As discussed in the "Further Applications" section of this collaboration (1.7), workers in the U.S. pay several types of taxes on income, including the FICA tax and the federal income tax. Your federal income tax rate is based on the amount of money you make. Income is broken into levels called tax brackets. The table below shows the tax brackets for 2021.[27]

Taxable Income	Tax
\$0–\$9,950	10% of taxable income
\$9,951–\$40,525	\$995.00 plus 12% of excess over \$9,950
\$40,526–\$86,375	\$4,664.00 plus 22% of excess over \$40,525
\$86,376–\$164,925	\$14,751.00 plus 24% of excess over \$86,375
\$164,926–\$209,425	\$33,603.00 plus 32% of excess over \$164,925
\$209,426–\$523,600	\$47,843.00 plus 35% of excess over \$209,425
\$523,601 and up	\$157,804.25 plus 37% of excess over \$523,600

(a) Calculate the tax for a person with taxable income of \$25,000.

(b) Calculate the tax for a person with taxable income of \$63,500. Write your answer rounded to two decimal places.

(c) Refer to your answer for (b). The total tax in (b) is what percentage of the person's taxable income? Round to one decimal place.

MAKING CONNECTIONS ACROSS THE COURSE

(8) In Collaboration 1.6, it was determined that the water footprint for a typical American is 2,841 m^3/year.

(a) A family of three would like to reduce their water footprint so that it is 75% of the typical American's water footprint. Which expression shows how they can calculate their target water footprint for one day? There may be more than one correct answer.

(i) $(3 \times 2{,}841 \times \frac{3}{4}) \div 365$

(ii) $2{,}841 \times 3 \times 0.75$

(iii) $3 \times 2{,}841 \times 0.75 \div 365$

(iv) $2{,}841 \times 3 \times \frac{4}{3} \div 365$

(v) $2{,}841 \times 3 \div 0.75$

(vi) $2{,}841 \div (365 \times 0.75) \times 3$

(vii) $3 \times (2{,}841 \times 0.75) \div 365$

(b) One thing a person can do to reduce his or her water footprint is to use less water every day. If each American were to reduce his or her daily water use by 2 m3 (528 gallons), how would you calculate the new annual water footprint for a typical American?

There may be more than one correct answer.

(i) 2,841 − 2 × 365

(ii) (2,841 – 2) × 365

(iii) 2,841 – (2 × 365)

(iv) 2,841 × 365 – 2

(v) 2,841 – 365 × 2

(vi) (2,841 ÷ 365 – 2) × 365

(vii) 2,841 ÷ 365 – 2

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[26] https://www.irs.gov/pub/irs-prior/f1040sse--2021.pdf

[27] https://taxfoundation.org/2021-tax-brackets/

1.9: Interpreting Statements about Percentages

INTRODUCTION

Mathematicians like to shorten things. Sometimes a capital letter (like C or Q) is used as a common variable to indicate that you are talking about two similar quantities. The numbers 1 and 2 indicate that the quantities represent two different quantities, but their values might still be the same. For example, if C1 represents students in one class and C2 represents students in a second class, it is possible that C1 = C2, even though they represent different things. **Note**: Remember, an inequality means that two quantities are not equal. Also, remember the inequality symbol (> or <) always points to the smaller of the two quantities.

Quick Check-in:

Please discuss the following questions in your group:

- What specific concept from Unit 1.7 do you feel fairly confident about?
- Are there any specific concepts in Unit 1.7 that you find challenging?

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- percentages involve a numerator (comparison value) and a denominator (reference value).

By the end of this collaboration, you should be able to

- correctly identify the quantities involved in a verbal statement about percentages.
- convert between ratios and percentages.
- convert between the decimal representation of a number and a percent.
- read and use information presented in a two-way table.

PROBLEM SITUATION: THE LANGUAGE OF PERCENTAGES

The World Health Organization (WHO) is the part of the United Nations that oversees health issues in the world. The WHO leads numerous studies on tobacco use around the world. In its study called, “Gender and Tobacco,” the organization learned that tobacco use among females is increasing.[28] For example, recent research shows that just as many young girls smoke as young boys. The report is filled with information about percentages of females who smoke, percentages of males who smoke, and the percentage of smokers who start smoking by age 10. The language used to describe this information can be difficult to understand; therefore, pay close attention to the language used to describe a percent at the beginning of this collaboration. This will help you to understand the findings reported in the WHO study.

Consider the following two quantities:

- Quantity 1 (Q1): The percentage of females who smoke.
- Quantity 2 (Q2): The percentage of smokers who are female.

(1) (a) Are these two quantities equal (Q1 = Q2)?

(b) Could Q1 be greater than Q2 (Q1 > Q2)?

(c) Could Q1 be less than Q2 (Q1 < Q2)?

(d) Explain your reasoning.

(2) What information would you need to calculate these percentages?

Smoking on Two College Campuses

Questions 3 and 4 present two situations with data about smoking. You can use these situations to test your ideas from Questions 1 and 2.

(3) Suppose a study on smoking was conducted at Midland University. The following table indicates the results of the study.

	Males	Females
Smokers	4,572	5,362

Nonsmokers	10,284	12,736

(a) What percentage of females smoke at Midland University? Round to the nearest percent.

(b) What percentage of smokers at Midland University are female? Round to the nearest percent.

(4) Suppose a study was conducted at Northwest College. The following table indicates the results of the study.

	Males	**Females**
Smokers	1,256	536
Nonsmokers	1,028	1,053

(a) What percentage of females smoke at Northwest College? Round to the nearest percent.

(b) What percentage of smokers at Northwest College are female? Round to the nearest percent.

(c) A newspaper stated that 40% of the male students at Northwest College are smokers. Is that claim reasonable?

(5) In 2021, the World Health Organization conducted a study about the prevalence of tobacco use worldwide.[29] The organization reported that 2.1% of the adult females in China smoke tobacco products. In the United States, 15.2% of adult females smoke.

(a) Out of 100 adult females in China, about how many are smokers?

(b) Out of 1,000 adult females in China, about how many are smokers?

(c) Out of 100 adult females in the United States, about how many are smokers?

(d) Out of 1,000 adult females in the United States, about how many are smokers?

(e) Suppose you read that 478 out of 1,000 males in China smoke. Based on this data, what percentage of males in China smoke?

(f) Are there more female smokers in China or the United States? Discuss in your group before recording your answer in the space below.

FURTHER APPLICATIONS

The following questions are included in Exercise 1.8. After you complete (a) and (b) below, be sure to transfer your answers to Exercise 1.8, Question 5.

(6) A teacher has collected data on the grades his students received in his morning and afternoon classes. The following tables show two different ways to represent the same data as percentages.

Table 1

	Grade A	**Grade B**	**Grade C**	**Grade D**	**Grade F**
Morning Class	12.5%	25.0%	37.5%	6.3%	18.7%
Afternoon Class	14.3%	20.0%	37.1%	8.6%	20.0%

Table 2

	Grade A	**Grade B**	**Grade C**	**Grade D**	**Grade F**
Morning Class	44.4%	53.3%	48.0%	40.0%	46.2%
Afternoon Class	55.6%	46.7%	52.0%	60.0%	53.8%

(a) Which table could be used to answer the following question: What percentage of the students who received an “A” is in the morning class? Explain your answer.

(b) Which table could be used to answer the following question: What percentage of the students in the morning class received an “A”? Explain your answer.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[28] https://apps.who.int/iris/handle/10665/43657

[29] https://www.who.int/publications/i/item/9789240039322

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1.9.1: Preparation 1.9

(1) Which of the following represents 0.02%? There may be more than one correct answer.

(i) 2

(ii) 0.02

(iii) 0.0002

(iv) 2 out of 100

(v) 0.2 out of 100

(vi) 0.02 out of 100

(vii) 2 out of 1,000

(viii) 2 out of 10,000

(2) Which of the following is equivalent to 4%? There may be more than one correct answer.

(i) 0.04

(ii) $\frac{1}{25}$

(iii) $\frac{4}{100}$

(iv) $\frac{40}{100}$

(v) 2 out of 5

(vi) 4 out of 100

(3) Which of the following is correct? There may be more than one correct answer.

(i) A percent is one part out of 100 parts.

(ii) A percent can be converted into a decimal number by dividing that percent number by 100.

(iii) A percent can be converted into a decimal number by moving the decimal point two places to the left and removing the percent sign (%).

(iv) 50% means 50 per 100 or $\frac{50}{100} = 0.5$.

(4) Which of the following is the percent estimate of 1/3, rounded to the nearest hundredth of a percent?

(i) 0.33%

(ii) 3.3%

(iii) 33.3%

(iv) 33.33%

(5) The quantity 1,352 is what percent of 40,929? Round to the nearest tenth of a percent.

Self-Regulating Your Learning: The Work Phase

Earlier, you read in detail about what it means to effectively *plan* for your learning. That involved accounting for time and effort, your confidence (self-efficacy), study strategies, and learning goals. In this lesson, we will discuss the second phase of self-regulated learning (SRL), the ***work*** phase.

As the name implies, the *work* phase of SRL is where you are actually working on the problem or assignment. However, it is more than just getting the assignment done. In this phase, you monitor or pay attention to a variety of things. For example:

- What you are or are not **understanding** (and when);
- Which **strategies** you are using;
- Which ones are working, and which ones are not;

- What **emotions** and feelings you are experiencing, both positive and negative;
- When you should **seek help** from others.

Let’s explore each of these in a little more detail.

Understanding: Self-regulated learners monitor what they understand and what they do not. This is done by frequently asking yourself: “Do I understand this?” or “Could I explain this to someone?” The goal is to monitor your understanding so that you may adapt your strategies, especially if you get stuck. Being honest about your understanding is important because it can help you progress successfully on a problem, or make you aware of your learning strengths and weaknesses. Sometimes, people talk about this as “thinking about your thinking.” Researchers call it *metacognition*.

Strategies: In learning about the SRL *plan* phase, you discovered that it can be useful to think about multiple strategies *before* you start working on a problem. In the work phase of SRL, having multiple strategies in mind (both those you have used before and those you plan to try) can help when you get stuck. You can stop, think about how the problem is progressing, and try another strategy that you think might work. Self-regulated learners often make mental notes about which strategies work in which situations, and which ones are easiest to use. Evaluating strategies allows you to become better at solving a variety of problems.

Emotions: Self-regulated learners know how to monitor their emotions—especially negatives ones such as frustration or anger—so that these emotions do not cause them to give up on a problem. When they start feeling frustrated, self-regulated learners often do things such as try new strategies, seek help, or engage in *positive self-talk*. This is saying things to yourself such as: “I know I can do this if I choose the right strategies and put in the effort, even if it is challenging.” The opposite is called *negative self-talk*, which involves saying things such as: “I am never going to get this! What is the point?” Monitoring and controlling your emotions, especially the negative ones, can be challenging and may require a lot of practice, but the benefits are worth it.

Seeking Help: With practice and experience, self-regulated learners know when it is beneficial to stop working and find someone else with whom they can discuss the problem. There is nothing wrong with getting help when you are learning something new. Some people think that asking for help means you do not have ability, but the truth is that knowing when to seek help is part of being an effective learner. Seeking help can save you time because you avoid the added frustration of making a lot of effort without making any progress. If you are spending a lot of time on a problem, you have tried several strategies without success, or you are not able to control negative emotions, stop and write down your questions. Bring the written questions with you and discuss the problem with someone else as soon as you can. Help could come from your instructor during his or her office and/or virtual office hours, a campus learning center, or other classmates.

During the *work* phase, you are required to juggle two things at once: (1) working on the problem or assignment, and (2) monitoring your progress (e.g., *thinking about your thinking*). This process takes practice; however, it is important to master if you want to become a self-regulated learner. Thinking about how you are working makes the work easier and gives you information for the SRL *reflect* phase. You will explore the *reflect* phase later. Until then, practice these work strategies while you are working on problems and class assignments.

After Preparation 1.8 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 1.8, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
understand the word percent and the notation used to describe percentages (%).	
use a calculator to divide two numbers and interpret the resulting decimal representation as a percent.	
calculate and estimate percentages.	

1.9.2: Exercise 1.9

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) A percentage is calculated by dividing one number by another number.

(ii) Smoking is a major health problem in the United States and China.

(iii) A percentage is a comparison of two numbers. To understand the meaning of a percentage, it is important to know what two quantities are being compared.

(iv) The percentage of students who are parents is the same as the percentage of parents who are students.

DEVELOPING SKILLS AND UNDERSTANDING

(2) Data from the *National Center for Education Statistics* (NCES)[30] provides a statistical snapshot of the proportion of bachelor's degrees conferred in different fields of study in 2019-20. A total of approximately 2,000,000 awarded degrees were included in the study. The table below displays the total number of students who majored in each of the following fields of study.

Field of Study	Number of Students who Majored in Field	Percentage of Students who Majored in Field
Psychology	120,000	(i)
Social Sciences/History	161,200	(ii)
Biology/Biomedical Sciences	126,600	(iii)
Computer/Information Science	97,000	(iv)
Engineering	128,300	(v)
Visual and Performing Arts	92,300	(vi)
Business	387,900	(vii)
Health Professions	257,300	(viii)
Communication/Journalism	91,800	(ix)

(a) Complete the table (i) to (ix) by filling in the *percentages* of students who majored in each field of study. Round to one decimal place where necessary.

(b) What was the *most popular* major in 2019–20?

(i) Engineering

(ii) Business

(iii) Health Professions

(iv) Psychology

(c) Fill in the blanks to complete the following statements.

(i) About ________ out of every 100 students in 2019–20 majored in the most popular field.

(ii) About ________ out of every 1,000 students in 2019–20 majored in the most popular field.

(3) Select the answer(s) that correctly complete the statement from the list below: A *New York Times* story reported that 10% of male high school dropouts are in jail or detention centers.[31] According to this statistic, about ________ in every ________ male high school dropouts is (are) in jail or juvenile detention. There may be more than one correct answer.

(i) 1 in every 10

(ii) 1 in every 10%

(iii) 0.1 in every 100

(iv) 1 in every 100

(v) 10 in every 100

(4) A teacher has collected data on the grades his students received in his morning and afternoon classes. The following tables show two different ways to represent the same data as percentages.

Table 1

	Grade A	Grade B	Grade C	Grade D	Grade F
Morning Class	12.5%	25.0%	37.5%	6.3%	18.7%
Afternoon Class	14.3%	20.0%	37.1%	8.6%	20.0%

Table 2

	Grade A	Grade B	Grade C	Grade D	Grade F
Morning Class	44.4%	53.3%	48.0%	40.0%	46.2%
Afternoon Class	55.6%	46.7%	52.0%	60.0%	53.8%

(a) Which table could be used to answer the following question: What percentage of the students who received an “A” is in the morning class?

(b) Which table could be used to answer the following question: What percentage of the students in the morning class received an “A”?

(c) What are the reference values in Table 1?

(i) The number of students in a certain class.

(ii) The number of students who earned a certain grade.

(iii) The number of students in a certain class who earned a certain grade.

(iv) The total number of students in both classes.

(d) What are the reference values in Table 2?

(i) The number of students in a certain class.

(ii) The number of students who earned a certain grade.

(iii) The number of students in a certain class who earned a certain grade.

(iv) The total number of students in both classes.

MAKING CONNECTIONS ACROSS THE COURSE

(5) Congratulations! You won a lottery prize and have a taxable income of $1,025,400. Use the table below to answer the following questions.

Taxable Income	Tax
$0–$9,950	10% of taxable income
$9,951–$40,525	$995.00 plus 12% of excess over $9,950
$40,526–$86,375	$4,664.00 plus 22% of excess over $40,525
$86,376–$164,925	$14,751.00 plus 24% of excess over $86,375
$164,926–$209,425	$33,603.00 plus 32% of excess over $164,925

$209,426–$523,600	$47,843.00 plus 35% of excess over $209,425
$523,601 plus	$157,804.25 plus 37% of excess over $523,600

(a) How much tax will you pay on your winnings? Round to the nearest dollar.

(b) Which of the following expressions can be used to calculate the tax?

(i) $1,025,400 – 0.37($501,800 – $157,804.25)

(ii) 0.37 + $501,800($1,025,400 – $157,804.25)

(iii) ($1,025,400 + $157,804.25) – ($523,600 × 0.37)

(iv) $157,804.25 + 0.37($1,025,400 – $523,600)

(v) $1,025,400 – 0.37($1,025,400 – $523,600)

After Exercise 1.8 (survey)

The work phase of regulating your learning includes checking your understanding. One of the ways to do this is by asking yourself: “Can I explain this to someone?” Check your understanding by reading Question 5(b) from Exercise 1.8 and explaining the answer below. In this question, you considered how much tax you would pay on lottery winnings of $1,025,400.

Write your explanation below.

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[30] https://nces.ed.gov/programs/coe/indicator/cta

[31] http://www.nytimes.com/2009/10/09/education/09dropout.html?_r=0

1.10: Percentages and Probabilities

INTRODUCTION

In this collaboration, we will be making quantitative statements about data that can be displayed in two-way frequency tables. Two-way frequency tables are used to display data based on two categorical variables, where each cell shows how many observations fall in each category.

Consider the following example. Out of 170 students who took a Chemistry 201 test,

- 20 students passed it even though they didn't complete the corresponding Test Review assignment provided by the instructor
- 80 students passed after completing the corresponding Test Review assignment provided by the instructor
- 60 students didn't pass it, and they didn't complete the corresponding Test Review assignment provided by the instructor
- 10 students didn't pass it even though they completed the corresponding Test Review assignment provided by the instructor

The given dataset of 170 students can be displayed in the following two-way frequency table.

	Completed the Review Assignment	Didn't complete the Review Assignment	Total
Passed the test	80	20	100
Didn't pass the test	10	60	70
Total	90	80	170

In your group, can you think of other examples of data that can be displayed in a two-way frequency table like the one above?

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- a percent has different uses, including being used to express the likelihood (or probability) of a certain event.
- it is important to select the correct comparison value and reference value when calculating percentages.

By the end of this collaboration, you should be able to

- extract relevant information from a table.
- select the appropriate values to calculate probabilities.

PROBLEM SITUATION: USING PERCENTAGES TO DESCRIBE THE ACCURACY OF MEDICAL TESTS

Some athletes use performance-enhancing drugs (PEDs) to improve how they do in sports. Schools, sports leagues, and other sports organizations usually do not allow the use of PEDs. These groups can administer athletes a blood or urine test to determine if the athletes are using drugs. In this problem situation, you will calculate the *probability* that such a test gives correct results and the probability that such a test gives incorrect results. **Probability** means the chance that something happens. Probabilities are often reported in percentages (%).

Five hundred athletes have undergone a test to determine if they use PEDs. A positive (+) test result indicates that the athlete is using a PED. A negative (–) test result indicates the athlete is not using these drugs. However, this test is not 100% accurate. This means that some errors may be present in the test results. The table below shows how often the test correctly determines if athletes used PEDs.

	Athletes Using PEDs	Athletes Not Using PEDs	Total
Positive Test Result	9	5	(a)
Negative Test Result	1	485	**486**
Total	**10**	(b)	**500**

Use the numbers in the table to answer the following questions. Be careful of what numbers you use for the numerator and denominator in your calculations.

(1) The table is missing one row total and one column total. Discuss with your group what the missing numbers should be. Fill in the missing totals (a) and (b).

(2) You might want to know: “If an athlete is using PEDs, what is the probability that this test gives a positive result?” Use the steps below to answer that question.

(a) How many athletes are using PEDs?

(b) How many of the athletes using PEDs received a positive test result?

(c) If an athlete is using PEDs, what is the probability that this test gives a positive result?

(3) The table shows that one athlete who was using PEDs received a negative test result. This means the test incorrectly identified this single athlete. What percentage of athletes who are using PEDs incorrectly test negative?

Hint: Think about the ratio of the number of incorrect negative results compared to the number of athletes who were using PEDs.

(4) You might want to know: “If an athlete is not using PEDs, what is the probability that this test gives a negative result?” Use the steps below to answer that question.

(a) How many athletes are not using PEDs?

(b) How many of the athletes not using PEDs received a negative test result?

(c) If an athlete is not using PEDs, what is the probability that this test gives a negative result?

(5) Think about a situation in which a school principal receives a positive result on a PED test for a student athlete. Answer these questions:

(a) What percentage of athletes not using PEDs incorrectly test positive?

Hint: Think about the ratio of the number of incorrect positive results compared to the number of those who are not using PEDS.

(b) How should the principal think about this percentage? What recommendation would you make to the principal about what she should do when she receives a positive result on a PED test for a student athlete?

(6) Are your results from Question 2(c) and Question 3 related? Are your results from Question 4(c) and Question 5(a) related? If yes, briefly explain.

(7) Refer to the table to answer questions (a) - (c).

(a) You might also want to know: “What percentage of all positive test results are from athletes who are actually using PEDs?” Round your answer to one decimal place.

Hint: Think about the ratio of the number of correct positive results compared to the number of all positive test results.

(b) Now answer “What percentage of all positive test results are from athletes who are NOT using PEDs?”

(c) Does your answer to part (b) surprise you? Does this make you rethink your answer for Question 5(b) above? Now what recommendation would you make to the principal about what she should do when she receives a positive result on a PED test for a student athlete?

(8) You might also want to know: “What percentage of all negative test results are from athletes who are not using PEDs?” **Hint**: Think about the ratio of the number of correct negative results compared to the number of all negative test results.

Use the table to answer that question. Round to one decimal place.

(9) You can use a percentage to show how *accurate* the PED test was. A test is **accurate** when it produces very few mistakes or errors. Pick one percentage that you think best describes how accurate the test was. Explain what this percentage says about the test and why you picked this percentage.

(10) Now, think about how to use a percentage to show how *inaccurate* the test was. A test is **inaccurate** if it produces many errors. Pick one percentage to show how inaccurate the test was. Discuss in your group what this percentage says about the test and why you picked this percentage.

FURTHER APPLICATIONS

You encountered the following dataset in this lesson:

	Athletes Using PEDs	Athletes Not Using PEDs	Total
Positive Test Result	9	5	**14**
Negative Test Result	1	485	**486**
Total	**10**	**490**	**500**

You will encounter a similar dataset in Question 4 of Exercise 1.9. Here is the dataset for easy reference:

	Athletes Using PEDs	Athletes Not Using PEDs	Total
Positive Test Result	90	4	**94**
Negative Test Result	10	396	**406**
Total	**100**	**400**	**500**

(11) In this question, you should refer to the dataset from the collaboration as “Population 1 (P1),” and the dataset from Exercise 1.9, Question 4, as “Population 2 (P2).”

(a) A **prevalence rate** is the percentage of people in a population who have a certain disease or behave in a certain way. Find the prevalence rate of athletes using PEDs for P1 and P2. Another way to say this is, “What percentage of the population of athletes used PEDs?” Put your answers in the first row of the following table.

	P1	P2
Prevalence rate	(a)	(f)
If an athlete is using PEDs, what is the probability that this test gives a positive result?	(b)	(g)
If an athlete is **not** using PEDs, what is the probability that this test gives a positive result?	(c)	(h)
What percentage of all positive test results are from athletes who are using PEDs?	(d)	(i)
What percentage of all positive test results are from athletes who are **not** using PEDs?	(e)	(j)

(b) Record your answers for P1 and P2 in (a) - (j) in the table above.

(c) What does the information in the table indicate about how prevalence can affect test outcomes? Discuss in your group. Write your answer using the Writing Principle.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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1.10.1: Preparation 1.10

According to the World Health Organization (WHO), “Every person is at risk of foodborne illnesses.”[32] A foodborne illness is an illness a person gets from eating food that has spoiled or been contaminated in some way.

(1) Every year, nearly one in 10 people around the world fall ill after eating contaminated food. This means that _________ % of people will likely suffer from foodborne diseases each year.

(2) In 1994, an outbreak of illness due to ice cream contaminated with the bacteria salmonella occurred in the U.S. The outbreak affected an estimated 224,000 people.

(a) If the total population in the United States at that time was 260,000,000, which is the best estimate for the percentage of people who were affected?

(i) About 0.1%

(ii) About 1%

(iii) About 5%

(iv) About 10%

(v) About 25%

(b) Complete the following statement: _____ out of every 100,000 people in the United States were affected by the 1994 salmonella outbreak. Round to the nearest whole person.

(3) A report about foodborne illnesses indicated “About 1 egg out of every 20,000 contains salmonella inside the shell.” This means that

(i) 0.005% of eggs contain salmonella.

(ii) about 1% of eggs contain salmonella.

(iii) more than 1% of eggs contain salmonella.

(iv) approximately 50 out of every 100,000 eggs contain salmonella.

(4) The World Health Organization article mentioned in question 1 states that of the 420,000 deaths each year from eating contaminated food, 125,000 are children under 5 years of age. What percentage of those who died were under the age of 5? Round to the nearest tenth of percent.

After Preparation 1.9 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 1.9, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
understand the word percent and the notation used to describe percentages (%).	
use a calculator to divide two numbers and interpret the resulting decimal representation as a percent.	
calculate percentages.	

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[32] https://www.who.int/health-topics/foodborne-diseases

1.10.2: Exercise 1.10

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main mathematical ideas of the collaboration?

- (i) A probability is a percentage in which the chance of an event is measured as a ratio out of 100.
- (ii) Medical tests are far less accurate than most people think.
- (iii) To calculate a percentage, divide the comparison value by the reference value.
- (iv) Probabilities are not related to percentages.

DEVELOPING SKILLS AND UNDERSTANDING

(2) About 12.6% of drivers in the United States are uninsured.[33] There were approximately 229 million drivers in the United States in 2019.[34] How many of these drivers were likely uninsured?

- (i) 28,854
- (ii) 28,854,000
- (iii) 288,540,000
- (iv) 2,885,400,000

(3) A test is administered to 500 athletes to determine if they are using performance-enhancing drugs (PEDs). It is assumed that a positive test result indicates that the athlete is using PEDs, and a negative test result indicates that the athlete is not using PEDs. However, this test is not 100% accurate, so some errors occur. The following table shows the test results for a group of athletes.

	Athletes Using PEDs	**Athletes Not Using PEDs**	**Total**
Positive Test Result	90	4	**94**
Negative Test Result	10	396	**406**
Total	**100**	**400**	**500**

Use the information in the table to answer the following questions.

(a) How many athletes are using PEDs?

(b) If an athlete is using PEDs, which of the following describes the chance that this test will give a positive result? There may be more than one correct answer.

- (i) 9%
- (ii) 90%
- (iii) 9 out of 100
- (iv) 90 out of 100

(c) What is the chance of the test giving a negative result for an athlete using PEDs?

(d) If an athlete is not using PEDs, what is the chance that this test will give a negative result?

- (i) 39.6%
- (ii) 99%
- (iii) 4 out of 396
- (iv) 4 out of 400

(e) What is the chance of the test giving a positive result for an athlete not using PEDs?

(4) A hospital tracks the number of cases (patients) that come into its Emergency Room during each eight-hour shift. The cases are listed in categories based on the severity of the illness or injury. The categories from least severe to most severe are: stable, serious,

and critical. The following table gives the data for a week.

(a) Complete the missing blanks (i), (ii), and (iii) in the table.

Shift	Stable	Serious	Critical	Total
8:00 a.m. – 3:59 p.m.	250	120	45	**415**
4:00 p.m. – 11:59 p.m.	270	230	105	(i)
12:00 a.m. – 7:59 a.m.	(ii)	175	95	**460**
Total	**710**	(iii)	**245**	**1480**

(b) A nursing supervisor ranks the shifts based on two different criteria. Which shift received the highest percentage of the total critical cases? For each shift, find the percentage of **critical cases compared to total critical cases**. Round to the nearest one percent.

Shift	Percentage of Total Critical Cases	Rank
(i) 8:00 a.m. - 3:59 p.m.		
(ii) 4:00 p.p. - 11:59 p.m		
(iii) 12:00 a.m. - 7:59 a.m.		

(c) Rank the shifts from highest to lowest.

(d) Which shift has the highest percentage of **critical cases compared to that shift's total cases**? Round to the nearest one percent.

Shift	Percentage of Cases Compared to Shift's Total Cases	Rank
(i) 8:00 a.m. - 3:59 p.m.		
(ii) 4:00 p.p. - 11:59 p.m		
(iii) 12:00 a.m. - 7:59 a.m.		

(e) Rank the shifts from highest to lowest.

(f) The hospital schedules nursing staff based on the number and severity of expected cases. One goal of scheduling is to have more experienced nurses work on critical cases. The nursing supervisor considers the overall number of experienced nurses and the ratio of experienced nurses to the less experienced nurses. Using these data and the tables you created in the questions above, which of the following conclusions could be drawn?

(i) The highest *number* and the highest *ratio* of experienced nurses should be scheduled during the 12:00 a.m. to 7:59 a.m. shift.

(ii) The highest *number* and the highest *ratio* of experienced nurses should be scheduled during the 4:00 p.m. to 11:59 p.m. shift.

(iii) The highest *number* of experienced nurses should be scheduled during the 4:00 p.m. to 11:59 p.m. shift. The highest *ratio* of experienced nurses should be scheduled during the 12:00 a.m. to 7:59 a.m. shift.

(iv) The highest *number* of experienced nurses should be scheduled during the 12:00 a.m. to 7:59 a.m. shift. The highest *ratio* of experienced nurses should be scheduled during the 4:00 p.m. to 11:59 p.m. shift.

MAKING CONNECTIONS ACROSS THE COURSE

(5) You can save time and work by using spreadsheets to perform calculations. To do this, you use formulas (as seen in Exercise 1.4). In this question, you will use a spreadsheet to calculate Marianne Lopez's self-employment taxes, which were discussed in Collaboration 1.7.

(a) Use the spreadsheet below and the information from Collaboration 1.7 to fill in the blanks (i) - (iv) for cells B2–B5.

Marianne Lopez spreadsheet. A2 = revenue. A3 = Expenses. A4 = % of income that is taxed. Q5 = Tax rate (decimal) and A7 self-employment tax. Answer spaces in B2, B3, B4, B5, and B7.

(b) Which of the following is a formula for cell B7 that will calculate the self-employment tax. Remember that the formula will start with an equal sign (=). In a spreadsheet, you use an asterisk (*) to indicate multiplication. So B3 × B4 would be written as B3*B4.

(i) = B4*B5*(B2-B3)

(ii) = B4*B3*(B2-B5)

(iii) = (B5-B4)*B2*B3

(iv) = B5*B3*(B4-B3)

(c) Suppose Marianne made a mistake in calculating her expenses. The expenses should be $4,371. Enter this new value into the spreadsheet. What is the new amount for the self-employment tax? Remember that since we are working with monetary values here, include both dollars and cents in your answer.

Self-Regulating Your Learning: The Reflect Phase

The last phase in regulating your learning is the ***reflect*** phase. In this phase, after you finish a problem or assignment, you intentionally reflect on how your learning and problem solving went. You gather information about yourself, about studying, and about learning in general. This information is then used to improve future learning situations. Here are some things that self-regulated learners reflect upon:

- Confidence (self-efficacy)
- Strategy selection
- Time and effort
- Emotions
- Causes of success and setbacks

Confidence (Self-Efficacy): Regulating your learning means you continuously pay attention to how much you believe that you can succeed at what you are trying to accomplish. This is important after you complete a problem because it allows you to plan for what you will need to do as you move forward. If you rate your confidence low, then you would benefit from spending more time practicing and studying, and you may decide to get additional help. If you do not stop to think about how things went, it is easy to just move on to the next concept, thinking that you understand something you do not. Also, it is important to give yourself credit for your successes. You will feel better about the time and effort you spent, and you will be more motivated to continue to work hard.

Strategy Selection: When regulating your learning, look back at the strategies you used when working on a problem or assignment. Make note of what seemed to work well for certain problems and what strategies seemed easier. This information will guide your plan phase in preparing for a new assignment. You can practice this by asking yourself: “What worked well and what did not?”

Time and Effort: Pay attention to the time and effort it took to complete an assignment. Reflect on whether you planned accurately. When doing this, try to identify the types of things you may need more time for in the future, or what might be more of a challenge. This information will be used during the plan phase.

Emotions: For successful learning, it is important to manage your emotions, especially your frustration and anxiety. One way to do this is to ask yourself what caused you to become frustrated or anxious and think about what helped you overcome those feelings. This gives you tools to deal with those feelings when they come up again.

Causes of Success and Failure: One of the most common challenges for students is correctly identifying the reasons for both their successes and their setbacks. Students often blame *external* factors for shortcomings. When faced with challenges, students often blame the teacher (“She does not explain well”); or maybe the book (“It is hard to read and it is confusing”); or the test itself (“It was full of trick questions”). The problem with blaming external factors is that it gives you little control over your own learning outcomes. On the other hand, by considering *internal* factors—ones you can control—then you are in charge of your learning outcomes. For example, when facing a setback, ask yourself:

- Did I spend enough time on this assignment?

- Did I use the right strategies?
- Did I seek help when I needed it?
- Did I put in the work and effort that was really required?

These types of self-reflection questions help you to understand yourself better and assist you in becoming a more effective learner.

---

[33] https://www.iii.org/fact-statistic/facts-statistics-uninsured-motorists

[34] https://www.fhwa.dot.gov/policyinformation/statistics/2019/dl1c.cfm

1.10.3: Stress Reappraisal

(to be completed before the Module 1 exam, as instructed)

As you might expect, taking a math test can be a very stressful experience. Before taking the Module 1 exam, we are going to complete a short reading exercise designed to help you perform well.

In the following pages, you will be presented with scientific research. Because scientific research articles can be long, complex, and difficult to understand we had students like you summarize the research articles discussed below.

The research presented in the following materials was conducted with people with anxiety disorders to help them cope with feelings of overwhelming anxiety. More specifically, materials provide you with information about how stress can help improve performance.

While you read each summary, please try to think about how stress can help you. Then, after each one, you will be asked to write a short summary using your own words describing how the information presented can help maximize your performance on the Module 1 exam. The information you write here will be included in messages to future Quantway and Statway students designed to provide students with the confidence to thrive in testing situations.

The following summary is of Jamieson and Mendes' 2010 study that appeared in the *Journal of Experimental Psychology*:

> In stressful situations, people experience changes in their body. They might experience these changes as "unsettled feelings" or "butterflies in their stomach," and conclude that they are nervous. However, the bodily changes that happen during stress can be good, not just bad. For instance, scientists have found that feelings of "butterflies" or nervousness indicate that the body is gathering resources to meet the demands of the situation. In other words, the body needs energy to perform and stress helps deliver this energy to where it is needed.
>
> Stress can be "good" or "bad," and the type of stress we experience depends on our perceptions and beliefs. To demonstrate how the body and mind work together, imagine you are a skier staring down a steep, icy slope with no other way off the mountain than going down this dangerous trail. Regardless of whether you like skiing, this situation would cause stress. Expert skiers may experience the stress as "excitement" if they believe they can handle the difficult trail, whereas novices are more likely to experience the stress as "fear" if the difficulty of the trail exceeds their skill level. Thus, the skier's response (excitement vs. fear) depends on how they perceive the situation and their body's responses.
>
> The negative consequences of stress are the direct result of people's beliefs that feeling anxious will make them perform poorly. However, our research from people with anxiety disorders indicates that **stress does not hurt performance, but can actually help** because our brain releases chemicals that help us think quickly. So, during the Module 1 exam, try and view stress as a coping tool.

In your own words please briefly describe how this information can help you perform well on the Module 1 exam:

____

____

____

____

The following summary is of Nock et al.'s (2011) study that appeared in the *Journal of Clinical Psychology*:

> Stress is a normal physical reaction that helps you face the challenges in your life. It is not harmful. In fact, if we did not have stress reactions we could not survive. If stress is helpful, then why do most people see it as a negative experience?
>
> Research from patients coping with anxiety disorders indicates that negative reactions to stressful situations like taking an exam in class or going on a job interview are the result of how we think about stress (also known as "cognitive appraisals"). When the "*fight or flight*" system activates, our brain searches for potential sources of harm. However, in modern society there is often no physical threat. When no explanation can be found, the search turns inwards and the brain invents explanations such as, "There must be something wrong with me." Nothing could be further from the truth.
>
> During stressful situations remember that your body's responses are good. Increased heart rate, sweating, and heavy breathing all help deliver oxygen where it is needed. Remember, if you find yourself experiencing stress during tests, focus on the benefits of your body's responses.

In your own words please briefly describe how this information can help you perform well on the Module 1 exam:

____

____

____

____

The following is an illustrative diagram that shows the biological changes that happen when we experience stress. Please take a minute to note where the changes occur and how these help us do well.

Illustration showing how stress affects the brain, heart and breathing, indicating the following: Brain: Stress releases chemicals that help you think more quickly & solve problems better. Maybe you've even heard of one of these chemicals before: Adrenaline. Heart: Stress increases your heart rate so it can deliver fuel to your brain to help you think. Your heart sends oxygen to fuel your brain. Breathing: Stress speeds up your breathing. You might even feel short of breath. This is okay because it means your body has lots of oxygen to help you do well.

In your own words please briefly describe how this information can help you perform well on the Module 1 exam:

____

____

____

____

Great job! You have finished the reading exercise.

Remember during the Module 1 exam, we ask that you try to remind yourself that your body's responses to the stressful testing situation will help you to perform well.

Good luck!

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1.11: Module 1 Review Survey

During any learning experience, it is important to frequently and accurately assess what you do and do not know. This is especially true before a quiz or test or when ending a module.

Math is different from many subjects. In math, you often have to show you can complete a problem, not just remember facts or choose the right answer. Every math student has had the experience of looking at work they have previously completed or examples done in the book and thinking, “I know how to do that,” only to get home or into a test and not be able to do a similar problem. To check your understanding accurately, you must do problems that represent the concepts and skills you need to know. If you take time to accurately assess what you know, you can cut down on your study time. You can dedicate your study time to learning only the concepts and skills you need to understand better.

ASSESSING YOUR UNDERSTANDING

The table on the following page lists the Module 1 concepts and skills you should understand. After completing it, you will be able to prioritize your review time more effectively.

Assess your understanding:

- Go through the topics list and locate each concept or skill in the Module 1 student workbook or in the online activities.
- If you recently used the skill and feel confident that you did it correctly, rate your understanding a 4 or 5.
- If you remember the topic but could use more practice, rate your understanding a 3.
- If you cannot remember that skill or concept, or still find it difficult, rate your understanding a 1 or 2.

Now that you have done an initial rating of your understanding, it is time to begin reviewing. The goal is to have a confidence rating of 4 or 5 on all the topics in the table when you have finished your review of Module 1.

Things to consider for reviewing:

1. Start at the beginning of the module and review the material in the Preparations, Collaborations, and Exercises, and your notes on the skills and concepts you rated 3 or below.
2. Select a few questions to do.
3. Once you have finished, remember to check the solutions. If you are not sure if you have done the problems correctly, check with your instructor, other classmates, your previous work or work with a tutor in the learning center or online.
4. Rate your confidence on this skill again. If you understand the concept better, rate yourself higher. Begin a list of topics that you want to review more thoroughly.

For topics that you need to review more thoroughly, make a plan for getting additional assistance by studying with classmates, visiting your instructor during office or virtual office hours, working with a tutor in the learning center or online, or looking up additional information on the Internet.

Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Module 1 Concept or Skill	Rating
Working with and Understanding Large Numbers	
Place value and naming large numbers (1.1)	
Scientific notation (1.6)	
Calculations with large numbers (1.6)	
Relative magnitude and comparison of numbers (1.6)	
Estimation and Calculation	
Rounding (1.1)	
Fractions and decimals (1.3)	
Relationship of multiplication and division (1.4)	
Order of operations (1.4)	

Properties that allow flexibility in calculations: Distributive Property, Commutative Property (1.4)	
Perform multistep calculations (1.7)	
Percentages and Ratios	
Estimations with fraction and percent benchmarks (1.2, 1.3)	
Calculate percentages (1.3)	
Write and understand ratios (1.6)	
Calculate percentages from a two-way table (1.8, 1.9)	
Use percentages as probabilities and ratios (1.9)	

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- 2.9: How Does Your City Stack Up?
  - 2.9.1: Preparation 2.9
- 2.10: Module 2 Review Survey

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2.1: How Crowded Are We?

INTRODUCTION

In this collaboration, you will learn about a way of measuring how crowded the people feel in the place where they live. This measurement is called **population density**.

To help you prepare, think about visiting a shopping mall on a Tuesday at 11:00 AM in May and on a Saturday at 11:00 AM in December. Discuss in your group which day would be more crowded. Come up with some ways that you might measure how "crowded" a place is. What happens if the rooms or places you are considering are different sizes?

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- population density is a ratio of the number of people per unit area.
- proportionality can be used to compare population densities.

By the end of this collaboration, you should be able to

- calculate population densities.
- determine if two population density ratios are proportional to each other.
- compare and contrast populations via their densities.

PROBLEM SITUATION 1: USING RATIOS TO MEASURE POPULATION DENSITY

In Collaboration 1.2, you learned Earth's human population has grown from about 1 billion people to about 7.9 billion in the last two centuries. However, populations in different regions do not always grow uniformly. For example, populations tend to increase in areas where people already live close enough to one another to find mates. On the other hand, crowded populations decrease when deadly diseases, such as smallpox or Ebola virus, sweep through them. In this lesson, you will compare geographic regions by their **population densities**. The population density of a geographic region is a ratio of the number of people living in that region to the area of the region. Population density ratios are typically reported in a way that makes it easy to compare the ratio to a standard area measurement, such as one square mile or one square kilometer.

Example: Imagine 200 people standing in an open field that measures one mile by one mile (also known as one square mile). The population density of the open field could be thought of as 200 people per one square mile. This can be expressed as a ratio. The expression below shows the population density of the open field as a ratio in fraction form.

$$\frac{200\ people}{1\ square\ mile}$$

(1) A **proportion** is the equality of two ratios. Quantities are proportional if they satisfy a proportion. Picture the open field from the example that has a population of 200 people per one square mile. Then, picture a second open field. The length of each side of the second open field is double the length of each side of the first open field (making it 2 miles by 2 miles). The amount of people standing in the second open field is double the amount of people standing in the first open field. Is the population density of the second field equal to the population density of the first field?

(2) An island in the archipelago nation known as the Maldives is threatened by global warming. The island, called the Ari Atoll, is approximately one square mile in size, and has a population of 1,260 people. What is the population density per square mile?

(3) Imagine that global warming caused flooding in Ari Atoll, so the island is now half of its former size. However, the same number of people still live in Ari Atoll.

(a) Before doing any calculations, decide if you think the population density of Ari Atoll increased, decreased, or stayed the same after the flooding. Explain your answer.

(b) Calculate the population density of Ari Atoll after the flood. Try this on your own first before sharing in your group.

(4) Suppose that since everyone is now living closer together, the birth rate on Ari Atoll increases rapidly. Soon the population has quadrupled. What is the population density now?

(5) Picture the open field from the example that contains 200 people and measures one square mile. Now picture another open field that measures five miles by five miles and has exactly the same population density as the first open field.

(a) How many people are standing in this field?

(b) Write a mathematical expression that shows that the population density of the first open field and the population density of the second open field are equal to each other.

(6) Consider a tiny island in the Atlantic Ocean. There are 50 survivors shipwrecked there. When they first arrived on the island, they all set up camp near the waterfall in the middle of the island. About six months later, a professor who was among the survivors claims the island can only support a population density of one person per acre (an acre is 43,560 square feet, which is a little less than the size of an American football field). If the survivors then decide to space their campsites out evenly across the island, how will this affect the overall population density of the island? Discuss with your group.

PROBLEM SITUATION 2: MAKING COMPARISONS WITH POPULATION DENSITY

How crowded is China compared to the United States?

(7) In 2022, in the United States, approximately 333,000,000 people occupied 3,794,000 square miles of land. In China, approximately 1,412,000,000 people lived on 3,705,000 square miles of land. Use this information to answer the following questions.

(a) A student carefully calculates the population densities of China and the United States. He decides that China is less dense than the United States. Using your estimation skills, decide if you think this student's calculation is correct.

(b) At a lecture, you hear someone claim that, in terms of population, China is more than four times as dense as the United States. Using your estimation skills, decide if you think this statement is correct.

(c) Calculate more precisely the densities (per square mile) of the Chinese and U.S. populations. Based on your calculation, how many times more dense is the more crowded population? Be ready to share your calculations during the class discussion.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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2.1.1: Preparation 2.1

(1) The Medical Center in Houston, Texas, is bound by US-59 to the north, I-610 to the west (west loop), I-610 to the south (south loop), and Hwy 288 to the east. The highways create a region that is roughly a three-mile by four-mile rectangle.

A diagram of the highways as described in sentences above.

(a) What are the correct units for the area of the region? There may be more than one correct answer.

(i) Miles

(ii) Square miles

(iii) mi^2

(iv) mi^3

(b) What is the area of the Medical Center?

(2) Which of the following represent(s) the number 8.4 billion? There may be more than one correct answer.

(i) 840,000,000,000

(ii) 8,400,000,000

(iii) 8.4×10^9

(iv) 8.4×10^{10}

(v) 8,400 million

(3) Certain words and phrases indicate that division is required to compute an answer. For example, *per*, *ratio*, and *divide into* indicate division. There are also different symbols for division, including the division sign (÷) and the fraction bar (/). Which of the following does *not* indicate division?

(i) To compare gasoline usage, compute miles per gallon.

(ii) Convert 1/4 to a decimal.

(iii) To compute a bill, total all charges.

(iv) To compute how fast water flows past a meter, compute gallons per minute.

(v) To compute how to share tips among five waiters, compute dollars per person.

(4) You drive 310 miles on 15 gallons of gas. Select the statement(s) that correctly summarize(s) the situation. There may be more than one correct answer.

(i) Your gas mileage is about 15 miles per gallon.

(ii) Your gas mileage is about 20 miles per gallon.

(iii) You can drive a little more than 15 miles on a gallon of gas.

(iv) You can drive a little more than 20 miles on a gallon of gas.

After Preparation 2.1 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 2.1, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
understand the concept of area.	
comprehend numbers up to the billions place.	
use a calculator to divide numbers.	

interpret fractions as division.	
interpret a decimal number.	

2.1.2: Exercise 2.1

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration? The questions refer to the following quantities:

50 people/mi^2	20 ft/5 sec	34%

(i) All three of the figures are ratios.

(ii) The first two figures are ratios.

(iii) The second figure could be written as 4 ft/sec.

(iv) The first figure is a population density.

Writing About Connections Among Mathematical Ideas

In this module you will be asked to write statements about connections among the mathematical ideas in lessons. For this first assignment, you are given statements to choose from. In future assignments, you may want to refer back to this as an example of how to write about connections.

(2) Refer back to Collaboration 1.6, Question 6. Choose the statement that explains how the mathematical ideas in Question 6 connect to the work in Collaboration 2.1.

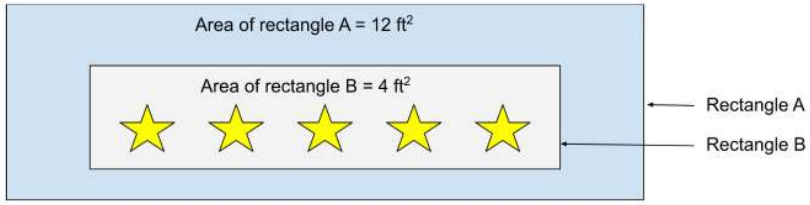
(i) In Collaboration 1.6, you learned about the water footprints of the United States and China. In Collaboration 2.1, you used the population density of the United States and China. Both lessons compared the United States and China.

(ii) In Collaboration 1.6, you had to calculate the water footprint per capita in the United States. This is like splitting up all the water equally among the people. In Collaboration 2.1, you calculated the population density of the United States. This is like splitting up all the people equally over an area of land. These are both ratios.

(iii) In Collaboration 1.6, you used the population of a country to find the water footprint per capita. In Collaboration 2.1, you used the population of a country to find a population density. Both lessons used population in their calculations.

DEVELOPING SKILLS AND UNDERSTANDING

(3) Use the picture below to answer the following questions.


Area of rectangle A = 12 ft^2
Area of rectangle B = 4 ft^2
Rectangle A
Rectangle B

(a) What are the units for the density of the stars in **Rectangle B**? Select the proper units.

(i) stars

(ii) stars/ft

(iii) stars^2/ft

(iv) stars/ft^2

(b) What is the density of the stars in **Rectangle B**? Write your answer as a fraction or as a number rounded to the nearest hundredth. Do not give your answer as a mixed fraction. Include the proper units.

(c) What are the units for the density of the stars in **Rectangle A**? Note that the stars in Rectangle B are also in Rectangle A. Select the proper units.

(i) stars

(ii) stars/ft

(iii) stars^2/ft

(iv) stars/ft^2

(d) What is the density of the stars in **Rectangle A**? Note that the stars in Rectangle B are also in Rectangle A. Write your answer as a fraction or as a number rounded to the nearest hundredth. Do not give your answer as a mixed fraction. Include the proper units.

(e) Suppose three new stars were added in the gray part of Rectangle A. Which of the following statements would be correct?

(i) The density of Rectangle A would increase. The density of Rectangle B would stay the same.

(ii) The density of Rectangle A would decrease. The density of Rectangle B would stay the same.

(iii) The density of Rectangle A would increase. The density of Rectangle B would increase.

(iv) The density of Rectangle A would decrease. The density of Rectangle B would decrease.

(v) The density of Rectangle A would stay the same. The density of Rectangle B would increase.

(4) Wikipedia states the following about Anchorage, Alaska: "The Anchorage metropolitan area … accounts for more than half the state's population."[1]

(a) Calculate the population density for Anchorage, based on a 2020 population of 398,328 people living on 1,968.6 square miles. Round to the nearest person per square mile.

(b) Wikipedia also says that the small Alaskan town of Ketchikan has the densest population in Alaska. Ketchikan had a population of 8,192 in 2020 and a land area of 4.9 square miles. Calculate the population density of Ketchikan. Round to the nearest person per square mile.

(5) The following information comes from the collaboration:

	Population	Land Area (sq. miles)
United States	333,000,000	3,794,000
China	1,412,000,000	3,705,000

In India, about 1,406,500,000 people live on 1,269,000 square miles of land.[2] Which of the following statements is **false**?

(i) The population density of India is approximately three times the population density of China.

(ii) The population density of India is approximately 12 times the population density of the United States.

(iii) The population densities of these three countries ranked from smallest to largest are the United States, India, and China.

(iv) The population density of the United States is approximately 88 people per square mile. That is smaller than the population densities for China and India.

(6) Which of the following population densities are equivalent to a density of 20 people/mi2? There may be more than one correct answer.

(i) 1 person/0.05 mi^2

(ii) 1 person/0.2 mi^2

(iii) 1 person/0.5 mi^2

(iv) 100 people/5 mi^2

(v) 200 people/2 mi^2

(7) One way to measure a country's economy is per capita gross domestic product, or per capita GDP. This is the value of all the products and services produced in a country over the course of a year divided by its population.

(a) According to WorldData.info, in 2021, the United States had a per capita GDP of \$70,249.[3] If the population was about 333 million, which of the following is a reasonable estimate for the GDP of the United States?

(i) $2.3 trillion

(ii) $21 billion

(iii) $21 trillion

(iv) $60 billion

(v) $60 trillion

(b) Also according to WorldData.info, in 2021, China had a per capita GDP of $12,556 and a population of around 1,412,000,000. Which of the following is a reasonable estimate for the GDP of China?

(i) $1.3 trillion

(ii) $17 billion

(iii) $17 trillion

(iv) $50 billion

(v) $50 trillion

MAKING CONNECTIONS ACROSS THE COURSE

(8) In April 2020, the population of Nebraska was 1,961,504, and its population density was 25.5 people per mi^2. The population of New Hampshire was 1,377,518, and its population density was 153.9 people per mi^2.[4] Which of the following statements is a valid interpretation of this information?

(i) Nebraska is approximately six times more densely populated than New Hampshire.

(ii) The population of Nebraska is approximately 71% of the population of New Hampshire.

(iii) New Hampshire is approximately six times more densely populated than Nebraska.

(iv) The population of New Hampshire is approximately 141% of the population of Nebraska.

(9) Cholera, a serious intestinal disease, broke out in London in the mid-19th century. Research by a physician named John Snow indicated that the disease was caused by germs in polluted water. A vaccine for cholera was then developed in the late 1800s. A vaccine is a drug that helps a person become immune to a disease. Scientists studied 818 people to determine the effects of a cholera vaccine.[5] The study lasted from 1894 to 1896.

	Infected	Not Infected
Vaccinated	3	276
Not Vaccinated	66	473

Use the information in the table to complete the following four statements. Round to the nearest tenth of a percent.

(a) ______% of those vaccinated were infected

(b) ______% of those not vaccinated were infected

(c) ______% of those infected were vaccinated

(d) ______% of those infected were not vaccinated

(e) Which of the following statements are correct based on the information in the table? There may be more than one correct answer.

(i) A person who was not vaccinated was 12 times more likely to get cholera than someone who got the vaccine.

(ii) A person who was not vaccinated was four times more likely to get cholera than someone who got the vaccine.

(iii) If a person was infected, it was four times more likely that (s)he was *not* vaccinated rather than vaccinated.

(iv) If a person was infected, it was more than 20 times more likely that (s)he was *not* vaccinated rather than vaccinated.

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[1] http://en.Wikipedia.org/wiki/Anchorage,_Alaska

[2] http://www.indiastat.com/default.aspx

[3] https://www.worlddata.info/largest-economies.php

[4] https://www.census.gov/quickfacts

[5] Major Greenwood and G. Udny Yule, “The Statistics of Anti-Typhoid and Anti-Cholera Inoculations, and the Interpretation of such Statistics in general,” *Proceedings of the Royal Society of Medicine* 8 (1915): 113-194.

2.1.3: Study Strategy Survey part 1

Study Strategies for Exams and Quizzes (Part 1)

We often do not stop to think about the study strategies we try; however, thinking about how you study and whether your study routine was effective can help your learning. This survey is designed to help you reflect upon your study habits. Answer the questions below to reflect upon the study habits you used for the Module 1 exam. If you did not complete a Module 1 exam, think about your most recent exam during another course.

1. What are some approaches to studying you tried for this exam?
2. What strategies were most effective and why?
3. What are some barriers to your studying?

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2.2: Shopping Smart

INTRODUCTION

With your group, share your thoughts about the following questions:

- Can you think of a time when you compared prices of items from different stores?
- Are larger quantities always a better deal?
- Can you think of a time when it is better to purchase in larger quantities?
- Can you think of a time when buying a larger quantity is not a good idea?

Quick Check-in:

Please discuss the following questions with your group:

- What specific concept from Preparation 2.1 do you feel fairly confident about?
- Are there any concepts from Preparation 2.1 that you find challenging?

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand

- the concept of proportional pricing in shopping for a variety of grocery items.
- how different sizes of the same product may not mean the same cost per unit weight.
- how a dollar can have different value (buying power) in different situations.

By the end of this collaboration, you should be able to

- calculate a unit rate.
- solve a proportion by first finding a unit rate and then multiplying appropriately.
- compare changes of unit prices even when the unit prices themselves are not given and cannot be found.

PROBLEM SITUATION 1: PURCHASING PRODUCTS PRICED BY WEIGHT

Georgia and Juan are planning a party for the weekend. Since they are college students, the amount that they can spend on food and paper products is limited, but they want to have the best party possible. Living in a metro area, they are able to go to a variety of grocery stores and compare prices. They have decided to pick Kroger and Byler's to compare prices for the products that they need. They have asked other students to bring some food and paper products to the party. Georgia and Juan are responsible for providing the chips and watermelon.

(1) Georgia went to Kroger to price Doritos® chips and found that she could buy a 10.5 ounce bag for $4.29. What is the price per ounce for these chips at Kroger?

(2) Juan went to Byler's and found a 13 ounce bag of Doritos chips for $5.12. What is the price per ounce for these chips at Byler's?

(3) From which store should they purchase their Doritos chips to get the best deal?

(i) Byler's

(ii) Kroger

Note: In this case, the price per ounce is known as the **unit price**. The unit price is a type of **unit rate**, which is used to more easily compare things of varying sizes of magnitude.

(4) Georgia wants to get the most chips for her money and noticed that the store also sells bags in three other sizes. If the price per ounce for chips in the other bags was the same as for the chips in the 10.5 ounce bag, how much would she expect to pay for each of the sizes below? **Note**: Discuss in your group before recording your answers in the spaces provided.

(a) 16.5 oz (party size) bag?

(b) 13.5 oz bag?

(c) 1.5 oz (individual serving size) bag?

(5) Georgia then noticed that the actual prices for the other bags were: $6.00 for the 16.5 oz bag, $5.40 for the 13.5 oz bag, and $1.00 for the 1.5 oz bag. How do these prices compare to her expectations and what does this indicate?

(6) Imagine the company that makes Doritos decides to change the size of the bag from 10.5 ounces to 12.5 ounces, but keeps the price ($4.29) the same.

(a) Will the unit price (price per ounce) remain the same for the new bag? If not, what will the new unit price be?

(b) How could the company ensure that the unit price stays the same after the change to a different bag size?

(7) Kroger and Byler’s used to sell a 10.5 oz bag of blue corn chips for the same price. Right before New Year’s, Kroger increased the price of this bag by 50% while Byler’s doubled the number of ounces in the bag and kept the same price. How are the new unit prices of blue corn chips at Kroger and Byler’s related?

PROBLEM SITUATION 2: PURCHASING PRODUCTS PRICED BY ITEM

Shopping for the best deal is different when buying products that are priced per item. Georgia wants to buy watermelon for the party and sees that each watermelon at Kroger costs $5.00, no matter how big it is.

(8) Do you think that the price per pound (or the ratio of dollars to pounds) is the same for every watermelon at Kroger? Explain your answer.

(9) Georgia selected three different watermelons from the Kroger bin and took them to a scale. She found that watermelon A weighed 6 pounds, watermelon B weighed 6.5 pounds, and watermelon C weighed 7.5 pounds. What is the price per pound for watermelons A, B, and C? Round your answers to the nearest cent. Remember that each watermelon was priced at $5.00.

(a) Watermelon A:

(b) Watermelon B:

(c) Watermelon C:

(10) Juan went to Byler’s, where he found that watermelons cost $0.76 per pound.

(a) How much would a 6 pound watermelon cost? Round your answer to two decimal places.

(b) How much would a 6.5 pound watermelon cost? Round your answer to two decimal places

(c) How much would a 7.5 pound watermelon cost? Round your answer to two decimal places

(d) If a watermelon costs the same at Byler's (price per pound) and at Kroger (price per watermelon), how much does it weigh?

(e) Using your answer from the previous question, would a 7 pound watermelon be less expensive at Byler's (price per pound) or at Kroger (price per watermelon)?

(i) Byler’s

(ii) Kroger

(11) The **buying power** of a dollar is the amount of goods or services a dollar can buy in a specific situation. Compare the relative buying power of a dollar with respect to the various watermelons A, B, and C at Kroger. Think about this on your own first before sharing your ideas with your group. Write your answer as a sentence using clear and specific language.

(12) One way of measuring the buying power of your money is calculating how much you can buy with one dollar. How can Georgia maximize the buying power of her money at Kroger? By buying lighter or heavier watermelons? Remember that all watermelons had the same price ($5). Explain your answer.

(13) How much watermelon per dollar does Georgia get when she buys watermelons A, B, and C?

___________ pounds per dollar when she buys watermelon A

___________ pounds per dollar when she buys watermelon B

___________ pounds per dollar when she buys watermelon C

(14) How are the results you obtained in Question 9 related to the ones you just obtained in Question 13?

FURTHER APPLICATIONS

Georgia and Juan have $40 to spend on their contribution of chips and watermelon to the party. They estimate that 30 people will attend the party. Since they want to get the most that they can for their money, they have decided to buy the 13.5 oz bag of Doritos chips for $4.99 and the largest watermelons they can find where the watermelons are priced at $5.00.

(15) According to one chip company's recommendations, for a good party, you should buy about 1.5 ounces of chips per person.

(a) If Georgia and Juan buy the 13.5 oz bags of chips, how many people will one bag feed?

(b) How many bags will be needed for the 30 people at the party?

(c) What will be the total cost for these bags of chips?

(d) How much money is left to spend on the watermelons?

(16) Georgia and Juan figure that one of the larger watermelons would feed 8 people. Each watermelon costs $5.00.

(a) How many watermelons will they need for 30 people?

(b) What will be the total cost of the watermelons?

(c) Will they have enough money to purchase the watermelons?

(d) Will they have any money left from the budget that they had set up of $40? If so, how much will they have left?

(17) In 2020, in the laundry room at Ramona's apartment building, a person could do one load of washing for $1.60. This wash lasted 40 minutes. This year, the landlord raised both the price of washing one load and the amount of washing minutes by 15%. Did the price of a minute of washing increase, decrease or remain the same? Justify your answer.

(18) At Ramona's building, the price of drying clothes for 56 minutes is $2.00. At Peter's building, one can buy drying intervals of 10 minutes for 40 cents each.

(a) Where is it more expensive to dry clothes? Justify your answer.

(b) How much would the landlord have to charge at Ramona's building for 56 minutes of drying if he or she wanted to make it as cheap (or as expensive) as at Peter's building?

(c) Debbie wants to dry her comforter for a whole hour. Where can she do it more cheaply, at Peter's building or at Ramona's? Justify your answer.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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2.2.1: Preparation 2.2

(1) The following problems have to do with multiplying and dividing by powers of 10. Look for patterns and ways to find the answers mentally without a calculator or writing the problem down. Check your answers with a calculator if you wish.

(a) 0.32 × 10

(b) 3.2 × 10

(c) 32 × 10

(d) 32 × 100

(e) 51,000 × 10,000

(f) 900×10^4

(g) 1.3×10^7

(h) 0.32 ÷ 10

(i) 3.2 ÷ 10

(j) 3,200,000 ÷ 10

(k) 5,500,000 ÷ 1,000

(l) 83,000,000 ÷ 10,000,000

(m) $67 \div 10^4$

(2) When you multiply 58,000,000,000 × 10,000, the calculator display will show: 5.8 E14. Which of the following represent(s) the same number as the number displayed on your calculator? There may be more than one correct answer.

(i) 5.8 × 14

(ii) 5.8×10^{14}

(iii) 58×10^{14}

(iv) 580,000,000,000,000

(v) 5,800,000,000,000,000

(3) The United States Department of Agriculture (USDA) publishes a “National Fruit and Vegetable Retail Report” every week.[6] This shows the national average costs of fruits and vegetables from store advertisements. The weekly report on January 20, 2023 showed the following prices for Golden Delicious apples:

Commodity	Variety	Unit	Price (in US$)
Apples	Golden Delicious	per pound	1.29
Apples	Golden Delicious	3 lb bag	3.83

If these were the prices at your local grocery store, which would cost you more?

(i) Three pounds of Golden Delicious apples, bought per pound.

(ii) A 3-lb bag of Golden Delicious apples.

(iii) They would cost the same amount.

(4) The same report gave the prices for navel oranges:

Commodity	Variety	Unit	Price (in US$)
Oranges	Navel	each	0.90
Oranges	Navel	per pound	1.32

Oranges	Navel	4 lb bag	4.95

If these were the prices at your local grocery store, which purchase would cost the least?

(i) Four navel oranges.

(ii) Four pounds of navel oranges, bought per pound.

(iii) A 4-lb bag of navel oranges.

(iv) All three would cost the same amount.

After Preparation 2.2 (survey)

You should be able to do the following things for the next class. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Unit 2.2, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
calculate a unit rate.	
make general sense of proportionality.	

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[6] http://www.ams.usda.gov/mnreports/fvwretail.pdf

2.2.2: Exercise 2.2

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) There are two good places to buy watermelon: Byler's and Kroger.

(ii) The cost of a bag of chips depends on how big it is.

(iii) All chips cost the same in terms of price per ounce.

(iv) The cost of chips per ounce represents a unit rate found by dividing the cost of the bag by the number of ounces it weighs.

DEVELOPING SKILLS AND UNDERSTANDING

(2) In this collaboration, you used ratios:

$$\frac{cost\ of\ chips\ in\ dollars}{ounces\ of\ chips} = price\ of\ chips\ per\ ounce$$

$$\frac{weight\ of\ watermelon}{price\ of\ watermelon} = weight\ of\ watermelon\ per\ dollar$$

Which of the following are ratios? There may be more than one correct answer.

(i) 252 miles

(ii) 67 hours

(iii) 10 miles/hour

(iv) 5 pounds/$3

(v) $98

(3) Calculate the gas mileage of a car that drives 283 miles on 12.3 gallons of gas. Round to the nearest mile per gallon.

(4) A car drives 630 miles on 35 gallons of gas. How far can it drive on 12 gallons?

(5) A jar holds 128 fluid ounces of juice. The label says the jar has 16 servings. How many fluid ounces are needed for 80 servings?

(6) According to the oil company BP, in 2021, the United States used 18,684,000 barrels of oil a day, and worldwide, people used around 94,088,000 barrels of oil per day.[7] This includes oil used for (among other things) fuel and manufacturing.

(a) If there were 333 million people in the United States in 2021, what was the daily consumption rate per capita (barrels per person) in the United States? Round to the nearest hundredth of a barrel.

(b) If there were 7.89 billion people in the world in 2021, which of the following statements would be correct? The U.S. rate of oil consumption per capita was about:

(i) 2 times the world rate.

(ii) 5 times the world rate.

(iii) 10 times the world rate.

(iv) 50 times the world rate.

(c) There are 42 gallons in a barrel of oil. Which of the following statements is true? The average American is responsible for about:

(i) 0.5 gallons of oil use per day.

(ii) 2.5 gallons of oil use per day.

(iii) 5 gallons of oil use per day.

(iv) 50 gallons of oil use per day.

MAKING CONNECTIONS ACROSS THE COURSE

(7) People often confuse the words *million*, *billion*, and *trillion* when speaking. An estimate can help you decide if the speaker uses the correct word. Consider this situation: A speaker says, “The U.S. federal debt is $18 billion dollars. That’s over $56,000 for every person in the country.”

Select the correct statement from the choices below. (**Note**: When you say the numbers are *consistent*, you mean that they make sense in relation to each other. Assume there are approximately 321 million people in the U.S.)

(i) The two numbers in the statement are consistent with each other.

(ii) The two numbers in the statement are not consistent. If the debt is $56,000 per capita, the total debt must be $18 million.

(iii) The two numbers in the statement are not consistent. If the debt is $56,000 per capita, the total debt must be $18 trillion.

(8) Terrence is very careful about tracking his gas mileage. Every time he fills his gas tank, he records how much gas he buys and the number of miles he has driven. He puts this information into a spreadsheet so he can easily calculate his gas mileage in miles per gallon.

Screen capture of data in an excel sheet, indicating the following in cells: A1 = Number of Miles B1 = Gallons of Gas A2 = 255 B2 = 13.2 A3 = 136 B3 = 5.9 A4 = 297 B4 = 13.5 A5 = 114 B5 = 5.3

(a) Select the formula that would calculate Terrence’s gas mileage.

(i) = (A2 + A3 + A4 + A5) / (B2 + B3 + B4 + B5)

(ii) = A2 + A3 + A4 + A5 / B2 + B3 + B4 + B5

(iii) = (B2 + B3 + B4 + B5) / (A2 + A3 + A4 + A5)

(iv) = B2 + B3 + B4 + B5 / A2 + A3 + A4 + A5

(b) Terrence is planning a long road trip of about 1,000 miles. The average price of gas is $3.85/gallon. Based on the data in the spreadsheet, calculate how much he should budget for gas. Round to the nearest dollar.

(9) The following table was created to compare two internet service plans, Plan A and Plan B. The monthly bill for each plan was computed based on the expected download speeds. While the actual monthly bills are not given, their differences are given in the last column.

Data (Mb)	Plan A	Plan B	Plan A – Plan B
100	*	*	−7.5
500	*	*	−5
750	*	*	−2.5
1000	*	*	0
2000	*	*	10.5

(a) The first entry in the last column is −7.5. Which of the following statements explains what this tells you about Plan A and Plan B?

(i) Plan A costs $7.50 more than Plan B for someone who has a 100Mb plan.

(ii) Plan B costs $7.50 more than Plan A for someone who has a 100Mb plan.

(iii) The Plan A customer used the internet 7½ fewer Mb than the Plan B customer.

(iv) The Plan B customer used the internet 7½ fewer Mb than the Plan A customer.

(b) Further down the last column is the entry 10.5. What does this tell you about Plan A and Plan B? Write your answer as a complete sentence using the Writing Principle.

(c) When are the two plans equal? Write your answer as a complete sentence using the Writing Principle.

[7] https://www.bp.com/content/dam/bp/business-sites/en/global/corporate/pdfs/energy-economics/statistical-review/bp-stats-review-2022-full-report.pdf

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2.3: Measuring Population Change

INTRODUCTION

The table below shows the apportionment of representatives (seats) after the 2020 census. Make note of the table legend and the colors used for the states, discuss with your group the questions below.

Legend:	Gained two seats Gained one seat No change **Lost one seat**

State	Seats		State	Seats		State	Seats		State	Seats		State	Seats
AK	1		HI	2		ME	2		NJ	12		SD	1
AL	7		IA	4		**MI**	**13**		NM	3		TN	9
AR	4		ID	2		MN	8		NV	4		TX	38
AZ	9		**IL**	**17**		MO	8		**NY**	**26**		UT	4
CA	**52**		IN	9		MS	4		**OH**	**15**		VA	11
CO	8		KS	4		MT	2		OK	5		VT	1
CT	5		KY	6		NC	14		OR	6		WA	10
DE	1		LA	6		ND	1		**PA**	**17**		WI	8
FL	28		MA	9		NE	3		RI	2		**WV**	**2**
GA	14		MD	8		NH	2		SC	7		WY	1

- Which states have the largest populations?
- Which states have small populations?
- Which states are growing fastest and growing slowest?
- What does it mean to be growing fastest in terms of representation?

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- a relative change is different from an absolute change.
- a relative measure is always a comparison of two numbers.

By the end of this collaboration, you should be able to

- calculate a relative change.
- explain the difference between relative change and absolute change.

PROBLEM SITUATION: HOW THE CENSUS AFFECTS THE HOUSE OF REPRESENTATIVES

Every 10 years, the United States conducts a census. The census tells us how many people live in each state. You can also find out how much populations have changed over time from the census data. The original purpose of the census was to decide on the number of representatives each state would have in the House of Representatives. Census data continue to be used for this purpose, but now there are many other uses for the data. For example, governments may use the data to plan for public services such as fire stations and schools.

In this problem situation, your instructor will ask your team to explore data from one of two census regions: 1) South Atlantic States or 2) Mountain States. There are tables on the next page that show the population of each state in these census regions in 2010 and 2020. You will be asked to calculate the population growth (in people) as a percentage for each state in your assigned region, and for the region as a whole. You will examine how this affects the number of representatives each state has in the House of Representatives.

The **absolute change** in a state's population tells us by how many people the population has changed. The **relative change** is the change as it compares to the earlier population. Often relative change is given as a percentage. See the table below for an example using the population change in California from 2010 to 2020.

	2020 Population (in millions)	2010 Population (in millions)	Absolute Change (in millions)	Relative Change (in percent form)
California	39.58	37.27	$39.58 - 37.27 = 2.31$	$2.31 \div 37.27 * 100\% = 6.2\%$

Use the following data to answer Questions 1–6.

South Atlantic States

	2020 Population	2010 Population	Absolute Change	Relative Change (in percent form)
Florida	21,570,527	18,801,310		
Georgia	10,725,274	9,687,653		
North Carolina	10,453,948	9,535,483		

Mountain States

	2020 Population	2010 Population	Absolute Change	Relative Change (in percent form)
Arizona	7,158,923	6,392,017		
Colorado	5,782,171	5,029,196		
Nevada	3,108,462	2,700,551		

(1) Calculate the absolute change in the population of each state. Write your answers in the appropriate table.

(2) Calculate the relative change in the population of each state. Write your answers in percent form in the table above.

(3) List the state in each region that changed the most in absolute population.

(4) List the state in each region that had the largest relative increase in population.

(5) Explain why the lists in Question 3 and Question 4 are not the same.

(6) Use the previous information to calculate the following absolute and relative changes in total population. Do the calculations on your own first before sharing in your group.

(a) The absolute change in total population for the three Mountain Region states.

(b) The relative change in total population for the three Mountain Region states.

(c) The absolute change in total population for the three South Atlantic Region states.

(d) The relative change in total population for the three South Atlantic Region states.

(7) Let's now consider a state we have not explored yet: Illinois. Interestingly, the population of the state of Illinois actually decreased between 2010 and 2020. Illinois had a population of 12,864,380 in 2010. Its population was 12,822,739 in 2020.

(a) What was the absolute change in Illinois' population?

(b) What was the relative change in Illinois' population? Write a statement that describes the relative change in Illinois' population between 2010 and 2020. Think about this on your own first before sharing with your group. Round your answer to the nearest hundredth of a percent.

FURTHER APPLICATIONS

(8) In 2019, the U.S. Congress debated cutting the federal budget. The goal was to reduce the national debt, which was $23 trillion.

(a) If one group wanted to reduce the budget by $500 billion. How large is this change relative to the national debt?

(b) If another group wanted to reduce the budget by $200 billion. How large is this change relative to the national debt?

(c) If a politician wanted to argue for the larger cut, do you think the politician should use the absolute or the relative change to justify his or her position? Why?

(d) If a politician wanted to argue for the smaller cut, do you think the politician should use the absolute or the relative change to justify his or her positions? Why?

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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2.3.1: Preparation 2.3

(1) Calculate the following values.

(a) What percent of 20 is 2?

(b) 28 is what percent of 80?

(c) What percent is 18/24?

(2) A school principal of 810 students needs to determine what percent of students passed and did not pass a statewide examination. Round to the nearest percent.

(a) If 550 students passed the exam, what percent passed the test?

(b) What percent did not pass the test?

(3) A laptop computer that you want to purchase was originally priced at $1,225. You will receive a 20% student discount, and the sales tax rate is 8%. How much money will you pay for the laptop?

After Preparation 2.3 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 2.3 you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can...	**Rating from 1 to 5**
find a percent of a number.	

Self-Regulated Learning: Reflect

How much time and effort per week is this course taking? Is it what you expected? Is there anything you need to adjust in your weekly schedule to make sure you are successful?

2.3.2: Exercise 2.3

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Absolute change is measured as a quantity (for example, an increase of \$3). Relative change is measured as a percentage compared to the reference value (for example, an increase of 3%).

(ii) To find relative change, subtract the original number from the new number and divide by the original.

(iii) Consider this situation: Quantity 1 increases by 15%. Quantity 2 increases by 20%. Quantity 2 must have increased by a larger amount than Quantity 1.

(iv) The population of a state determines how many representatives it has in the House of Representatives.

DEVELOPING SKILLS AND UNDERSTANDING

(2) The following headlines all refer to change. Identify the change as *absolute* or *relative*.

(a) “Enrollments at Northeastern University are expected to increase by 1,500!”

(i) Absolute change

(ii) Relative change

(b) “Another 14% tuition increase is expected.”

(i) Absolute change

(ii) Relative change

(c) “A new proposal has sales tax rates dropping from 3% to 1%, a drop of only 2%.”

(i) Absolute change

(ii) Relative change

(d) “A new proposal has sales tax rates dropping from 3% to 1%, a 67% drop!”

(i) Absolute change

(ii) Relative change

(3) The two questions below refer to data taken from the U.S. Census.[8] The dollar values take into account the changes in the economy over the years (i.e., inflation).

(a) A typical high-income household in 1990 earned \$191,028. A similar household in 2019 earned \$286,138. What was the relative increase in income for these households from 1990 to 2019? Round to the nearest one percent.

(b) A typical middle-income household in 1990 earned \$47,707. A similar household in 2019 earned \$56,700. What was the relative increase in income for these households from 1990 to 2019? Round to the nearest one percent.

(4) After the U.S. Congress passed temporary tax cuts in 2010, persons in the U.S. who earned \$50,000 per year had a lower tax rate. On average, these persons retained 2% more of their income than before the cuts. The 2010 law also gave the same persons a separate \$850 credit on their federal tax return.

(a) How much extra money did a typical person earning \$50,000 receive due to the federal tax cuts?

(b) This extra money represents what percent of the typical person’s salary? Round to the nearest tenth of a percent.

(5) Due to the same 2010 law, persons earning \$500,000 per year were taxed at a lower rate on the first \$106,800 they earned, saving them 2% of this portion of their salary. The same persons also received \$14,250 more on their federal tax return. Fill in the blanks to complete the statement below.

(a) A person earning \$500,000 a year received \$___________ more of their income. Round to the nearest dollar.

(b) A person earning \$500,000 a year received ___________% more of their income. Round to the nearest tenth of a percent.

MAKING CONNECTIONS ACROSS THE COURSE

(6) In Collaboration 1.4, you used results from the 2020 Consumer Expenditure Survey on how Americans spend their income. A summary of this information is given in Table 1.[9]

Table 1: Percentages of Average Annual Household Expenditures

Housing	34.91%
Food	11.93%
Transportation	16.02%
Everything Else	37.14%

Which pie chart best represents the data given in Table 1?

(i) Circle graph showing the average annual household expenditures, shown from highest to lowest: - Everything else - Housing - Transportation - Food	(iii) Circle graph showing the average annual household expenditures, shown from highest to lowest: - Housing - Everything else - Transportation - Food
(ii) Circle graph showing the average annual household expenditures, shown from highest to lowest: - Everything else - Housing - Food - Transportation	(iv) Circle graph showing the average annual household expenditures, shown from highest to lowest: - Housing - Everything else - Food - Transportation

(7) Many egg producers keep chickens in small cages that do not allow the chickens to move. Some people believe that this is unhealthy, so they buy eggs from chickens that are not caged 24-hours a day. These are sometimes called “free-range” chickens. The United States Department of Agriculture (USDA) allows chickens to be called free-range as long as the chickens spend some of their time outside. The European Union (EU), however, has several additional restrictions. One of these is that the farmers must provide enough outside area so that if all the chickens were outside, the density of chickens would be no more than 0.25 chickens/sq meter.[10]

(a) How many square meters does the EU require for one chicken?

(b) A farmer in the United States wants to meet the EU guidelines. She measures her area in yards (1 sq meter = 1.196 sq yards). How many square yards does she need for 1,100 chickens?

(i) 301.1 square yards

(ii) 4,400.0 square yards

(iii) 4,812.2 square yards

(iv) 5,262.4 square yards

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[8] https://www.census.gov/data/tables/time-series/demo/income-poverty/historical-income-households.html

[9] https://www.bls.gov/opub/reports/consumer-expenditures/2020/home.htm

[10] http://en.Wikipedia.org/wiki/Free_range_eggs

2.4: Picturing Data with Graphics

INTRODUCTION

You saw the graph below in Preparation 2.4:

Line graph titled Average Household Income (2021 Dollars). X-axis is Years ranged from 2010 to 2022 in increments of 2. Y-axis is Average Household Income ranged from $60,000 to $74,000 in increments of $2,000. At 2011, income is around $60,000. At 2012, income is around $60,500. At 2013, income is around $62,500. At 2014, income is around $62,000. At 2015, income is around $65,000. At 2016, income is around $67,000. At 2017, income is around $68,000. At 2018, income is just over $68,000. At 2019, income is around $73,000. At 2020, income is just under $72,000. At 2021, income is about $71,000.

Based on the graph, discuss the following questions in your group:

- How much did the Average Household Income increase from 2011 to 2019?
- How much did the Average Household Income decrease from 2019 to 2021?

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- the scale on graphs can change the perception of the information they represent.
- to fully understand a pie chart, the reference value must be known.

By the end of this collaboration, you should be able to

- calculate relative change from a line graph.
- estimate the absolute size of the portions of a pie chart given its reference value.
- use data displayed on two graphs to estimate a third quantity.

PROBLEM SITUATION 1: READING LINE GRAPHS

Compare Graph 1 (from Preparation 2.4) and Graph 2 below.

Graph 1

Line graph titled Average Household Income (2021 Dollars). X-axis is Years ranged from 2010 to 2022 in increments of 2. Y-axis is Average Household Income ranged from $60,000 to $74,000 in increments of $2,000. At 2011, income is around $60,000. At 2012, income is around $60,500. At 2013, income is around $62,500. At 2014, income is around $62,000. At 2015, income is around $65,000. At 2016, income is around $67,000. At 2017, income is around $68,000. At 2018, income is just over $68,000. At 2019, income is around $73,000. At 2020, income is just under $72,000. At 2021, income is about $71,000.

Graph 2

Line graph titled Average Household Income (2021 Dollars). X-axis is Years ranged from 2010 to 2022 in increments of 2. Y-axis is Average Household Income ranged from $0 to $74,000 in increments of $10,000. At 2011, income is around $60,000. At 2012, income is around $60,500. At 2013, income is around $62,500. At 2014, income is around $62,000. At 2015, income is around $65,000. At 2016, income is around $67,000. At 2017, income is around $68,000. At 2018, income is just over $68,000. At 2019, income is around $73,000. At 2020, income is just under $72,000. At 2021, income is about $71,000.

(1) What do you notice about the two graphs?

(2) Based on these two graphs, would it be fair to say that the average household income was significantly higher in 2021 than it was in 2011?

PROBLEM SITUATION 2: READING BAR GRAPHS

In Preparation 2.4, you were given two pairs of statements about Jeff's housing expenditures. The statements are below for reference. You were asked to consider how both pairs of statements could be true. You were also asked to determine when Jeff spent more on housing. Your class will now discuss how understanding the answers to these questions can help you understand the gross domestic product (GDP) of a country. The gross domestic product is the value of all goods that are produced in a country in a certain time period, typically one year.

Pair 1	Pair 2
In 2000, Jeff spent $900 per month on housing. In 2020, Jeff spent $1,800 per month on housing.	In 2000, Jeff spent 20% of his income on housing. In 2020, Jeff spent 10% of his income on housing.

(3) Share your ideas from Preparation 2.4 with your group. Record two examples below.

(4) Think about the statement, "The 2020 military budget is way out of hand and has never been higher." You will use Graphs 3 and 4 to evaluate the statement.[12]

Graph 3

Vertical bar graph showing the US National Debt Trillions of Dollars from 1960 to 2020. Y-axis represents the amount in billions of dollars, ranging from 0 to 900. X-axis represents the years, ranging from 1960 to 2050. 1960 = just over 0 1970 = in between 0 and 100 1980 = in between 100 and 200 1990 = just over 300 2000 = about 300 2010 = in between 700 and 800 2020 = just under 800.

Graph 4

Vertical bar graph showing the US National Debt as a percent of GDP from 1960 to 2020. Y-axis represents the percent of GDP, ranging from 0 to 10. X-axis represents the years, ranging from 1960 to 2020. 1960 = about 9 1970 = about 8 1980 = about 5 1990 = about 6 2000 = about 3 2010 = about 5 2020 = about 4

(4) Is the statement true? Based on what information?

PROBLEM SITUATION 3: READING PIE (CIRCLE) CHARTS

Refer to the graphs below,[13,14] to answer Questions 5 and 6. This pair of graphs cannot be used to predict that the number of non-Hispanics in the United States is expected to decline between 2020 and 2060.

Circle graph showing the U.S. Hispanic Population for 2020. Hispanic = 19% Not Hispanic = 81%	Circle graph showing the Projected U.S. Hispanic Population for 2060. Hispanic = 28% Not Hispanic = 72%

(5) The U.S. population in 2020 was around 330,000,000. In 2060, the U.S. population is expected to be around 404,000,000. Estimate the number of Hispanic and non-Hispanic Americans at each time.

(6) Does your work in Question 5 confirm or contradict the notion that this pair of graphs cannot be used to predict that the number of non-Hispanics in the United States is expected to decline between 2020 and 2060? Explain.

FURTHER APPLICATIONS

(7) Write two different statements that compare the U.S. Hispanic population in 2020 to the projected U.S. Hispanic population in 2060.

Write your two statements here:

1.

2.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[12] https://www.macrotrends.net/countries/USA/united-states/military-spending-defense-budget

[13] https://www.census.gov/quickfacts/fact/table/US/PST045221

[14] https://www.census.gov/content/dam/Census/library/publications/2020/demo/p25-1144.pdf

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2.4.1: Preparation 2.4

Data are increasingly presented in a variety of forms intended to interest you and invite you to think about the importance of the data and how they might affect your lives. The following are some of the types of common displays:

- Pie charts
- Scatterplots
- Histograms and bar graphs
- Line graphs
- Tables
- Pictographs

What questions should you ask yourself when you study a visual display of information?

- What is the title of the chart or graph?
- What question is the data supposed to answer? (For example: How many males versus females exercise daily?)
- How are the columns and rows labeled? How are the vertical and horizontal axes labeled?
- Select one number or data point and ask, “What does this mean?”

Use the following chart to help you understand what some basic types of visual displays of information tell you and what questions they usually answer.

This looks like a …	This visual display is usually used to …	For example, it can be used to show …
pie chart	• show the relationships between different parts compared to a whole.	• how time is used in a 24-hour cycle. • how money is distributed. • how something is divided up.
line graph	• show trends over time. • compare trends of two different items or measurements.	• what is increasing. • what is decreasing. • how the cost of gas has increased in the last 10 years. • which of these foods (milk, steak, cookies, eggs) has risen most rapidly in price compared to the others.
histogram or bar graph	• compare data in different categories. • show changes over time.	• how a population is broken up into different age categories. • how college tuition rates are changing over time.
table	• organize data to make specific values easy to read. • break data up into overlapping categories.	• the inflation rates over a period of years. • how a population is broken into males and females of different age categories.

You will use the following information and your responses to Questions 9 - 11 in your next collaboration. Be sure to take notes.

(1) Use Graph 1 to answer the following questions.[11] (Note: The vertical axis starts at $60,000 instead of $0.)

Graph 1

Line graph titled Average Household Income (2021 Dollars). X-axis is Years ranged from 2010 to 2022 in increments of 2. Y-axis is Average Household Income ranged from $60,000 to $74,000 in increments of $2,000. At 2011, income is around $60,000. At 2012, income is around $60,500. At 2013, income is around $62,500. At 2014, income is around $62,000. At 2015, income is around $65,000. At 2016, income is around $67,000. At 2017, income is around $68,000. At 2018, income is just over $68,000. At 2019, income is around $73,000. At 2020, income is just under $72,000. At 2021, income is about $71,000.

(a) Select the best phrase to complete this sentence: The numbers on the **horizontal axis** (the line across the bottom of the graph) represent…

(i) income from $2,010 to $2,022.

(ii) income from $2,010,000 to $2,022,000.

(iii) years from 2010 to 2022.

(iv) years and income.

(b) Select the best phrase to complete this sentence: The numbers on the **vertical axis** (the line along the left of the graph) represent…

(i) years from $60,000 to $74,000.

(ii) income for a household or family from $60,000 to $74,000.

(iii) income for one person from $60,000 to $74,000.

(iv) income in 2021 from $60,000 to $74,000.

(c) A good estimate of the average household income in 2016 is

(i) $66,000

(ii) $66,200

(iii) $66,800

(iv) $67,300

(d) A good estimate of the average household income in 2012 is

(i) $60,000

(ii) $60,500

(iii) $61,000

(iv) $61,400

(e) Use the estimates from (c) and (d) to calculate the relative change in the average household income from 2012 to 2016. Round to the nearest one percent. Indicate if the change is an increase or decrease.

(i) The relative change is…

(ii) The change is…

(2) Two pairs of statements about Jeff's housing expenditures are given below. How can both pairs of statements be true? When did Jeff spend more on housing? Be prepared to discuss your answers in class.

Pair 1	Pair 2
In 2000, Jeff spent $900 per month on housing. In 2020, Jeff spent $1,800 per month on housing.	In 2000, Jeff spent 20% of his income on housing. In 2020, Jeff spent 10% of his income on housing.

After Preparation 2.4 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 2.4, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	Rating from 1 to 5
read a line graph.	
read a bar graph.	
read a pie graph.	
calculate relative change.	

Self-Regulated Learning: Reflect

Which problems from this lesson do you feel you understood well? Which ones might you find it beneficial to talk to your teacher or someone else about?

When planning to do this homework assignment, how accurately did you predict how long the assignment was going to take?

Name two strategies you used when solving problems in this assignment.

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[11] https://www.census.gov/data/tables/time-series/demo/income-poverty/historical-income-households.html

2.4.2: Exercise 2.4

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) It is important to consider the gross national product when considering the size of the national debt. They relate to each other just as a person’s personal debt relates to his/her income.

(ii) In Graph A below, you can find the quantity represented by “Part A1” by multiplying the percentage for the section times the total quantity represented by the circle.

(iii) In the graphs below, you know that “Part A2” represents a larger quantity than “Part B2” because the piece of the graph is larger.

(iv) In the graphs below, you cannot compare the quantities represented by the parts because you do not know the reference values.

Circle graph showing Graph A, indicating the following: Part A1 = Smaller portion Part A2 = Larger portion.

Circle graph showing Graph B, indicating the following: Part B1 = Smaller portion Part B2 = Larger portion

DEVELOPING SKILLS AND UNDERSTANDING

(2) Use the graph[15] below to answer the following questions. (**Note**: The median is the middle number of a data set when the numeric values are arranged in order of size.)

Line graph showing median new house cost in the US. X-axis is year ranged from 2008 to 2024 in increments of 2. Y-axis is price in dollars ranging from 210000 to 450000. At 2010, price is about 220k. At 2012, price is just under 250k. At 2014. Price is about 290k. At 2016, price is about 310k. At 2018, price ia bout 330k. At 202, price is about 340k. At 2022, price is a little more than 450k.

(a) Estimate the percentage increase in new housing prices from 2010 to 2022. (Choose the best answer.)

(i) The prices increased from around $220,000 to around $450,000, about a 49% increase.

(ii) The prices increased from around $220,000 to around $230,000, about a 16% increase.

(iii) The prices increased from around $220,000 to around $450,000, about a 105% increase.

(iv) The prices increased from around $220,000 to around $230,000, about a 14% increase.

(b) Estimate the percentage increase in new housing prices from 2011 to 2013. (Fill in the blanks.)

The prices increased from around ___________ to ___________. This is about a _________% increase.

(c) Estimate the percentage increase from 2020 to 2022. (Fill in the blanks.)

The prices increased from around __________ to ___________. This is about a _________% increase.

(3) South Central Bank has a policy that limits the amount of debt customers may have in order to receive a loan. The following pie chart shows the *highest* percentage of debt that the bank will allow.

Circle graph showing the South Central Bank has a policy that limits the amount of debt customers. Debt = 36% Other Expenses = 64%

(a) What is the reference value in this situation?

(i) Other expenses

(ii) Debt

(iii) Total of other expenses and debt

(iv) Other expenses minus debt

(b) Three graphs are given below. Each graph represents a loan customer. Each customer’s debt is broken into three categories: car, credit card, and mortgage (the loan on a house). Which customer(s) meet the bank policy on the limit of the amount of debt? There may be more than one correct answer.

(i) Circle graph showing the Customer A. Car = 4% Credit Card = 2% Mortgage = 30% Other Expenses = 64%

(ii) Circle graph showing the Customer B. Car = 8% Credit Card = 4% Mortgage = 35% Other Expenses = 53%

(iii) Circle graph showing the Customer C. Car = 5% Credit Card = 5% Mortgage = 10% Other Expenses = 80%

(4) The following two pie charts show how two families spend their money. The Alvarez family has a take-home pay of $3,650 per month and the Martinez family has a take-home pay of $7,300 per month.

Circle graph showing the Alvarez family spending. Food = 18% Housing = 33% Gasoline = 6% Utilities = 11% Entertainment = 16% Savings = 16%

Circle graph showing the Martinez family spending. Food = 14% Housing = 34% Gasoline = 3% Utilities = 8% Entertainment = 27% Savings = 14%

(a) Select the statement that best compares how much the families spend on gasoline.

(i) The Alvarez Family spends more on gasoline than the Martinez Family.

(ii) The two families both spend $200 on gasoline.

(iii) Both families spend around $200 on gasoline. This is 6% of the Alvarez budget, but only 3% of the Martinez budget because the Martinez family starts with about twice as much money as the Alvarez family.

(iv) The Alvarez family spends more on gasoline than the Martinez family because 3% of $7,300 is more than 6% of $3,650.

(b) Estimate the amount each family spends on food and housing. (Fill in the tables.)

Alvarez Family	
Food	*(i)*
Housing	*(ii)*

Martinez Family	
Food	*(iii)*
Housing	*(iv)*

(c) Both families spend about the same percentage of their income on housing. The family with the ________ income can afford a house that has twice the payment and maintenance costs.

(i) Higher

(ii) Lower

MAKING CONNECTIONS ACROSS THE COURSE

(5) In 2020, a total of 2,738 teens (ages 13–19) were killed in car crashes in the United States. The number of teen males who were killed was 1,866 and the number of teen females who were killed was 864.[16]

A reporter used this information to write, “About 32%, or one out of every three girls will be killed in a car crash.” Which of the following statements is the best critique of this statement?

(i) The statement is correct because 864 is 32% of 2,738, which is very close to one-third. The ratio of 1 out of 3 is a good estimate of the percentage.

(ii) The statement is incorrect because 864 is 32% of 2,738, which is very close to three-tenths. The ratio of 1 out of 3 is not a good estimate of the percentage.

(iii) The statement is incorrect because the reference value is the number of teens killed, not all teen girls. It would be correct to say that 32% of teens killed in car crashes are female.

(iv) The statement is incorrect because 864 is about 50% of 1,866. The ratio of 1 out of 3 is not a good estimate of the percentage.

(6) The following graphs show how much two families spend on tickets to professional baseball games each year.

Vertical bar graph showing how much two families spend on tickets to professional baseball games in 2022. Y-axis represents the amount, ranging from $0 to $7,000. X-axis represents each family. Cobb family = $6,000 Wagner family = in between $1,000 and $2,000

Vertical bar graph showing how much two families spend on tickets to professional baseball games in 2022 (percent of income). Y-axis represents the percentage, ranging from 0% to 6%. X-axis represents each family. Cobb family = 3% Wagner family = 5%

Choose the true statement based on these graphs.

(i) The Wagners must have a higher income than the Cobbs. They spend less on baseball tickets, but it is a higher portion of their total income.

(ii) The Cobbs must have a higher income than the Wagners. They spend more on baseball tickets, but it is a smaller portion of their total income.

(iii) The Wagners must have a higher income than the Cobbs. They spend more on baseball tickets, but it is a smaller portion of their total income.

(iv) The Cobbs must have a lower income than the Wagners. They spend more on baseball tickets, but it is a smaller portion of their total income.

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[15] https://www.census.gov/construction/nrs/historical_data/index.html

[16] https://www.cdc.gov/transportationsafety/teen_drivers/teendrivers_factsheet.html

2.5: Risk Reduction

INTRODUCTION

Answer the following questions in your group:

- What do the words, "absolute" and "relative" mean to you when you are thinking about numbers?
- What does the "risk" of something happening mean when thinking about medical issues?

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- the change in a quantity can be expressed as an absolute change and a relative change.
- there is often ambiguity in the English language when talking about the change of a quantity that is represented by a percent (e.g., many rates).

By the end of this collaboration, you should be able to

- create graphs that show both absolute and relative changes in a rate (percent).
- compute absolute and relative changes.

PROBLEM SITUATION: THE EFFECT OF REDUCING RISK

A new drug advertises that it “reduces the risk of heart attack by 50%.” In order to better understand the benefits of this drug, you will examine heart attack risk for two different groups.

Group 1 consists of individuals in Africa who • are 40 years old, • do not have Diabetes Mellitus, • smoke tobacco, • have high cholesterol, and • have high blood pressure.	Group 2 consists of individuals in Africa who • are 40 years old, • do not have Diabetes Mellitus, • do not smoke, • have low cholesterol, and • have high blood pressure.

The World Health Organization reports that individuals in Group 1 have a 40% chance (or risk) of suffering a heart attack within 10 years. The same report indicates that individuals in Group 2 have a 10% risk of suffering a heart attack within 10 years.

(1) What are the differences between individuals in Group 1 and Group 2? Take a minute to think about this on your own before sharing in your group.

(2) The following questions are designed to help you understand heart attack risk in Group 1.

(a) If 500 people from Group 1 are observed for 10 years, how many individuals would you expect to suffer a heart attack within this time period?

(b) Which of the following graphs best shows the risk of heart attack within 10 years for 500 Group 1 individuals? (**Note**: There are four graphs to choose from.)

Vertical bar graph showing the 10-year heat attack risk for 500 group 1 individuals. Y-axis represents the number of individuals, ranging from 0 to 350. X-axis represents the types of groups. Number of individuals expected to suffer a heart attack = 200 Number of individuals expected not to suffer a heart attack = 300

Vertical bar graph showing the 10-year heat attack risk for 500 group 1 individuals. Y-axis represents the number of individuals, ranging from 0 to 500. X-axis represents the types of groups. Number of individuals expected to suffer a heart attack = in between 0 and 100 Number of individuals expected not to suffer a heart attack = in between 400 and 500

Graph 1 Graph 2

Vertical bar graph showing the 10-year heat attack risk for 500 group 1 individuals. Y-axis represents the number of individuals, ranging from 0 to 500. X-axis represents the types of groups. Number of individuals expected to suffer a heart attack = 50 Number of individuals expected not to suffer a heart attack = 450

Vertical bar graph showing the 10-year heat attack risk for 500 group 1 individuals. Y-axis represents the number of individuals, ranging from 0 to 400. X-axis represents the types of groups. Number of individuals expected to suffer a heart attack = 150 Number of individuals expected not to suffer a heart attack = 350

Graph 3 Graph 4

(c) If 500 people from Group 1 are treated with the new drug that is advertised to “reduce the risk of heart attack by 50%,” how many individuals would you expect to suffer a heart attack within the next 10 years? Discuss in your group.

(d) Look at the graph below. Discuss with your group how best to describe what this graph shows in one or two sentences.

Vertical bar graph showing the 10-year heat attack risk for 500 group 1 individuals taking new drug. Y-axis represents the number of individuals, ranging from 0 to 450. X-axis represents the types of groups. Number of individuals expected to suffer a heart attack = 100 Number of individuals expected not to suffer a heart attack = 400

(e) The graphs below show the correct answers for (b) and (d). The differences between the graphs show the effect of the new drug on 500 people from Group 1. But what, specifically, do these graphs show that was reduced by 50%? Discuss in your group.

Vertical bar graph showing the 10-year heat attack risk for 500 group 1 individuals. Y-axis represents the number of individuals, ranging from 0 to 350. X-axis represents the types of groups. Number of individuals expected to suffer a heart attack = 200 Number of individuals expected not to suffer a heart attack = 300

Vertical bar graph showing the 10-year heat attack risk for 500 group 1 individuals taking new drug. Y-axis represents the number of individuals, ranging from 0 to 450. X-axis represents the types of groups. Number of individuals expected to suffer a heart attack = 100 Number of individuals expected not to suffer a heart attack = 400

(f) There are two ways to report the expected drop in heart attacks due to the new drug. The **absolute change** is a subtraction of the old outcome from the new outcome. It is typically reported as a number. **Relative change** is a ratio of the amount of change in comparison to the old outcome. It is typically reported as a percentage. Report both the absolute and relative changes in the number of individuals who are expected to suffer heart attacks.

(3) The following questions are designed to help you understand heart attack risk in Group 2.

(a) If 500 people from Group 2 are observed for 10 years, how many individuals would you expect to suffer a heart attack within this time period?

(b) Which of the following graphs best shows the risk of heart attack within 10 years for 500 Group 2 individuals? (**Note**: There are four graphs to choose from.)

Vertical bar graph showing the 10-year heat attack risk for 500 group 2 individuals. Y-axis represents the number of individuals, ranging from 0 to 500. X-axis represents the types of groups. Number of individuals expected to suffer a heart attack = 50 Number of individuals expected not to suffer a heart attack = 450

Vertical bar graph showing the 10-year heat attack risk for 500 group 2 individuals. Y-axis represents the number of individuals, ranging from 0 to 500. X-axis represents the types of groups. Number of individuals expected to suffer a heart attack = 100 Number of individuals expected not to suffer a heart attack = 400

Graph 5 Graph 6

Vertical bar graph showing the 10-year heat attack risk for 500 group 2 individuals. Y-axis represents the number of individuals, ranging from 0 to 350. X-axis represents the types of groups. Number of individuals expected to suffer a heart attack = in between 150 and 200 Number of individuals expected not to suffer a heart attack = in between 300 and 350

Vertical bar graph showing the 10-year heat attack risk for 500 group 2 individuals. Y-axis represents the number of individuals, ranging from 0 to 350. X-axis represents the types of groups. Number of individuals expected to suffer a heart attack = 200 Number of individuals expected not to suffer a heart attack = 300

Graph 7 Graph 8

(c) If 500 people from Group 2 are treated with the new drug and are observed for 10 years, how many individuals would you expect to suffer a heart attack within this time period?

(d) Look at the graph below. Discuss with your group how best to describe what this graph shows in one or two sentences.

Vertical bar graph showing the 10-year heat attack risk for 500 group 2 individuals taking new drug. Y-axis represents the number of individuals, ranging from 0 to 500. X-axis represents the types of groups. Number of individuals expected to suffer a heart attack = in between 0 and 50 Number of individuals expected not to suffer a heart attack = in between 450 and 500

(e) The graphs on the next page show the correct answers for (b) and (d). The differences between the graphs show the effect of the new drug on 500 people from Group 2. What, specifically, is being reduced by 50% in these graphs?

(f) There are two ways to report the drop in heart attacks due to the new drug. The *absolute change* refers to the actual number of fewer individuals suffering heart attacks as a result of using the new drug, while the *relative change* (or *percent* change) refers to this change as a percentage. Report both the absolute and relative change in the number of individuals who suffered heart attacks.

(4) Suppose a third group of people have only a 0.5% chance of suffering a heart attack. How do you think the absolute and relative changes in the number of individuals (out of 500) who are expected to suffer a heart attack will compare to your earlier calculations?

(5) Based on the previous calculations, critique the statement: “A drug which reduces the risk of heart attack by 50% will most likely save many lives.”

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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2.5.1: Preparation 2.5

(1) A digital camera is regularly priced at \$350 and it is marked 30% off the regular price. What is the new price for the digital camera?

(i) \$105

(ii) \$245

(iii) \$320

(iv) \$338.40

(2) The American Diabetes Association reports the following information:

In general, if you are a man with type-1 diabetes, the odds of your child getting diabetes are 1 in 17. If you are a woman with type-1 diabetes and your child was born before you were 25, your child's risk is 1 in 25; if your child was born after you turned 25, your child's risk is 1 in 100.

Rank the risk of having a child with diabetes from 1 (highest risk) to 3 (lowest risk).

______ Mother with type-1 diabetes who gave birth after turning 25

______ Father with type-1 diabetes

______ Mother with type-1 diabetes who gave birth before turning 25

(3) Mark each statement true or false.

(a) A man with type-1 diabetes has a child. The probability that the child will also have diabetes is about 6%.

(i) True

(ii) False

(b) A 20 year-old woman with type-1 diabetes has a child. The probability that the child will also have diabetes is about 25%.

(i) True

(ii) False

(c) A 30-year-old woman with type-1 diabetes has a child. The probability that the child will also have diabetes is about 1%.

(i) True

(ii) False

Self-Regulated Learning

An aspect of the *work* phase is always checking your understanding. One way to check your understanding is to explain how you solved a problem. Explain how you solved Question 1.

Reflect phase: After checking your understanding, on a scale from 1 to 5, how confident are you that you answered Question 1 correctly? Be sure to incorporate this confidence rating in the table at the end of the assignment.

After Preparation 2.5 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 2.5, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
read and interpret bar graphs.	
understand the word *percent* and the notation used to describe percents (%).	

use a calculator to divide two numbers and interpret the resulting decimal representation as a percent.	
understand that a percent may be used to express the likelihood (or probability) of a certain event.	

2.5.2: Exercise 2.5

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) To calculate a percentage, move the decimal in the percentage rate two places to the left and multiply times the reference value.

(ii) Smoking increases your chance of a heart attack.

(iii) Consider this situation: 20% of a population gets a certain illness. A vaccine can reduce the infection rate by 10%. Out of 100 people, the reference value for the 10% reduction would be the full population (or 100 people).

(iv) Consider this situation: 20% of a population gets a certain illness. A vaccine can reduce the infection rate by 10%. Out of 100 people, the reference value for the 10% reduction would be the 20 people who are projected to get the illness.

DEVELOPING SKILLS AND UNDERSTANDING

(2) Think back to your work from Question 2 and 3 in Collaboration 2.5 to answer the following questions.

(a) In class, you used a group with 500 people. Repeat the work with a group of 100 people.
Group 1 has a 40% risk of heart attack and Group 2 has a 10% risk. Complete the table.

	Group 1	**Group 2**
Number in group	100	100
Number who are expected to have a heart attack without the drug		
Number who are expected to have a heart attack with the drug		
What percentage of the group is expected to have a heart attack with the drug?		

(b) Select the true statement:

(i) The size of the group does not change anything about the results for the group.

(ii) The size of the group changes the number of people who are expected to have a heart attack with the drug, but it does not change the percentage.

(iii) The size of the group does not change the number of people who are expected to have a heart attack with the drug, but it does change the percentage.

(iv) The size of the group changes the number and the percentage of people who are expected to have a heart attack with the drug.

(3) The University of Washington is a large public university, while the University of Puget Sound is a small private university.

(a) If both of these universities are expecting 12% more freshmen next year, which university will see the greatest absolute change in the number of freshmen? (Choose the best answer.)

(i) Since the University of Washington is larger, they have more freshmen to begin with. Thus, a 12% increase will not translate to a larger absolute change.

(ii) Since the University of Washington is larger, they have more freshmen to begin with. Thus, a 12% increase will translate to a larger absolute change.

(iii) Both will undergo the exact same absolute change.

(iv) Since the University of Puget Sound is smaller, they have fewer freshmen to begin with. Thus, a 12% increase will translate to a larger absolute change.

(b) Which university will see the greatest relative change in the number of freshmen?

- (i) University of Washington
- (ii) University of Puget Sound
- (iii) Both universities will undergo the same relative change.

(4) Suppose that the populations of the United States and China both undergo the same absolute change in one year. Which undergoes the larger relative change?

- (i) The United States would undergo the smaller relative change, since the population of China is so much larger than that of the United States.
- (ii) China would undergo the larger relative change since the population of the United States is much smaller than that of China.
- (iii) The United States would undergo the larger relative change since the population of the United States is so much smaller than that of China.
- (iv) The two countries would undergo the exact same relative change.

(5) Suppose that the populations of the United States and China both increase by 12 million people in one year. What would be the relative change for each country? Answer to the nearest tenth of a percent.

(a) If you assume that the U.S. population is about 333 million, an increase of 12 million would result in a relative change of ___________%.

(b) If you assume that China has about 1.4 billion people, an increase of 12 million would result in a relative change of ___________%.

(6) Employees at a certain company currently have to pay 3% of their health care costs, while the company pays the rest. Next year, however, employees will have to pay 6% of their health care costs. Express this change as an absolute change (in percentage points) and as a relative change.

(a) Absolute change: ____________ percentage points.

(b) Relative change: _____________%.

MAKING CONNECTIONS ACROSS THE COURSE

(7) The graph below is based on information from the 2020 U.S. Census.[17] Use the graph to answer the questions.

Bar chart showing US population by age in 2022. X-axis is age ranged from 0-9 to 100+ in increments of 9 years. Y-axis is population in hundreds of thousands ranging from 0 to 500. 0-9 = just under 400. 10-19 = about 44. 20-29 = about 450. 30-39 = about 450. 40-49 = about 410. 50 to 59 = about 425. 60-69 = about 385. 70-79 = just under 250. 80-89 = about 100. 90-99 = about 25. 100+ = about 2.

(a) Which is the best estimate of the total population for age group 0–9?

- (i) 400
- (ii) 40,000
- (iii) 40,000,000
- (iv) 400,000,000

(b) Which is the best estimate for the total population of age group 70–79?

- (i) Two hundred and four
- (ii) Two hundred and four thousand
- (iii) Two hundred and four million
- (iv) Twenty four million

(c) Which statement is the most accurate?

- (i) There are fewer than 10,000,000 people age 70 and older.
- (ii) There are between 10,000,000 and 20,000,000 people age 70 and older.

(iii) There are between 20,000,000 and 30,000,000 people age 70 and older.

(iv) There are between 30,000,000 and 40,000,000 people age 70 and older.

(d) An advertiser is considering two advertising campaigns for a product. Campaign A is most effective for ages 20–39, while Campaign B is most effective for ages 30–49. Which campaign should the advertiser choose to reach the most people?

(i) Campaign A

(ii) Campaign B

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[17] https://www.census.gov/data/tables/time-series/demo/popest/2020s-national-detail.html

2.6: What Is Average?

INTRODUCTION

The terms *mean*, *median*, and *mode* are three different kinds of averages. In your group, discuss how you would describe each of these terms to someone who didn't know what they meant.

- Mean:
- Median:
- Mode:

Imagine a place that has varying daily temperatures. The following table shows the temperatures for one week. What is the mean temperature?

Monday	62°
Tuesday	90°
Wednesday	106°
Thursday	10°
Friday	50°
Saturday	0°
Sunday	50°

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- numerical data can be summarized using measures of central tendency.
- the mean and median statistics for a set of data can provide different snapshots of the data.
- conclusions derived from statistical summaries are subject to error.
- a spreadsheet can be used to organize data.

By the end of this collaboration, you should be able to

- calculate the mean and median for numerical data.
- create a data set that meets certain criteria for measures of central tendency.

PROBLEM SITUATION: SUMMARIZING DATA ABOUT CREDIT CARDS

A revolving line of credit is an agreement between a consumer and lender that allows the consumer to obtain credit for an undetermined amount of time. The debt is repaid periodically and can be borrowed again once it is repaid. The use of a credit card is an example of a revolving line of credit.

According to CreditCards.com, U.S. consumers own more than 455 million credit cards. About 98% of the total U.S. revolving debt is made up of credit card debt. Average credit card debt per household with a credit card is $5,525. In the US alone, there are more than $4 trillion in transactions annually. It is estimated that there are 12,000 credit card payment transactions made every second.

According to the College Finance survey, released in 2021, 38% of college students are behind on their payments and 45% are only paying their minimum amount. The *median* debt carried by college students was $3,280. Recall that the **median** is the middle number of a data set when the numeric values are arranged in order of size. In 2020, 79% of all American consumers had at least one credit card, but only 55% carried a balance. If a credit card user carries a balance (e.g., does not pay the monthly debt in full) the credit card company assesses a finance charge (interest) for the use of their money. This can be avoided by paying the balance in full.[18]

In the first part of this collaboration, you will use this information about credit cards to learn about some ways to summarize quantitative information.

The word **mean** (the sum of all numbers in the data set divided by the number of numbers in the data set) was introduced in Preparation 2.5 and will be used throughout this collaboration. (**Note**: There are actually several different kinds of means. The one you will discuss in this collaboration is the arithmetic mean. An arithmetic mean is the same as an average.)

(1) The population of the United States is slightly more than 330 million people. There are about 120 million households in the United States. What is the mean number of credit cards per capita? What is the mean number of credit cards per household?

(2) Consider the statement: “Average credit card debt per household with a credit card is $5,525” How would you explain this statement? Discuss in your group.

(3) Recall that “average credit card debt per household with a credit card is $5,525.” Imagine you ask four groups of five households what their credit card debt is. The amount of debt for each household in each group is shown in the table.

Screen capture of data in an excel sheet, indicating the following in cells: Group A Group B Group C Group D $0 $5,525 $6,600 $27,625 $14,800 $5,525 $4,375 $0 $700 $5,525 $5,125 $0 $6,900 $5,525 $7,200 $0 $5,225 $5,525 $4,325 $0

(a) Find the mean debt of each group of households. Make sure the value you found is reasonable given the values in the table.

(b) Create your own data set of the debt of five other households. This data set should have a mean debt of $5,525. You can choose the credit card debt for the individual college seniors, but the mean debt should be $5,525. Write your data in the table below.

Your Data

(c) Find the median of each set of data including the one you created.

Group A	Group B	Group C	Group D	Your Data

(4) The introductory information in this problem situation gives data credit card debt carried by freshmen. We are told that the median debt carried by college students was $3,280. Create a data set of six students so that the data set has the same median reported for all college students. Enter your data in the table below.

Debt of College Students

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[18] https://www.creditcards.com/statistics/ownership-statistics/

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2.6.1: Preparation 2.6

(1) Find the sum of each set of numbers, as well as the size n of each set of numbers (n = number of numbers in the set). Use these exercises to practice techniques for adding numbers quickly. Try to do the problems without a calculator. Think about ways in which grouping the numbers might make them easier to add.

(a) 25, 35, 19, 31

(i) Sum:

(ii) n:

(b) 101, 73, 49, 27, 24, 36

(i) Sum:

(ii) n:

(c) 25, 25, 25, 30, 30, 30, 32, 32, 32

(i) Sum:

(ii) n:

Averages: A Measure of Center

The following information will be used in Collaboration 2.6.

People often talk about “averages,” and you probably have an idea of what is meant by that. Now, you will look at more formal mathematical ways of defining averages. In mathematics, you call an average a **measure of center** because an average is a way of measuring or *quantifying* the center of a set of data. There are different measures of center because there are different ways to define the center.

Figure A **Figure B** **Figure C** **Figure D**	Dotted plot showing Figure A, as described in the surrounding paragraphs.

Think about a long line of people waiting to buy tickets for a concert. (Figure A shows a line about 100-feet long and each dot represents a person in the line.) In some sections of the line people are grouped together very closely, while in other sections of the line people are spread out. How would you describe where the center of the line is?

Would you define the center of the line by finding the point at which half the people in the line are on one side and half are on the other (see Figure B)? Is the center based on the length of the line even though there would be more people on one side of the center than on the other (see Figure C)? Would you place the center among the largest groups of people (see Figure D)? The answer would depend on what you needed the center for. When working with data, you need different measures for different purposes.

Mean (Arithmetic Average): The mean is what most people call the “average.” To find the average of numeric values, first find the sum of the values, then divide the sum by the number of values.

Example: Find the mean of 18, 23, 45, 18, 36.

First find the sum of the numbers: 18 + 23 + 45 + 18 +36 = 140.

Then divide the sum by 5 because there are 5 numbers: 140 ÷ 5 = 28.

The mean is 28.

Median: To find the median of numeric values, arrange the data in order of size. If there is an odd number of values, the median is the middle number. If there is an even number of values, the median is the mean of the two middle numbers.

Example (data set with an odd number of values): Find the median of 18, 23, 45, 18, 36.

Write the numbers in order: 18, 18, 23, 36, 45.

There is an odd number of values, so the median is the number in the middle.

The median is 23.

Example (data set with an even number of values): Find the median of 18, 23, 45, 18, 12, 50.

Write the numbers in order: 12, 18, 18, 23, 45, 50.

There is an even number of values, so there is no one middle number.

Find the median by finding the mean of the two middle numbers:

- 18 + 23 = 41
- 41 ÷ 2 = 20.5

The median is 20.5.

Mode: To find the mode, find the number(s) that occur(s) most frequently. There may be more than one mode.

Example: Find the mode of 18, 23, 45, 18, 36.

The number 18 occurs twice, more than any other number, so the mode is 18.

After Preparation 2.6 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 2.6, you should understand the concepts and demonstrate these skills.

Skill or Concept: I can …	**Rating from 1 to 5**
perform basic operations using quantities as integers, fractions, or decimals with the aid of technology.	
identify the mean, median, and mode of a small data set.	

In the question above, if you or another student had any ratings below 3, what are some things that you or the student could do to increase confidence while planning for Collaboration 2.6? Please describe at least two ways.

2.6.2: Exercise 2.6

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Any of the three measures of central tendency (mean, median, and mode) are good representations of data. It does not matter which one you use.

(ii) U.S. college students carry far too much credit card debt.

(iii) The mean is calculated by adding all the numbers and dividing by the number of data points.

(iv) The mean, median, and mode all give important information about a data set, but they do not give a complete picture of the data set.

DEVELOPING SKILLS AND UNDERSTANDING

(2) Use the following data set to answer the questions.

13 15 20 20 20 20 20 20 20 23 27 31

(a) What is the mean? Round to two decimal places.

(b) What is the mode?

(c) What is the median?

(d) What fraction of the numbers in the data set is less than the median?

(e) What fraction of the numbers in the data set is greater than the median?

(f) Which of the following statements is correct?

(i) The median is the middle of a data set. Half of the data points are always less than the median, and half are always greater than the median.

(ii) The median is the middle of a data set. When listed in increasing order, there are the same number of data points to the left and to the right of the median..

(iii) The median is the middle of a data set. At least half of the data points are always equal to the median.

(iv) The median is not the middle of a data set. You cannot predict the distribution of the numbers in relationship to the median.

(3) Consider the statement “In the U.S. in 2021, there were $4.1 trillion in credit card transactions.”

(a) What was the daily average dollar amount of transactions in 2021? Round to the nearest hundred million dollars.

(b) How many dollars were spent in credit card transactions each day?

(i) $11,232,876

(ii) $1,460,000,000

(iii) $6,849,315,068

(iv) It is impossible to know.

(4) Students at Dover Community College have a mean credit card debt of $3,600 with a median of $1,500. Students at Ralton Community College have a mean credit card debt of $3,000 with a median of $2,800. Which of the following statements are true based on this information? There may be more than one correct answer.

(i) Less than half of Dover students have debt more than $3,600.

(ii) No more than half of Ralton students have debt less than $2,800.

(iii) About three-fourths of Dover students have debt less than $3,600.

(iv) The largest debt of the Ralton students is less than the largest debt of the Dover students.

(v) The total debt of Ralton students is less than the total debt of Dover students.

(5) Decide whether the following statements are true or not, based on the information provided. Be prepared to explain your reasoning.

(a) The median of 25 numbers is 55. Twelve of the numbers must be greater than 55.

(i) True

(ii) False

(b) The average of 11 numbers is 130. None of the 11 numbers are more than 260.

(i) True

(ii) False

(c) The average of 25 numbers is 100, and the median of those 25 numbers is also 100. The mode of the 25 numbers must be 100.

(i) True

(ii) False

(d) The mean of 45 numbers is 70. If you pick any group of 10 numbers from the 45, the mean will be 70.

(i) True

(ii) False

(e) The average of 42 numbers is 20. The sum of all 42 numbers is 840.

(i) True

(ii) False

(f) The average of 49 numbers is 100. If a 50th number is added and the average remains at 100, the 50th number must have been 100.

(i) True

(ii) False

(6) Rio Blanca City Hall publishes the following statistics on household incomes of the town's citizens. (The mode is given as a range.)

Mean: $257,000 Median: $65,000 Mode: $20,000–$30,000

Which measure would be the most useful for each of the following situations?

(a) State officials want to estimate the total amount of state income tax paid by the citizens of Rio Blanca.

(i) Mean

(ii) Median

(iii) Mode

(b) The school district wants to know the income level of the largest number of students.

(i) Mean

(ii) Median

(iii) Mode

(c) A businesswoman is thinking about opening an expensive restaurant in the town. She wants to know how many people in town could afford to eat at her restaurant.

(i) Mean

(ii) Median

(iii) Mode

MAKING CONNECTIONS ACROSS THE COURSE

(7) Use Figures 1 and 2 for the following questions.

Figure 1

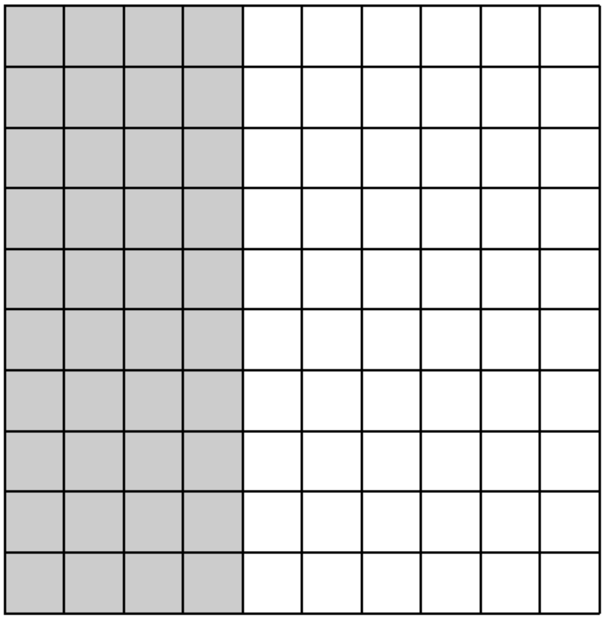
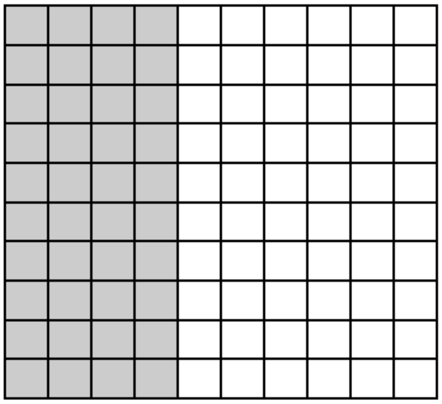


Figure 2



(a) Select the figure(s) with 40% shaded.

  (i) Figure 1

  (ii) Figure 2

  (iii) Both Figure 1 and Figure 2

  (iv) Neither Figure 1 nor Figure 2

(b) Which of the following statements are correct? There may be more than one correct answer.

  (i) The shaded area of Figure 1 is larger than the shaded area of Figure 2 because Figure 1 is larger than Figure 2.

  (ii) The shaded area of Figure 1 is the same as the shaded area of Figure 2 because they are both 40% of the square.

  (iii) The shaded area of Figure 1 is the same proportion of the figure as the shaded area of Figure 2 because they are both 40% of the square.

(c) These figures illustrate what important concept?

(i) Percentages cannot be used for comparisons unless the reference values are equal.

(ii) Percentages compare measures relative to the size of the reference values, but do not give information about absolute measures.

(iii) Percentages are a ratio out of 100, so they can always be compared directly. In other words, 60% of one value is equal to 60% of another value.

(8) Animal population densities are important to those who manage wildlife. This is particularly true when animals pose risks to humans. Managing the population of grizzly bears in Yellowstone National Park is one example. Grizzly bears roam Yellowstone Park, which is 3,472 square miles. Additionally, the bears roam a surrounding area that is 40% as large as Yellowstone. Although their territories overlap considerably, each bear needs at least 10 square miles to himself/herself to call home.[19]

(a) Calculate the total available land in and around Yellowstone for grizzly bears to roam. Round to the nearest square mile.

(b) Estimate the maximum number of bears that can be supported on the available land.

(c) If there are about 3 adult female bears for every 2 adult male bears, then approximately how many male bears can be supported in and around Yellowstone Park?

(d) How many female bears?

Bear Sightings

The bar graph below shows the number of bear sightings (all bears, not only grizzlies) in Yellowstone National Park from 2015 to 2019.[20] Because the number of sightings is affected by the number of visitors to the park, these data are shown as well.[21]

Note: Because the number of visitors each year is much larger than the number of bear sightings, the data are plotted on different scales. The scale on the left (ranging from zero to 4,300,000) is for the number of visitors, shown in gray. The scale on the right (ranging from zero to 2000) is for the number of bear sightings, shown in black.

Vertical bar graph showing the number of bear sightings (all bears, not only grizzlies) in Yellowstone National Park from 2015 to 2019. Left y-axis represents the number of visitors, ranging from 0 to 4,500,000. Right y-axis represents the number of reported bear sightings, ranging from 0 to 2000. X-axis represents the years, ranging from 2015 to 2019. 2015: Number of visitors = about 400,000; Number of bear sightings = about 1200. 2016: Number of visitors = just over 400,000; Number of bear sightings = about 1200. 2017: Number of visitors = a little over 400,000; Number of bear sightings = just under 1200. 2018: Number of visitors = about 425,000; Number of bear sightings = about 1400. 2019: Number of visitors = about 400,000; Number of bear sightings = about 1800.

(9) (a) In what year was the percentage of visitors who saw bears the highest?

(b) In what year was the percentage of visitors who saw bears the lowest?

(c) Estimate the average number of bear sightings over these five years.

(i) 1100

(ii) 1200

(iii) 1400

(iv) 1600

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[19] https://www.usgs.gov/publications/yellowstone-grizzly-bear-investigations-2020-annual-report-interagency-grizzly-bear

[20] https://www.nps.gov/yell/learn/management/upload/2019-BEAR-REPORT-FINAL_reduced.pdf

[21] https://www.statista.com/statistics/254231/number-of-visitors-to-the-yellowstone-national-park-in-the-us/

2.7: Making Good Decisions with Good Statistics

INTRODUCTION

Imagine you have been offered a job at one of the following companies. Which one would you pick based only on the two salary descriptions below, and why? Discuss in your group and be prepared to support your reasoning.

- Company A: average salary $105,000 per year
- Company B: median salary $80,000 per year

Position	Company A	Company B
Secretary	$45,000	$52,000
Sales representative	$47,000	$70,000
Sales representative	$52,000	$75,000
Security officer	$53,000	$80,000
Security officer	$53,000	$81,000
Warehouse manager	$55,000	$86,000
CEO	$240,000	$286,000
	Mean (average) salary $105,000	Median salary $80,000

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- each statistic—the mean, median, and mode—is a different summary of numerical data.
- conclusions derived from statistical summaries are subject to error.
- measures of central tendency can be used to make decisions.

By the end of this collaboration, you should be able to

- make good decisions using information about data.
- interpret the mean, median, or mode in terms of the context of the problem.
- match data sets with appropriate statistics.

PROBLEM SITUATION 1: MAKING SENSE OF MEASURES OF CENTRAL TENDENCY

Examine the following three advertisements.

Employment Opportunities

Sales Positions Available! We have immediate need for five enthusiastic self-starters who love the outdoors and who love people. Our salespeople make an average of $1,000 per week. Come join the winning team. Call 555-0100 now!	**Are you above average?** Our company is hiring one person this month—will you be that person? We pay the top percentage commission and supply you leads. Half of our sales force makes over $3,000 per month. Join the *Above Average Team*! Call 555-0127 now! We are!	**NEED A NEW CHALLENGE?** Join a super sales force and make as much as you want. Five of our nine salespeople closed FOUR homes last month. Their average commission was $1,500 on each sale. Do the math—this is the job for you. Making dreams real— call 555-0199

(1) (a) Identify any measures of central tendency and how they are used in each advertisement.

(b) For each advertisement, create a set of data that fits the description. Note that this exercise is similar to the one in Collaboration 2.6 where you made a list of credit card debts for the five college students.

(2) In which job would you expect to earn the most money? Discuss in your group.

PROBLEM SITUATION 2: UNDERSTANDING TRENDS IN DATA

The median and average sales price of new homes sold in the United States from 1963–2022 are shown in the following graph.[23]

Line graph showing the median and average sales prices of new homes sold in the United States annual data from 1963 to 2021. Y-axis represents the prices, ranging from $0 to $600,000. X-axis represents the years, ranging from 1963 to 2021. Median data line is shown starting in between $0 and $100,000 in 1963, increasing steadily to finish just in between $400,000 and $500,000 in 2021. Average data line is shown starting in between $0 and $100,000 in 1963, increasing steadily to finish just in between $500,000 and $600,000 in 2021.

(3) Examine the graph and write at least three statements about the data. Recall the Writing Principle: Use specific and complete information.

(4) Table 1 gives a sample data set of home prices that matches the data shown in Graph 1 for the year 1977. And five possible data sets for the year 2019 are given in Table 2. Use your knowledge of mean and median to answer the following questions *without* calculating the mean of the data sets. There may be more than one correct answer to any of the questions.

Table 1: Sales Prices of a Sample of New Homes Sold in the United States in 1977

1977 Sales Price
$35,300
$41,100
$43,400
$48,800
$62,700
$70,600
$77,500

Table 2: Possible Data Sets for 2019

Set A	Set B	Set C	Set D	Set E
$325,300	$164,000	$185,900	$74,000	$154,000
$321,100	$185,000	$203,300	$95,000	$170,000
$323,400	$205,000	$210,200	$125,000	$200,000
$328,800	$320,000	$226,400	$320,000	$320,000
$342,700	$325,000	$268,100	$342,000	$330,000
$350,600	$549,000	$291,800	$350,000	$715,000
$357,500	$890,000	$312,500	$351,000	$749,000

(a) Which of the data sets could represent the data in the graph?

(b) Which of the data sets would likely have a mean that is less than the median?

(c) Which of the data sets would likely have a mean and median that are close together?

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[23] https://www.census.gov/construction/nrs/data/series.html

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2.7.1: Preparation 2.7

(1) The graph below is based on U.S. Census data.[22]

Line graph showing the median and average sales prices of new homes sold in the United States annual data from 1963 to 2021. Y-axis represents the prices, ranging from $0 to $600,000. X-axis represents the years, ranging from 1963 to 2021. Median data line is shown starting in between $0 and $100,000 in 1963, increasing steadily to finish just in between $400,000 and $500,000 in 2021. Average data line is shown starting in between $0 and $100,000 in 1963, increasing steadily to finish just in between $500,000 and $600,000 in 2021.

(a) What was the average or mean price of a new home in 2020?

(b) What was the median price of a new home in 1984?

(c) Which of the following intervals had the largest increase in the median price?

- (i) 1981–1984
- (ii) 1996–1999
- (iii) 2002–2005
- (iv) 2019–2022

(d) In what two-year period was the largest drop in the average price?

- (i) 1990–1992
- (ii) 2002–2004
- (iii) 2007–2009
- (iv) 2009–2011

After Preparation 2.7 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 2.7, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
perform basic operations using quantities such as integers, fractions, or decimals with the aid of technology.	
find the mean, median, and mode of a set of numeric data.	
read a line graph.	

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[22] https://www.census.gov/construction/nrs/data/series.html

2.7.2: Exercise 2.7

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Home prices in 2022 were more than 20 times what they were in the 1960s.

(ii) The mean and median of a data set are always very close together, but the mode might be very different.

(iii) When using averages or measures of central tendency, it is always important to ask questions about what the different types of measurements do and do not tell you about the data set.

(iv) The median is the middle number when a data set is listed in order. If there is an even number of points in the data set, the median is found by finding the mean of the middle two numbers.

(2) Refer back to Exercise 2.4, which contains a graph about housing prices.

Line graph showing median new house cost in the US. X-axis is year ranged from 2008 to 2024 in increments of 2. Y-axis is price in dollars ranging from 210000 to 450000. At 2010, price is about 220k. At 2012, price is just under 250k. At 2014. Price is about 290k. At 2016, price is about 310k. At 2018, price ia bout 330k. At 202, price is about 340k. At 2022, price is a little more than 450k.

Which of the following explains what the data for 2020 tells you about the cost of a house.

(i) The median house price in 2020 was about $335,000. This means the most common cost of a house in 2020 was $335,000.

(ii) The median house price in 2020 was about $335,000. This means that about half the houses cost $335,000 or less and half cost $335,000 or more.

(iii) The median house price in 2020 was about $335,000. This means exactly half the houses cost $335,000.

(iv) The median house price in 2020 was the same as the modal house price in 2020, which was about $335,000.

DEVELOPING SKILLS AND UNDERSTANDING

(3) The first advertisement discussed in class states that the salespeople make an average of $1,000 per week. Suppose there are nine salespeople. What would the ninth person need to earn for the mean to be $1,000 if the other eight salespeople earned $550, $600, $600, $800, $950, $950, $1,000, and $1,100?

(4) The second advertisement states that half the salespeople make more than $3,000 per month. Suppose there are eight salespeople. What would the eighth person need to earn for the median to be $3,000 if the other seven salespeople earned $2,400, $2,500, $2,800, $2,800, $3,400, $3,400, and $3,800?

(5) Which statistic (mean, median, or mode) is most appropriate in each of the following situations?

(a) Tables in the dining hall are numbered 1 through 12 for students who eat there. The principal calls out a number for the table that will go through the buffet line first. The other tables follow in order of the table numbers. One student is sure the principal calls certain tables more often. She keeps track of which numbers are called over a 21-day period.

(i) Mean

(ii) Median

(iii) Mode

(b) The offensive line of a football team is larger than in previous years. The program will list a statistic to show this fact.

(i) Mean

(ii) Median

(iii) Mode

(c) A reporter is doing a story on the falling prices of homes in a large neighborhood. The reporter wants to demonstrate how the prices have fallen for all homes, not just the most expensive houses.

(i) Mean

(ii) Median

(iii) Mode

(6) Lines at the Department of Motor Vehicles are so long! A supervisor decided to conduct a study on the number of people standing in line. At the beginning of each hour for an entire week, the supervisor counted the number of people in line and recorded the number. At the end of the week, the supervisor made the frequency table below. Note that the first column shows the number of people in line at the beginning of the hour. The second column shows the number of times that length of line occurred in the 40 observations.

Number of People in Line	Frequency
1	1
2	0
3	2
4	2
5	2
6	4
7	4
8	4
9	8
10	4
11	9

(a) How many times did the supervisor observe six people in line?

(b) Which calculation could be used to determine how many people the supervisor observed standing in line all together?

(i) $1 \times 1 + 2 \times 0 + 3 \times 2 + 4 \times 2 + 5 \times 2 + 6 \times 4 + 7 \times 4 + 8 \times 4 + 9 \times 8 + 10 \times 4 + 11 \times 9$

(ii) 1 x 2 x 3 x 4 x 5 x 6 x 7 x 8 x 9 x 10 x 11

(iii) 1 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11

(iv) 1 + 2 + 2 + 2 + 4 + 4 + 4 + 8 + 4 + 9

(c) How many observations did the supervisor make?

(d) Determine the mode of the data in the frequency table.

(e) Determine the median of the data in the frequency table.

(f) Determine the mean of the data in the frequency table.

(g) The supervisor made observations again the following week. The table below shows the observations.

Number of People in Line	Frequency
1	3
2	4
3	5
4	4
5	5
6	1
7	2

8	1
9	2
10	6
11	8

The mean for the second week was 6.366, the median was 5, and the mode was 11. The supervisor wanted to make the argument that additional personnel were needed. Which of these arguments is correct?

(i) Most of the time, there are five or more people in line.

(ii) Most of the time, there are five people in line.

(iii) Most of the time, there are six people in line.

(iv) Most of the time, there are 10 or more people in line.

(7) Three descriptions of measures of central tendency are given below. They are labeled "A," "B," and "C." Descriptions of data sets are listed below that. Match each data set with a description of measures of central tendency by writing the letter in the blank. Choices may be used more than once.

A The mean and median are close together.

B The mean is much higher than the median.

C The median is much higher than the mean.

(a) _____ The data set has a large range with a few very high numbers and many small numbers.

(b) _____ The data set has a large range with the numbers evenly spaced.

(c) _____ The data set has a small range with most of the numbers grouped in the middle.

(d) _____ The data set has a large range with a few very low numbers.

(8) If you lived in Canada in 2021, you might have seen the following headline:

Canada Below G7 Average for Productivity!

Here is some information to help you understand this headline.

Productivity is a way to measure the economy of a nation. One way to measure productivity is by Gross Domestic Product (GDP) per person. You may recall from Unit 2.4 that GDP is the value of all the goods and services produced in a country.

The **G7** is a coalition of the major industrial democracies in the world. It includes the United States, United Kingdom, France, Germany, Italy, Canada, and Japan.

(a) Which of the following is most likely what the author of the headline wanted the reader to think?

(i) Canada's economy is weak and is falling behind other countries in the G7.

(ii) Canada's economy is strong and is leading other countries in the G7.

(iii) Canada's economy is very similar to other countries in the G7.

(iv) Canada's economy should not be compared to other countries.

(b) Which of the following can you conclude from the headline?

(i) Canada is less productive than half of the G7 nations.

(ii) There is at least one G7 nation that is more productive than Canada.

(iii) There is at least one G7 nation that is less productive than Canada.

(iv) None of the above.

(c) A graph of the GDP per person of the G7 nations is shown below. Find the mean of the GDP per person for the G7 nations. Round to the nearest hundred dollars.

Vertical bar graph showing the GDP per worker of the G7 nations in US dollars for 2021. Y-axis represents the US dollar amount in thousands, ranging from 0.0 to 120.0. X-axis represents the countries. Japan = 42.9 Italy = 45.9 Germany = 57.9 United Kingdom = 49.7 Canada = 42.1 France = 50.5 United States = 69.3

(d) Is the headline correct?

(e) Which of the seven G7 nations have "above average" productivity?

(f) Which of the following are correct conclusions based on the data in the graph? There may be more than one correct answer.

(i) Canada is in the top half of the G7 in productivity.

(ii) Canada's productivity is within $8,000 of all the G7 nations except for the United States and Japan.

(iii) Canada is far behind the G7 nations in productivity.

(iv) Canada cannot be compared to other G7 nations.

MAKING CONNECTIONS ACROSS THE COURSE

In Collaboration 2.8, you will explore themes about the *purchasing power of money*, the role of the *Urban Consumer Price Index*, and the concept of index numbers.

The Purchasing Power of Money

The **purchasing power of money** is the value of a currency, such as the U.S. dollar expressed in terms of the amount of goods and services that one unit of that currency can buy. The following questions will help you learn about it.

Gasoline costs have varied significantly in recent years. The American Petroleum Institute posted an update on gasoline prices for January 30, 2023.[24]

> **U.S. Pump Price Update—January 30, 2023**
>
> According to the Energy Information Administration (EIA), the average U.S. retail price for regular grade of gasoline rose this week, up by 7.4 cents (2.2 percent) from the previous Monday's price to $3.489 per gallon. This week's price was above the year-ago price, up by 3.6 percent from $3.368 per gallon. This price was $1.52 below the record price of $5.006 per gallon set on 6/13/22.

(9) (a) How much was the average retail price for one gallon of gasoline a week before this article was published?

(i) $3.518

(ii) $3.415

(iii) $3.563

(iv) $3.833

(b) What is the relative change in the retail price for one gallon of gasoline from June 13, 2022, to January 30, 2023?

(i) -25.6%

(ii) -30.3%

(iii) -43.5%

(iv) -150.1%

(c) If the retail price for one gallon of gasoline was $3.368 a year before, then what is the absolute change in the retail price on January 30, 2023?

(i) $0.121

(ii) $1.21

(iii) $0.360

(iv) $1.097

(10) A key idea from this lesson is that the purchasing power of a dollar is not constant. For example, the price of gasoline varies greatly, so the amount of gas you can buy with $1 varies over time. The **Urban Consumer Price Index (CPI-U)** is a tool designed to compare the price of goods and services. Comparisons made with this tool are made in terms of **base-year** dollars. The base year is the first year included in a comparison. You will be using the CPI-U in your next collaboration. The following questions will help you learn about it.

Refer to the “Consumer Price Index” page of the Bureau of Labor Statistics website, www.bls.gov/cpi, to answer the following questions. You can also use a search engine like Google to look up “Bureau of Labor Statistics CPI”. **Hint**: If you are on the homepage of the Bureau of Labor Statistics, find “Consumer Price Index” in the subjects tab at the top of the page (under the heading “Inflation & Prices”).

Go to the CPI Overview and read the “Coverage” section.

(a) Are user fees for services like water and sewer included in the CPI?

(i) Yes

(ii) No

(b) Are income taxes included in the CPI?

(i) Yes

(ii) No

Go to the CPI Overview and read the “Uses” section. The website states that the CPI affects the income of almost 90 million people because their incomes are tied to changes in the CPI

(c) Which option is the best estimate for the percentage of these people who receive Social Security benefits?

(i) A little less than 48%

(ii) More than 50%

(iii) Around 70%

(iv) 100%

(d) Which option is the best estimate for the fraction of these people who receive food stamps?

(i) About a third

(ii) More than half

(iii) About three-fourths

(iv) 100%

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[24] https://www.api.org/-/media/Files/Oil-and-Natural-Gas/Gasoline/US-gasoline-distillate-update.pdf

2.7.3: Study Strategy Survey part 2

Study Strategies for Exams and Quizzes (Part 2)

Look back on the answers you provided in Study Strategies Part 1 (in Unit 2.1). Note: you can do this by going to the learning path for Unit 2.1 and clicking to view your answers for the study strategies survey. What effective strategies will you employ for the next module exam?

Considering your schedule, when will you use each of these strategies? Where will you be? How long will you study for? What materials will you need?

Do you anticipate any barriers to your studying? If so, try and brainstorm ways to overcome those barriers.

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2.8: Has the Minimum wage Kept Up?

INTRODUCTION

In this collaboration, we will be discussing the concept of purchasing power and adjusting for inflation.

Many employers and government agencies periodically increase salaries/benefits to adjust for inflation (increasing cost of goods and services). Those adjustments are often referred to as COLA which stands for "Cost of Living Adjustment."

In the following example, Mei's employer doesn't offer cost-of-living adjustments. Does Mei's salary have the same buying power in 2021 as it did in 2019? Discuss with your group.

Year	Mei's Annual Salary	Mei's monthly expenses		
		Rent	Groceries	Transportation
2019	$65,000	$850	$390	$130
2020	$65,000	$890	$410	$130
2022	$65,000	$920	$440	$150

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- ratios provide a way of comparing the relative increase or decrease of two variables.
- index numbers are a way of comparing the relative size of a variable over time.

By the end of this collaboration, you should be able to

- use proportional reasoning to find the size of a variable that remains a constant proportion of another variable.
- use index numbers to find the value of a variable relative to time (or another variable).

PROBLEM SITUATION: THE BUYING POWER OF THE MINIMUM WAGE

You found from your work in Preparation 2.8 that the minimum wage did not increase enough from 1997 to 2021 to keep up with the cost of a movie ticket. Remember that the **minimum wage** is the minimum amount employers must pay their employees required by federal law. A movie ticket is only a single product that someone might buy. To compare the purchasing power of the minimum wage from 1938 to 2021, you need more information about more products. You will use a tool called an index number.

Index numbers are a way to compare the relative change or difference in a data set, such as the prices of products. The change can be measured over time or over different geographic regions. There are different types of index numbers. One type is a measure of average relative change in data. In this instance, one data point is called the base and is assigned the value of 100 (meaning it represents 100%). The other figures are adjusted in proportion to the base. The plural form of the word *index* is **indices**.

The following two questions will help you understand index numbers. You will begin by looking at one product: **the Big Mac**.

(1) You are going to use 1992 as the base year for our Big Mac Price Index. Prices of the Big Mac vary with time. In 1992, a Big Mac cost about $2.19.

(a) In 1968, a Big Mac cost about $0.49. Compare the prices by finding the ratio of the 1968 price to the 1992 price. Express the ratio as a percent. Round to the nearest percent.

(b) In 2021, a Big Mac cost about $4.93.[28] Compare the prices by finding the ratio of the 2021 price to the 1992 price. Express the ratio as a percent. Round to the nearest percent.

(c) Now let's think about how to use the answers from (a) and (b) to complete the Big Mac Price Index for the years 1968, 1992, and 2021. In parts (a) and (b), what did you use as the base? What is the index for 2021? Think about this in your group before completing the table.

Big Mac Price Index

Year	Index Number

1968	
1992	
2021	

The Value of the Dollar

Now you have seen how an index is created and what it means. You are now going to look at a real index that the U.S. government uses to track changes in the prices of many items over time. It is called the Consumer Price Index (CPI). You will use a version called the **Urban Consumer Price Index (CPI-U)**.

The Bureau of Labor Statistics publishes the CPI-U each month. The CPI is a measure of the weighted average of a “basket of consumer goods and services.” The CPI-U basket includes transportation, food, medical care, housing, apparel, recreation, and education. The CPI represents the cost of that basket of goods and services in *base-year* dollars. This allows us to compare the buying power of a dollar in one year as compared to another. If the CPI is more than 100, then average prices are higher than in the base year. If the CPI is less than 100, then average prices are lower than in the base year. Because the CPI compares prices over time, it is one of the most frequently used statistics for identifying periods of **inflation** or **deflation** (increase or decrease of prices over time). An increase in the CPI indicates *inflation* in prices. A decrease indicates *deflation*. The following table shows the indices for selected years from 1913 to 2021.[29]

Year	Index		Year	Index		Year	Index
1913	9.9		1949	23.8		2005	195.3
1914	10.0		1950	24.1		2006	201.6
1915	10.1		1981	90.9		2007	207.3
1921	17.9		1982	96.5		2008	215.3
1922	16.8		1983	99.6		2009	214.5
1923	17.1		1984	103.9		2010	218.1
1924	17.1		1985	107.6		2012	229.6
1925	17.5		1996	156.9		2014	236.0
1936	13.9		1997	160.5		2015	237.0
1937	14.4		1998	163.0		2016	240.0
1938	14.1		1999	166.6		2017	245.1
1939	13.9		2000	172.2		2018	251.1
1940	14.0		2001	177.1		2019	255.7
1946	19.5		2002	179.9		2020	258.8
1947	22.3		2003	184.0		2021	271.0
1948	24.1		2004	188.9			

(2) What year is used as the base for the CPI-U? What does the base mean?

(3) How does the 2021 dollar compare in value to the 1983 dollar?

(4) How does the 1949 dollar compare in value to the 1983 dollar?

(5) Who had more purchasing power? (Provide quantitative information to support your answer.)

- The person making minimum wage ($0.25 per hour) in 1938.
- The person making minimum wage ($7.25 per hour) in 2021.

FURTHER APPLICATIONS

(6) Find the price of gasoline in 1981 and in 2021. Give the sources for your information. Use the CPI-U table in the lesson to evaluate the statement, “Gasoline was more expensive in 2021 than in 1981.” Provide mathematical information to justify your explanation.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[28] https://www.economist.com/big-mac-index

[29] https://www.usinflationcalculator.com/inflation/consumer-price-index-and-annual-percent-changes-from-1913-to-2008/

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2.8.1: Preparation 2.8

The following information will be used in the next collaboration.

The United States has a federal **minimum wage**. This means that there is a law that requires employers to pay employees a minimum amount. Many states also have their own minimum wage laws that require a higher wage than the federal law. The minimum wage started in 1938 as part of a law that protected the rights of workers in many ways. The minimum wage in 1938 was set at $0.25 per hour.[25]

Since 1938, Congress has increased the minimum wage many times to account for inflation. **Inflation** is when prices increase over time. Maybe you have heard stories from older people about how a cup of coffee used to cost a dime or a gallon of gasoline was less than a dollar. Prices for these items have increased due to inflation. **Deflation** is when prices decrease over time.

When deciding if the minimum wage should be increased, people often talk about the purchasing power of the wage. In Lesson 2.8, you will answer the question: Did the minimum wage in 1938 have more purchasing power than the minimum wage in 2021? You will start by thinking about the specific example of buying a movie ticket for the following questions.

(1) The minimum wage in 1997 was $5.15 per hour. The minimum wage in 2021 was $7.25 per hour.

(a) What was the absolute change in the minimum wage from 1997 to 2021?

(b) What was the relative change in the minimum wage from 1997 to 2021? Round to the nearest tenth of one percent.

(2) The average price of a movie ticket in 1997 was $4.59.[26] Which statement is correct?

(i) In 1997, a person earning minimum wage had to work less than an hour to earn enough for a movie ticket.

(ii) In 1997, a person earning minimum wage had to work more than an hour to earn enough for a movie ticket.

(3) The average price of a movie ticket in 2021 was $9.57.[27] Which statement is correct? Be prepared to explain your answer in class.

(i) In 2021, a person earning minimum wage had to work less than an hour to earn enough for a movie ticket.

(ii) In 2021, a person earning minimum wage had to work more than an hour to earn enough for a movie ticket.

In Collaboration 2.8, you will be discussing the concept of purchasing power and adjusting for inflation.

Here are some examples of such adjustments:

- Social Security benefits: https://www.ssa.gov/cola/
- VA benefits: https://www.benefits.va.gov/compensation/rates-index.asp

After Preparation 2.8 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 2.8, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
calculate and interpret relative change.	
understand the minimum wage.	

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[25] https://www.dol.gov/general/aboutdol/history/flsa1938

[26] https://www.statista.com/statistics/187091/average-ticket-price-at-north-american-movie-theaters-since-2001/

[27] https://www.statista.com/statistics/187091/average-ticket-price-at-north-american-movie-theaters-since-2001/

2.8.2: Exercise 2.8

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) The Consumer Price Index (CPI) is an example of an index number that measures the cost of a basket of goods and services over time. It can be used to show inflation and deflation of prices.

(ii) Inflation is calculated by finding the relative change in the CPI.

(iii) The CPI is a ratio that compares the cost of goods to a base year in the form of a percentage. This is an example of how ratios are used for comparisons.

(iv) The CPI always increases over time.

DEVELOPING SKILLS AND UNDERSTANDING

(2) The price of a first-class stamp has increased rapidly since the 1970s. The graph below shows the price (in cents) of a first-class stamp from 1917 to 2022.[30]

Line graph showing the price (in cents) of a first-class stamp from 1914 to 2022. Y-axis represents the price (in cents), ranging from 0 to 60. X-axis represents the years, ranging from 1914 to 2014. Data line is shown starting in between 0 and 10 cents in 1914, remaining in between 0 and 10 cents until 1974, increasing steadily to finish just below 50 cents in 2014, and then to about 60 cents in 2022.

(a) Estimate the absolute increase in the price of a first-class stamp from 2012 to 2022.

(i) $0.05

(ii) $0.15

(iii) $0.25

(v) $0.33

(b) Estimate the relative increase in the price of a first-class stamp from 2006 to 2022.

(i) 10%

(ii) 25%

(iii) 33%

(iv) 50%

(c) Suppose the absolute increase in the price of a first-class stamp from 2022 to 2032 is the same as it was from 2012 to 2022. What would the cost of a first-class stamp be in 2032?

(i) $0.66

(ii) $0.75

(iii) $0.80

(iv) $0.90

(d) Suppose the relative increase in the price of a first-class stamp from 2022 to 2032 is the same as it was from 2012 to 2022. What would the cost of a first-class stamp be in 2032?

(i) $0.60

(ii) $0.66

(iii) $0.80

(iv) $0.90

Urban Consumer Price Index (CPI-U)

For Questions 3 and 4, refer to the CPI-U table in Collaboration 2.8. This table gives the cost of a representative basket of goods and services in base-year dollars (1983). The table shows the indices for selected years from 1913 to 2021. The Consumer Price Index (CPI-U) provides an indication of how the price of consumer goods changes over time.

(3) In 2006, a 12-ounce coffee at Starbucks cost \$1.46. In 2021, that same cup of coffee was \$1.85.

(a) What was the relative increase in the price of the cup of coffee from 2006 to 2021? Round to the nearest one percent.

(b) What was the relative increase in the CPI-U from 2006 to 2021? Round to the nearest one percent.

(c) Which of the following statements is true?

- (i) From 2006 to 2021, the price of coffee at Starbucks increased at a lower rate than the CPI-U.
- (ii) From 2006 to 2021, the price of coffee at Starbucks increased at the same rate as the CPI-U.
- (iii) From 2006 to 2021, the price of coffee at Starbucks increased at a higher rate than the CPI-U.

(4) The rent on a building in 2008 was \$1,350 per month. The landlord changes the rent relative to changes in the CPI-U. Use the CPI-U to calculate the rent in 2021. Round to the nearest dollar.

(5) In Exercise 2.7, you examined productivity data for the G7 nations. The graph below shows another way of comparing productivity data using an index. Use the graph to answer the following questions. **Note**: These data are not the same as in the earlier problem.

Vertical bar graph showing the comparison productivity data using an index. Y-axis represents the index (UK = 100), ranging from 80 to 140. X-axis represents the countries. Japan = in between 80 and 90 UK = N/A Canada = in between 100 and 110 Germany = in between 110 and 120 France = just over 100 Italy = in between 90 and 100 G7 excluding UK = at 100 US = about 140

(a) How does Japan’s productivity compare to that of the United Kingdom?

- (i) Japan has a higher rate of productivity.
- (ii) Japan has a lower rate of productivity.
- (iii) Japan has about the same rate of productivity.

(b) Which two countries have about the same rate of productivity?

(c) Which country is used as the base?

(d) What does the vertical scale represent?

- (i) Productivity in U.S. dollars
- (ii) Productivity in Euros
- (iii) Index numbers (percentages relative to a base)
- (iv) Population of each country

(e) Which of the following statements are true?

- (i) The U.S. productivity is about \$139.00 per worker.
- (ii) The U.S. productivity is about \$39.00 per worker more than the U.K. productivity.
- (iii) The U.S. productivity is 39% higher than the U.K. productivity.
- (iv) The U.S. productivity is 139% higher than the U.K. productivity.

(6) Data from the Census of Population and Housing surveys of 1974, ,1989, 2009, and 2021; the National Association of Realtors; and the U.S. Census Bureau are shown below.

	1974	1989	2009	2021
Median Family Income	\$13,020	\$34,400	\$60,600	\$70,780
Median Price of House	\$86,751	\$84,622	\$185,000	\$346,900

In which of these four years was a house most affordable?

- (i) 1974
- (ii) 1989
- (iii) 2009

(iv) 2021

MAKING CONNECTIONS ACROSS THE COURSE

Now you will be expected to conduct Internet searches to find information. You will begin to encounter questions, such as the next one, that require you to identify the information you need and find the information from a reliable source.

(7) According to Our World In Data, there were around 4,500,000 confirmed Covid-19 cases in Canada by the end of 2022. About 49,000 people in Canada had died of Covid-19 by the end of that year. In the United States, around 100,000,000 Americans were infected with Covid-19 by the end of 2022, and about 1,100,000 of those people died.

(a) Use estimation to identify which of the following statements are true. There may be more than one correct answer.

(i) There are more Covid-19 cases in Canada than in the United States.

(ii) The Covid-19 infection rate was higher in Canada than in the United States.

(iii) The death rate among people infected with Covid-19 is about the same in Canada as it is in the United States.

(iv) By the end of 2022, the United States had about 20 times as many Covid-19 cases as Canada and about nine times the population.

(b) Suppose that a new experimental drug being tested in Canada showed that it can decrease the death rate among people infected with Covid-19 by 18%. If this drug had been used by all Canadians infected with Covid-19, how many people would you have expected to die from Covid-19? Round to the nearest hundred people.

(c) Suppose that the drug from (b) has the same influence on Americans infected with Covid-19. If the drug had been used by all Americans infected with Covid-19, how many people would you have expected to die from Covid-19? Round to the nearest thousand people.

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[30] https://about.usps.com/who-we-are/postal-history/domestic-letter-rates-since-1863.htm

2.9: How Does Your City Stack Up?

INTRODUCTION

Occasionally on the radio or in the news you will see reports about the best medium size cities, or the safest towns, or best vacation spots. Do you ever wonder how those are determined? Typically, these reports are not based on a single factor, but take into consideration a number of different ways that a person might judge "best." This collaboration includes an exploration of the type of data that is used as a basis for those claims. This collaboration also demonstrates the convenience of spreadsheets when working with large data sets.

The following chart is one such data set:

City	Population (2019)	Cost-of-Living index (2022)	Job growth (2021)	Med. household income (2019)	Income growth (2015-19)
Anchorage, AK	288,000	148.6	-6.9%	$86,986	3.5%
Bridgeport, Stamford, Norwalk, CT	944,177	135.7	-5.3%	$103,421	7.5%
Columbia, SC	133,114	92.1	-2.8%	$55,310	6.4%
Danville, VA	39,070	76.1	-7.6%	$37,186	6.1%
El Paso, TX	681,728	85.9	-2.8%	$49,955	10.1%
Fargo, ND-MN	245,471	91.4	-3.6%	$63,551	8.9%
Grand Junction, CO	64,289	106.2	-2.2%	$57,301	10.5%
Honolulu, HI	347,397	194.8	-7.0%	$80,513	6.7%
Ithaca, NY	30,014	111.2	-0.9%	$56,947	7.6%
Joplin, MO	51,818	84.9	-1.1%	$47,768	8.8%
Knoxville, TN	187,603	85.3	-1.7%	$52,376	9.5%

In the table above, the column labeled the Cost of Living (COL) Index uses the national average for cost of living as the "base" for comparison. So a city with a COL of 100 would have a COL that is the same as the national average.

Discuss the following questions in your group.

- Which cities have a cost of living above the national average?
- Which cities have a cost of living below the national average?
- Name the city with a cost of living closest to the national average.

Look at the row for Columbia, SC. Answer the following questions in your group:

- What is the population of Columbia, SC? What is the number in words?
- Cost-of-Living Index: The CoL index for Columbia, SC is 92.1. State in your own words what this means.
- Median Household Income: What does a median of 55,310 mean? Why do you think a median was used versus an average/mean?
- Income growth: What do you think this means?

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- multiple sources and types of data are important for making judgments and decisions.

By the end of this collaboration, you should be able to

- draw appropriate conclusions from data.
- apply previous learning to new contexts.

PROBLEM SITUATION: DATA ON THE BEST CITIES

In this collaboration, you will compare cities based on various sources of data selected for the purpose of comparison. The following table includes data from 20 cities.

City	Population (2019)	Cost-of-Living index (2022)	Job growth (2021)	Med. household income (2019)	Income growth (2015-19)
Anchorage, AK	288,000	148.6	-6.9%	$86,986	3.5%
Norwalk area, CT	944,177	135.7	-5.3%	$103,421	7.5%
Columbia, SC	133,114	92.1	-2.8%	$55,310	6.4%
Danville, VA	39,070	76.1	-7.6%	$37,186	6.1%
El Paso, TX	681,728	85.9	-2.8%	$49,955	10.1%
Fargo, ND-MN	245,471	91.4	-3.6%	$63,551	8.9%
Grand Junction, CO	64,289	106.2	-2.2%	$57,301	10.5%
Honolulu, HI	347,397	194.8	-7.0%	$80,513	6.7%
Ithaca, NY	30,014	111.2	-0.9%	$56,947	7.6%
Joplin, MO	51,818	84.9	-1.1%	$47,768	8.8%
Knoxville, TN	187,603	85.3	-1.7%	$52,376	9.5%
Little Rock-North Little Rock, AR	741,960	86.8	-2.20%	$57,037	8.90%
Morgantown, WV	138,176	86.6	-2.80%	$49,120	7.50%
Olympia, WA	152,949	144.4	-4.40%	$78,757	8.80%
Portland-South Portland, ME	535,663	134.1	-3.50%	$70,406	6.90%
Reno-Sparks, NV	471,519	116.6	-6.10%	$64,124	10.60%
St. Cloud, MN	161,641	89.7	-2.30%	$60,087	7.30%
Topeka, KS	125,904	84.5	-1.20%	$54,783	7.40%
Worcester, MA	185,877	128.5	-5.10%	$64,171	8.80%
York-Hanover, PA	448,273	90.1	-1.70%	$63,067	5.80%

(1) A website lists both Anchorage, Alaska and Danville, Virginia on their list of “10 Best Places to Live Comfortably.”

(a) Use the information in the table to decide whether you are more likely to “live well” financially in Anchorage or Danville. Use estimation skills to justify your decision.

(b) Calculate how much a person in Danville would need to make in order to have the same “buying power” as someone earning the median income in Anchorage.

(2) Use the tables to answer the following questions comparing Olympia, Washington, and St. Cloud, Minnesota.

(a) In which of the two cities are there more families with an income above $60,000?

(b) In which of the two cities are there more families with an income between $60,000 and $78,000?

(3) Review the information for Reno-Sparks, Nevada. What do you think was happening in the job market from 2015 to 2019?

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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2.9.1: Preparation 2.9

In Collaboration 2.9, you will explore an example data set similar to those used by magazines to create their “best cities” lists for various parameters. A sample of this sort of data is given below:

Notes on the table:

- The cost of living index is based on the national average of 100.
- Population figures are from the United States Census Bureau (2019 estimates)
- Cost of living index figures are from Numbeo (as of March 2022)
- Job growth figures are from the Bureau of Labor Statistics (2021 annual averages)
- Median household income figures are from the United States Census Bureau (2019 estimates)
- Income growth figures are from the United States Census Bureau (2015-2019 estimates)

City	Population (2019)	Cost-of-Living index (2022)	Job growth (2021)	Med. household income (2019)	Income growth (2015-19)
Akron, OH	197,597	79.5	-5.3%	$42,452	8.3%
Albany-Schenectady-Troy, NY	886,188	100.4	-7.8%	$65,229	6.2%
Albany, GA	72,130	80.8	-4.7%	$32,981	8.6%
Albuquerque, NM	560,218	92.9	-1.5%	$51,949	6.3%
Alexandria, LA	47,334	82.8	-5.5%	$41,269	8.6%
Amarillo, TX	199,371	88.6	-2.1%	$55,032	10.4%
Ames, IA	66,258	89.7	-2.4%	$53,393	9.8%
Anchorage, AK	288,000	148.6	-6.9%	$86,986	3.5%
Anderson, SC	197,575	85.5	-3.5%	$47,081	8.9%
Ann Arbor, MI	123,851	96.4	-4.4%	$74,378	12.4%
Anniston-Oxford, AL	115,527	83.8	-4.9%	$38,784	8.3%
Asheville, NC	92,870	111.9	-2.1%	$56,343	10.1%
Auburn-Opelika, AL	166,836	87.7	-1.7%	$50,429	8.9%
Augusta-Richmond County, GA-SC	202,518	88.4	-4.8%	$47,982	8.8%

(1) Based on the sample given, identify each of the following values:

(a) Which city has the

(i) highest cost of living (COL)?

(ii) the lowest cost of living (COL)?

(b) What is the median household income in Ames, Iowa?

(c) What does the median tell you about the population of Ames, Iowa?

(d) Which city had the largest decrease in jobs?

(e) The Data USA estimated that in 2020, the Hispanic population in Amarillo, TX was approximately 66,200.[31] What percentage of Amarillo's population is Hispanic? Round to the nearest tenth of a percent.

(2) Find the mean and median of the populations in the list. You may use any appropriate technology, such as a spreadsheet or a calculator.

(a) What is the mean of the populations?

(b) What is the median of the populations?

After Preparation 2.9 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 2.9, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
perform basic operations using quantities such as integers, fractions, or decimals with the aid of technology.	
calculate a ratio and write the result as a percent.	
calculate and apply the measures of central tendency.	

---

[31] "Amarillo, TX," *Data USA*, accessed March 2, 2023. https://datausa.io/profile/geo/amarillo-tx/.

2.10: Module 2 Review Survey

As with Module 1, you should assess your understanding of Module 2 to prepare for the Module 2 test. Your instructor may give you specific assignments for your review in addition to this self-assessment.

Assessing Your Understanding

The table on the following page lists the Module 2 concepts and skills you should understand. After completing it, you will be able to prioritize your review time more effectively.

Assess your understanding:

- Go through the topics list and locate each concept or skill in the Module 2 student workbook or in the online activities.
- If you recently used the skill and feel confident that you did it correctly, rate your understanding a 4 or 5.
- If you remember the topic but could use more practice, rate your understanding a 3.
- If you cannot remember that skill or concept, or still find it difficult, rate your understanding a 1 or 2.

Now that you have done an initial rating of your understanding, it is time to begin reviewing. The goal is to have a confidence rating of 4 or 5 on all the topics in the table when you have finished your review of Module 2.

Things to consider for reviewing:

1. Start at the beginning of the module and review the material in the Preparations, Collaborations, and Exercises, and your notes on the skills and concepts you rated 3 or below.
2. Select a few questions to do.
3. Once you have finished, remember to check the solutions. If you are not sure if you have done the problems correctly, check with your instructor, other classmates, your previous work or work with a tutor in the learning center or online.
4. Rate your confidence on this skill again. If you understand the concept better, rate yourself higher. Begin a list of topics that you want to review more thoroughly.

For topics that you need to review more thoroughly, make a plan for getting additional assistance by studying with classmates, visiting your instructor during office or virtual office hours, working with a tutor in the learning center or online, or looking up additional information on the Internet.

Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Module 2 Concept or Skill	Rating
Using Ratios	
Understand meaning of equivalent ratios in context (2.1)	
Use units with ratios (2.1)	
Calculate a unit rate (2.2)	
Use ratios and proportionality to calculate new values (2.2)	
Interpret and use index numbers to calculate new values (2.8, 2.9)	
Applications of Percentages	
Calculate and interpret absolute change between two quantities (2.3, 2.5)	
Calculate and interpret relative change between two quantities (2.3, 2.5)	
Calculate and interpret absolute change between two percentages (2.3)	
Calculate and interpret relative change between two percentages (2.3)	

Make and interpret comparisons of absolute measurements versus relative measurements (2.4, 2.5)	
Graphical Displays	
Read and interpret pie graphs, bar graphs, and line graphs (2.4)	
Recognize distortion of graphs due to different scales (2.4)	
Calculate absolute and relative change from a graph (2.4)	
Measures of Central Tendency	
Calculate mean, median, and mode of a data set (2.6, 2.7)	
Interpret the meaning of and differences between the mean, median, and mode (2.6, 2.7)	

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3.1: Salary per Minute

INTRODUCTION

In 1998, “NASA lost its $125-million Mars Climate Orbiter because spacecraft engineers failed to convert from English to metric measurements when exchanging vital data before the craft was launched. A navigation team at the Jet Propulsion Laboratory used the metric system of millimeters and meters in its calculations, while Lockheed Martin Astronautics in Denver, which designed and built the spacecraft, provided crucial acceleration data in the English system of inches, feet, and pounds. As a result, JPL engineers mistook acceleration readings measured in English units of pound-seconds for a metric measure of force called newton-seconds.”

"People make mistakes all the time," said Carl Pilcher, the agency's science director for solar system exploration. "I think the problem was that our systems designed to recognize and correct human error failed us.”

Even though this was an expensive mistake ($125 million!), the mistake allowed the engineers to recognize that their system for catching errors (like conversion mistakes) was not good enough and had to be improved. Not all mistakes cost $125 million, but mistakes often provide an opportunity to make something better!

In this collaboration, we will be discussing how to convert units both between and within the metric and the US measurement systems. The procedure is called dimensional analysis, or the Unit Factor Method. Dimensional analysis is commonly used by healthcare professionals, computer scientists, or NASA engineers, just to mention a few professions.

Quick Check-in:

Please discuss the following questions with your group:

- What specific concept from Unit 2.9 do you feel fairly confident about?
- Are there any specific concepts from Unit 2.9 that you find challenging?

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- the units found in a solution may be used as a guide to the operations required in the problem—that is, factors are positioned so that the appropriate units cancel.
- units can add meaning to the numbers that result from calculations.

By the end of this collaboration, you should be able to

- write a rate as a fraction.
- use a unit factor to simplify a rate.
- use dimensional analysis to help determine the factors in a series of operations to obtain an equivalent measure.

PROBLEM SITUATION: USING DIMENSIONAL ANALYSIS

Dimensional analysis is a method of setting up problems that involves converting between different units of measurement. It is also called *unit analysis* or *unit conversion*. Many professionals—including pharmacists, dieticians, lab technicians, and nurses—use dimensional analysis. It is also useful for everyday conversions in cooking, finances, and currency exchanges. Many people can do simple conversions without dimensional analysis; however, they will likely make mistakes on more complex problems.

The advantage of using dimensional analysis is that it is a way to check your calculations. While it is always important that you develop your own methods to solve problems, this is a time when you are encouraged to learn and use a specific method. Once you have learned dimensional analysis, you can decide when to use it and when to use other methods.

(1) (a) According to Toyota’s website, a 2023 Prius can get an estimated 57 miles per gallon (mpg) in the city and 56 mpg on the highway.[1] How many miles will you be able to drive in the city if you have 4.5 gallons of gas?

(b) How many gallons of gas will you need to drive 3,450 miles on the highway? Round your answer to one decimal place.

Converting Your Paycheck

This is an example of how to use dimensional analysis to solve a problem.

Sample Question: Your paycheck for two weeks came out to $1,200. You work eight hours a day, five days a week. How much are you making per minute in cents? (**Hint**: You will need to use dimensional analysis to solve this problem.)

Answer: First, decide what you are looking for. Here you are looking for the rate of cents per minute, which can be written as:

$$\frac{c}{min}$$

Now, identify a rate that we know has the same numerator)the part of the fraction that represents a count of the number of parts) as the rate we are looking for. Since cents converts to dollars, we have:

$$\frac{100c}{\$1} \rightarrow \frac{c}{min}$$

Next, we want to cancel the “$” (since this unit is not in the answer we are looking for). To cancel the “$” we can multiply by a unit ratio with “$” in the *numerator*. Since $1,200 is the amount we make in two weeks, we can multiply by that ratio:

$$\frac{100c}{\$1} \times \frac{\$1200}{2\ weeks} \rightarrow \frac{c}{min}$$

Now, we need to cancel the unit “weeks”, so our next ratio must have “weeks” in the numerator:

100 cents over $1 times $1200 over 2 weeks times week over 5 days goes to cents over minutes.

Continuing this process, we finally get to the unit factor for “minute”:

100 cents over $1 times $1200 over 2 weeks times week over 5 days times day over 8 hours times hour over 60 minutes goes to cents over minutes.

(2) Is the resulting calculation reasonable? Discuss with your group.

(3) Many states have banned texting while driving because it is dangerous, but many people do not think that texting for a few seconds is that harmful. Suppose you are driving 60 miles/hour and you take your eyes off the road for four seconds. How many feet (ft) will you travel in that time? **Hint**: Start with the unit you are looking for (ft). Then, create a chain of ratios, starting with one where “ft” is in the numerator that will cancel all other units. Remember that there are 5,280 ft in one mile.)

(4) In Module 2 we examined population densities and used these to calculate projected populations. The population density of Tokyo[2] is 6,158 people per square kilometer (km). Use dimensional analysis to calculate how many people would live in the nation of Japan[3], which comprises an area of approximately 378,000 square km, if the entire nation was as dense as the city of Tokyo.

Dimensional analysis is an essential skill in dosage calculations, which are necessary for any nurse to master. The following two questions will be more difficult to answer without using dimensional analysis.

(5) Nurses are often required to calculate dosages. That is, they must check the order that a doctor has given for the administration of a drug and decide whether the dosage is correct. To calculate correctly they must convert between different metric units. For example, 1,000 milligrams (mg) = 1 gram (g); and 1,000 micrograms (mcg) = 1 mg.

Suppose a doctor has ordered a dose of 0.1 gram of a medication. The drug comes in a solution concentration of 200 mg per milliliter. How many milliliters of this solution is required?

(6) Now, calculate how many milliliters you would need to administer 500 mg from a dosage concentration of 1 g per 3 mL.

FURTHER APPLICATIONS

(7) Do an Internet search for “dimensional analysis” or “unit analysis” or “unit factor method.” Find at least one site that provides examples of how to make conversions using this technique.

(a) Record the site name and URL address.

(b) Copy one example of a conversion using dimensional analysis (as shown on the site).

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

---

[1] https://www.toyota.com/prius/

[2] https://www.metro.tokyo.lg.jp/ENGLISH/ABOUT/HISTORY/history03.htm

[3] https://www.cia.gov/the-world-factbook/countries/japan/#geography

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3.1.1: Preparation 3.1

There was no Exercise for Unit 2.9. However, the content presented here will prepare you for Collaboration 3.1.

Equivalent Fractions

Four circles showing the following: 1st circle: Shown as one whole 2nd circle: Shown as two halves (one half is highlighted) 3rd circle: Shown as four quarters (two quarters are highlighted) 4th circle: Shown as eight eighths (four eighths are highlighted) The same part of an object: 1/2 = 2/4 = 4/8

The same part of an object: 1/2 = 2/4 = 4/8

Two fractions are equivalent if they have the same value or represent the same part of an object.

For example, the figure shows that $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ all represent the same part of a whole. They are equivalent fractions.

Recall that the **denominator** of a fraction represents the number of parts into which the whole has been divided. The **numerator** represents a count of the number of parts.
So, $\frac{4}{8}$ means that the whole is divided into eight equal parts, and four of these parts are counted.

(1) Write another fraction that are equivalent to $\frac{1}{2}$, other than $\frac{2}{4}$ or $\frac{4}{8}$.

Simplest Form of a Fraction

The fraction $\frac{50}{100}$ is equivalent to $\frac{1}{2}$. Note that you can write

$$\frac{50}{100}=\frac{1\cdot 50}{2\cdot 50}=\frac{1}{2}\cdot 1$$

The above calculation shows that both 50 and 100 can be written as a number times 50. You say that 50 and 100 have a "common factor of 50." Another way to think of this is that the number "1" (written as $\frac{50}{50}$) is embedded in the fraction $\frac{50}{100}$.

The number "1" is a special number in mathematics because if you multiply any number by 1, you get a result that is equivalent to the original.

Simplify and Simplest Form

The word, **simplify** means to rewrite a fraction in an equivalent form with smaller numbers. The **simplest form** means that the fraction is written using the smallest possible whole numbers. In general, answers should always be given in simplest form unless the question specifically calls for a different form

For example:

By dividing $\frac{50}{50}$ to get 1, you simplify $\frac{50}{100}$ to $\frac{1}{2}$.

Caution! It is common to say that you "cancel" the 50 from the numerator (top number) and denominator (bottom number) and write the fraction in "reduced form." This language is misleading for two reasons. While numbers can be "canceled" by dividing by 1, it is better to think about the operation (add, subtract, multiply, divide) so you can understand why it works.

Second, the value of the simpler fraction is the same as the original fraction, but the word "reduced" implies that the "reduced fraction" represents a smaller quantity. It can be confusing, so the terminology *simplest form* makes more sense.

(2) Write each fraction in its simplest form.

(a) $\frac{24}{36}$

(b) $\frac{9}{12}$

(c) $\frac{20}{30}$

(d) $\frac{35}{28}$

(3) Compare the two fractions to determine if they are equivalent.

(a) $\frac{3}{6}$, $\frac{6}{16}$

(b) $\frac{3}{4}, \frac{6}{9}$

(c) $\frac{10}{8}, \frac{5}{4}$

(d) $\frac{10}{24}, \frac{5}{8}$

Multiplying Fractions

The fact that common factors in the denominator and numerator of a number can be divided to make 1 can be used to make multiplying fractions easier. Consider the following multiplication problem.

Image showing two-thirds times seven-eighths.

If you see that there is a common factor of 2 in the numerator and denominator before multiplying, you can divide the common factors first. This makes the multiplication easier because you have smaller numbers to work with and the simplification is complete.

Image showing multiplication procedure for two-thirds times seven-eighths.

This is an important concept when working with ratios with units. You will learn more about this below.

Dividing Fractions

The division of fractions is not the most intuitive of procedures. So, instead of asking you to memorize the division rule, let's start by looking at a few examples.

Suppose you have $48 to spend on going to the movies during a month. How many tickets can you buy in a month? A movie ticket costs $8. One way to think about this is that you want to know how many groups of $8 there are in $48, or 48 ÷ 8.

In the same way, suppose you had $10 to spend on downloading songs for a 1/2 dollar. (For the sake of the mathematics, you are going to express "a half of a dollar" as a fraction instead of as a decimal.) This means you want to know how many 1/2 dollars there are in $10. Your common sense probably tells you that the answer is 20 because every one dollar has two halves. So you multiplied 102. Look at this written as a calculation:

One over two and two over one are called reciprocals.

Division is the same as multiplying by the *reciprocal*. Here are more examples:

Four divided by a half is the same as four times 2 over one.

Perform the calculations indicated in Questions 4–7. Write your answer as a whole number or a fraction in reduced form. **Note**: If your answer is an improper fraction (the numerator is bigger than the denominator), leave it that way. Don't convert it to a decimal or a mixed number. Improper fractions are easier to work with and easier to type into a computer.

(4) $10 \cdot \frac{2}{5} =$

(5) $8 \div \frac{3}{4} =$

(6) $\frac{7}{8} \div \frac{5}{8} =$

(7) $\frac{7}{8} \cdot \frac{8}{5} =$

(8) Jarrod is helping with his daughter's school assembly. Every student will receive a decorative ribbon to wear. The ribbons have to be 2/3 of a foot long. A local store donates 30 feet of ribbon. How many decorative ribbons can Jarrod make?

(9) Lorinda has started to think about saving for retirement. She reads a recommendation that says she should save at least 3/10 of her income because she is over 40 years old. Lorinda makes $42,000 a year. How much should she save in one year according to this recommendation?

Conversion Factors

A fraction that is a ratio of quantities can be equivalent to 1, even when the numerator and denominator are not the same number. However, it is necessary that the numerator and denominator represent equivalent quantities. For example, the following fractions are all forms of 1:

16 ounces over 1 pound. 1 mile over 5280 feet. 60 minutes over 1 hour.

These types of ratios are sometimes called **conversion factors** because they can be used to convert between units.

(10) Complete the following fractions to make a conversion factor. Look up the conversion factor in a reference book or on the Internet if you do not know it.

(a) 1 yard over ? feet.

(b) ? fluid ounces over 1 gallon.

(c) ? inches over 1 yard.

Converting for Speed

A rate, like 35 miles per hour, can be expressed as a fraction: $\frac{35\ miles}{1\ hour}$. You can use the conversion factor of 60 minutes per 1 hour to convert the rate to the units of miles per 1 minute. The example below shows how to set up a multiplication problem with the rate and the conversion factor to convert miles per hour to miles per minute.

35 miles over 1 hour times 1 hour over 60 minutes.

Notice that the conversion factor is written so that the units of "hours" are in the numerator. This is because you want the "hours" to divide out in the same way that common factors divided out in the multiplication problems above. This leaves the units of miles/minute as shown below (on the right).

Multiplication of Fractions	Unit Conversion
$\frac{2}{5}\times\frac{15}{7}\rightarrow\frac{2}{3}\times\frac{5\times 3}{7}\rightarrow\frac{6}{7}$	35 miles over 1 hour times 1 hour over 60 minutes equals 35 miles over 60 minutes which equals 0.58 miles over 1 minute.

Both $\frac{35\ miles}{hour}$ and $\frac{0.58\ mile}{minute}$ are **unit rates**. Unit rates are ratios that have 1 unit in the denominator. Here are some other examples of unit rates:

- $\frac{35,000\ people}{square\ mile}$
- $\frac{2,500\ calories}{day}$
- $\frac{56,661\ views}{YouTube\ Video}$

You will use this concept in Collaboration 3.1.

This assignment included a large number of vocabulary words that are important. Make sure you fully understand each term listed below:

- Equivalent fraction
- Simplify, simplest form
- Common factor
- Reciprocal
- Unit rate
- Conversion factor

After Preparation 3.1 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = confident and 5 = very confident).

Before beginning Collaboration 3.1, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	Rating from 1 to 5
multiply two fractions.	
divide two fractions.	

understand that a fraction can be simplified by dividing common factors in the numerator and denominator (simplify fractions).	
understand that multiplying by 1 doesn’t change a value.	
be familiar with basic units of measure of length (feet, miles) and time (seconds, hours, minutes).	

3.1.2: Exercise 3.1

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Units can be used to set up conversion problems by using the fact that common factors in the numerator and denominator of a fraction divide out to one.

(ii) It is important to know how many cents you are making per minute.

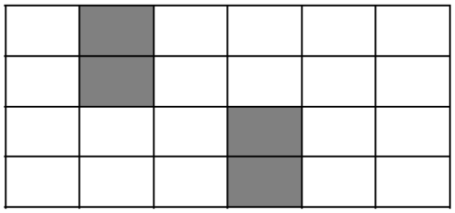
(iii) Units can be used to set up conversion problems by using the fact that common factors in the numerator and denominator of a fraction can be subtracted to equal zero.

(iv) It is possible to convert miles per hour to feet per second.

DEVELOPING SKILLS AND UNDERSTANDING

(2) Use Figure 1 for the following questions.

Figure 1



(a) (i) What fraction of Figure 1 is shaded?

(ii) Shade the same fraction of the area in Figures 2 and 3.

Figure 2

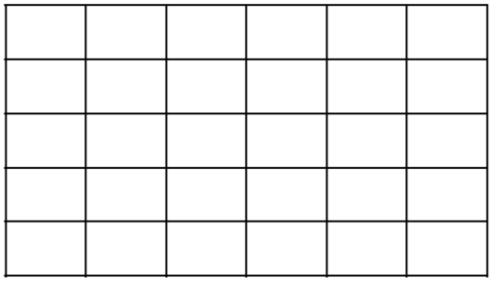
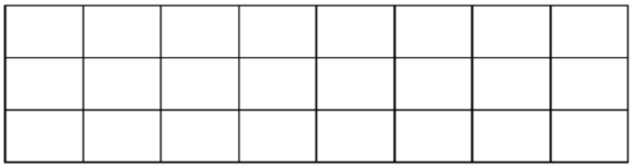


Figure 3



(b) How many boxes did you shade for Figure 2?

(c) How many boxes did you shade for Figure 3?

(3) As of March 2023, Florence Griffith-Joyner held the women's world record for the 100-meter dash. She set the record with a time of 10.49 seconds in 1988.[4] Which of the following calculations are correctly set up to convert this speed into miles per hour?

(i) $\frac{10.49\ seconds}{100\ meters} \cdot \frac{1\ meter}{3.28\ feet} \cdot \frac{5,280\ feet}{1\ mile} \cdot \frac{1\ minute}{60\ seconds} \cdot \frac{1\ hour}{60\ minutes}$

(ii) $\frac{100\ meters}{10.49\ seconds} \cdot \frac{3.28\ feet}{1\ meter} \cdot \frac{1\ mile}{5,280\ feet} \cdot \frac{60\ seconds}{1\ minute} \cdot \frac{60\ minutes}{1\ hour}$

(iii) $\frac{10.49\ seconds}{10\ meters} \cdot \frac{1\ meter}{3.28\ feet} \cdot \frac{5,280\ feet}{1\ mile} \cdot \frac{60\ seconds}{1\ minute} \cdot \frac{60\ minutes}{1\ hour}$

(iv) $\frac{100\ meters}{10.49\ seconds} \cdot \frac{3.28\ feet}{1\ meter} \cdot \frac{1\ mile}{5,280\ feet} \cdot \frac{1\ minute}{60\ seconds} \cdot \frac{1\ hour}{60\ minutes}$

(4) Find the answer to the conversion in the previous question. Round to the nearest tenth of a mile per hour.

(5) A 2023 Toyota Prius hybrid vehicle gets 56 mpg for highway driving. The tank holds 11.3 gallons of fuel.[5] Typically the low fuel warning light comes on when approximately two gallons of fuel remain in the tank. Which of the following calculations can be used to find the distance that can be traveled after the fuel light comes on and before the car runs out of gasoline?

(i) $\frac{1}{2\ gallons} \cdot \frac{56\ miles}{1\ gallon} = 28\ miles$

(ii) $\frac{2\ gallons}{1} \cdot \frac{56\ miles}{1\ gallon} = 112\ miles$

(iii) $\frac{11.3\ gallons}{1} \cdot \frac{56\ miles}{1\ gallon} = 632.8\ miles$

(iv) $\frac{2\ gallons}{1} \cdot \frac{1\ gallon}{56\ miles} = \frac{1}{28} miles$

MAKING CONNECTIONS ACROSS THE COURSE

(6) In Collaboration 1.6, you learned about a water footprint. Part of a person's water footprint is the water used for cleaning. In this question, you will calculate the cost of water for laundry and bathing. You will use the City of New York 2023 rate of \$9.87/100 cubic feet of water. Calculate the cost of each of the following based on this rate. Use the conversion factor of 7.48 gallons per cubic foot.[6]

(a) A standard washing machine uses approximately 50 gallons of water per load.[7] A household washes one load of laundry per week for 52 weeks. Find the total cost per year. Round to the nearest dollar per year.

(b) According to one study, the average American shower lasts for 8.2 minutes and uses 17.2 gallons of water.[8] A person showers once a day for a year. Find the total cost per year. Round to the nearest dollar per year.

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[4] https://olympics.com/en/news/evolution-womens-100m-world-record-flojo

[5] https://www.toyota.com/prius/

[6] https://www.nyc.gov/site/dep/water/residential-water-use.page

[7] ibid.

[8] ibid.

3.2: The Cost of Driving part 1 / The Cost of Driving part 2

The Cost of Driving, Part 1

INTRODUCTION

Greg owns a rental property in Beaufort, SC. His mortgage, insurance, and HOA fees total up to $1,400 per month. In addition, he's expecting the following maintenance expenses over the next year:

- Furnace replacement: $5,500
- Gutters replacement: $2,500
- Landscaping: $2,000

What's the minimum monthly rent Greg should be charging his tenants for the next year, just to break even? As a group, discuss your strategies for solving this problem.

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- units can be used in dimensional analysis to set up calculations.
- precision should be based on several factors, including the size of the numbers used and the precision of the original values. Rounding can produce large differences in results.

By the end of this collaboration, you should be able to

- solve a complex problem with multiple pieces of information and steps.
- use dimensional analysis.
- investigate how changing certain values affects the result of a calculation.

PROBLEM SITUATION: COST OF DRIVING

Jenna's job requires her to travel. She owns a 2020 Toyota 4Runner, but she also has the option to rent a car for her travel. In either case, her employer will reimburse her for the mileage using the rate set by the Internal Revenue Service. In 2023, this rate was set at 65.5 cents per mile.[9] Over the next two lessons you will explore the question of whether it would be better for Jenna to drive her own car or to rent a car.

(1) What do you need to know to calculate the cost of Jenna driving her own car?

(2) What do you need to know to calculate the cost of Jenna renting a car?

Skill Building

This section introduces skills that will help you with the problem situation.

(3) Gas mileage is rated for either city driving or highway driving. Most of Jenna's travel will take place on the highway. For one trip, she drives 150 miles and the price of gas is $3.50/gallon. Her 4Runner gets 18 miles/gallon. If Jenna rents a car, she can request a small, fuel-efficient model such as the Hyundai Elantra, which gets 40 miles/gallon.

(a) Use your estimation skills to compare the cost of gas for the two vehicles. Which one costs more? Explain your answer.

(b) Now, calculate the cost of gas for each vehicle. Think about the discussion on units in Preparation 3.2. Try this on your own first before sharing your ideas in your group. Round to two decimal places.

4Runner:

Elantra:

(4) Using the information below, calculate Jenna's total cost of driving a rental car for a round trip.

- Price of gas: $3.50/gallon
- Length of trip (one way): 193 miles
- Gas mileage of rental car: 40 miles/gallon
- Price of the rental car: $118.98 plus 15.3% tax (Gas is not included in the rental price and the car must be returned to the rental agency with a full tank.)

Work through the following series of calculations to find the total cost of Jenna driving a rental car for a round trip. Enter your answers rounded to two decimal places when necessary.

Round trip distance =

Amount of gas needed for round trip =

Price of gas for round trip =

Total price of rental car =

Total cost for round trip =

(5) Using the information below, calculate the total cost of Jenna driving her own car for a round trip.

- Price of gas: \$3.50 per gallon
- Length of trip (one way): 193 miles
- Gas mileage of Jenna's car: 18 miles per gallon

Maintenance costs for Jenna's car:

- General maintenance (oil and fluid changes): \$45 every 3,000 miles
- Tires: Tires for Jenna's car cost \$960; they are supposed to be replaced every 50,000 miles
- Repairs: The website Edmunds.com estimates repairs on a three-year-old 2020 4Runner will be approximately \$426 per year; this is based on driving 15,000 miles.[10]

Work through this problem in your group. Round to two decimal places in your calculations.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

The Cost of Driving, Part 2

INTRODUCTION

This is the formula that Jenna could use to figure out the total cost of renting a Hyundai Elantra:

$$R = \frac{g \times m}{40} + 137.18$$

Discuss in your group what you think happens to the value of R as g and m change.

R = total cost of renting car

g = price of gas

m = total miles

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- units can be used in dimensional analysis to set up calculations.
- precision should be based on several factors, including the size of the numbers used and the precision of the original values. Rounding can produce large differences in results.

By the end of this collaboration, you should be able to

- solve a complex problem with multiple pieces of information and steps.
- use dimensional analysis.
- investigate how changing certain values affects the result of a calculation.

PROBLEM SITUATION: COMPARING COSTS

Let us return to the problem situation from the previous collaboration (3.2, Part 1). Recall that Jenna's job requires her to travel. She owns a 2020 Toyota 4Runner, but she also has the option to rent a car for her travel. In either case, her employer will reimburse her for the mileage using the rate set by the Internal Revenue Service. In 2023, this rate was set at 65.5 cents per mile.[11]

(1) Calculate how much profit Jenna makes after she pays her expenses in the situation where she rents a car, and in the situation where she drives her own car. The information for both situations was used in Question 4 and 5 in Part 1. It is displayed again below. Enter your calculations in the table below. Round to two decimal places in your calculations.

Info from Q4, 3.2 part 1	Info from Q5, 3.2 part 1
• Price of gas: \$3.50/gallon • Length of trip (one way): 193 miles • Gas mileage of rental car: 40 miles/gallon • Price of the rental car: \$118.98 plus 15.3% tax (Gas is not included in the rental price and the car must be returned to the rental agency with a full tank.)	• Price of gas: \$3.50/gallon • Length of trip (one way): 193 miles • Gas mileage of Jenna's car: 18 miles per gallon Maintenance costs for Jenna's car: • General maintenance (oil and fluid changes): \$45 every 3,000 miles • Tires: Tires for Jenna's car cost \$960; they are supposed to be replaced every 50,000 miles • Repairs: The website Edmunds.com estimates repairs on a three-year-old 2020 4Runner will be approximately \$426 per year; this is based on driving 15,000 miles.
Total cost of driving a rental car for a round trip = **\$170.96**	Total cost of Jenna driving her own car for a round trip = **\$99.22**

	Total Cost to Jenna	Cost per mile	Jenna's Profit After Expenses
Jenna's own car	(a)	(c)	(e)
Rental car	(b)	(d)	(f)

Trip Length and Gas prices

Jenna's job requires her to make trips of various lengths. The price of gas tends to change over time. In mathematics, we call parameters that change **variables**. In the following questions you will explore Jenna's expenses of renting versus the cost of using her own car for making a single trip by varying the gas price and trip's length.

(2) In the table below, work in your group to enter:

- Jenna's cost of a round trip for each given gas price and trip length if Jenna decides to rent a car.
- Since her trips vary in length, it is useful for Jenna to know the cost per mile of renting a car for each trip.

Round all your calculations to two decimal places. Note: Complete the first row with the calculated costs from Question 1.

Gas Price	Round Trip Length	Cost for Jenna Renting a Car	Cost for Jenna Renting a Car (per mile)
\$3.50	386 miles	From Q1	From Q1
\$5.00	386 miles	(a)	(b)
\$3.50	772 miles	(c)	(d)
\$5.00	772 miles	(e)	(f)

(3) (a) In the table below, work in your group to enter:

- Jenna's cost of a round trip for each given gas price and trip length if Jenna drives her own car. Show your calculations in the table below.
- Since her trips vary in length, it is useful for Jenna to know the cost per mile of driving her own car for each trip. Write your results in the table below.

Round all your calculations to two decimal places. Note: Complete the first row with the calculated costs from Question 1.

Gas Price	Round Trip Length	Cost for Jenna Driving her Own Car	Cost for Jenna Driving her Own Car (per mile)
$3.50	386 miles	From Q1	From Q1
$5.00	386 miles	(a)	(b)
$3.50	772 miles	(c)	(d)
$5.00	772 miles	(e)	(f)

(b) In calculating the cost for each trip that Jenna makes, we perform similar calculations. Create a mathematical equation for the cost per mile of Jenna driving her own car. An equation is a statement of equality, meaning that it tells you that two expressions are equal to each other. Take a minute to think this through individually before sharing your ideas in your group. Round to four decimal places where necessary.

(c) Use the equation from (b) to calculate how much it would cost for Jenna to drive 1,000 miles if the price of gas is $4.00. Take a minute to think this through individually before sharing your ideas in your group.

(4) Using the work you have done in Questions 2 and 3 as a group, write a note to Jenna explaining how she can decide if it is better to drive her own car or get a rental. Your explanation should include information about what factors affect the cost of driving and why.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[9] https://www.irs.gov/newsroom/irs-issues-standard-mileage-rates-for-2023-business-use-increases-3-cents-per-mile

[10] https://www.edmunds.com/toyota/4runner/2020/cost-to-own/

[11] https://www.irs.gov/newsroom/irs-issues-standard-mileage-rates-for-2023-business-use-increases-3-cents-per-mile

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3.2.1: Preparation 3.2

Ratios and Unit Rates

Unit rates are ratios with a denominator of “1,” although they are not always written as fractions. For example, 60 mph written as a unit rate is $\frac{60\ miles}{1\ hour}$. The language “miles per hour” implies that the operation is miles divided by hours.

(1) Write each expression as a unit rate in fractional form.

(a) 23 mpg (miles per gallon)

(b) 5 gal/min (gallons per minute)

(c) $7.15/hr (dollars per hour)

(2) Convert each rate into a unit rate.

(a) $\frac{250\ miles}{12.5\ gallons}$

(b) $\frac{450\ miles}{8\ hours}$

(c) $\frac{\$382}{40\ hours}$

After Preparation 3.2 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 3.2, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	Rating from 1 to 5
use dimensional analysis in a contextual problem.	

3.2.2: Exercise 3.2

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Renting a car is more expensive than driving your own car.

(ii) A small difference in rounding does not make much difference in results.

(iii) A small difference in rounding can significantly change results when working with large numbers.

(iv) You can calculate the cost of gas per mile by dividing the price of gas by the gas mileage of the car.

DEVELOPING SKILLS AND UNDERSTANDING

(2) In planning a Thanksgiving vacation, you want to rent a car for a week and travel the Pacific Highway from San Francisco to San Diego. You want to return to San Francisco via Las Vegas, Death Valley, and Yosemite National Park. This trip covers approximately 1,500 miles. You plan to return the car with a full tank of gasoline. You are considering two options advertised by Hertz:

- Toyota Camry, 39 mpg highway, costs $505.59 plus taxes and fees, totaling $593.28.
- Toyota Prius Hybrid, 53 mpg highway, costs $679.49 plus taxes and fees, totaling $849.61.

In both cases, you must purchase gasoline, which costs $4.70 per gallon in California.

(a) Find the total cost of renting and driving the **Camry** for the trip:

(i) Total gasoline cost is $_________

(ii) Total cost is $_________

(b) Find the total cost of renting and driving the **Prius** for the trip.

(i) Total gasoline cost is $_________

(ii) Total cost is $_________

(3) The advertised mpg for new cars is based on a speed of 55 mph. As speed increases above 55 mph, the efficiency reduces dramatically.[12]

- 3% less efficient at 60 mph
- 17% less efficient at 70 mph

(a) Compare the efficiency of a Toyota Camry (39 mpg highway) versus a Toyota Prius (53 mpg highway) at speeds of 55 mph, 60 mph, and 70 mph. Round to the nearest tenth.

	55 mph	**60 mph**	**70 mph**
Camry	39 mpg		
Prius	53 mpg		

(b) If gasoline costs $4.70 per gallon, how much money would you save by going 60 mph versus 70 mph on the trip of 1,500 miles in the

(i) Camry? $_________

(ii) Prius? $_________

(c) How much longer, to the nearest minute, would it take to travel 300 miles at 60 mph versus 70 mph?

MAKING CONNECTIONS ACROSS THE COURSE

(4) The U.S. Census Bureau posted the following information in 2020:

Nearly 11.6 million children in the United States—16% of all children—live in families with incomes below the federal poverty level ($26,200 a year for a family of four).[13]

(a) The following graph can be used to illustrate the statement from the U.S. Census Bureau. Identify what each section and the full circle represent in terms of the context and the percentage. Section 1 is the darker section.

Circle graph showing two sections. Section 1 = 21% Section 2 = 79%

The full circle represents ____ and is _______%.

Section 1 represents ____ and is _______%.

Section 2 represents ____ and is _______%.

(b) Can the statement from the US Census Bureau be restated to say that 16% of people below the federal poverty level in the United States are children? Select the best answer.

(i) Yes, because the reference value in both statements is the children in the United States.

(ii) Yes, because both statements are about children living in poverty so the percentages are the same. The reference values do not matter.

(iii) No, because the percentage of children is really much higher because families really need much more money than the federal poverty level.

(iv) No, because the reference value in the first statement is the total number of children. The reference value in the second statement is the total number of people below the poverty level.

(c) Based on the information in the paragraph, which of the following is the best estimate for the number of children in the United States in 2020?

(i) 30 million

(ii) 75 million

(iii) 100 million

(iv) There is not enough information to make an estimate.

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[12] www.mpgforspeed.com

[13] https://www.census.gov/library/publications/2021/demo/p60-273.html

3.3: The Fixer Upper

INTRODUCTION

Recall the work done in Preparation 3.3 to calculate length, area, and volume. Read the definitions below for a quick refresher.

Length is the amount of much space an object takes up in **one dimension**. For example, a standard computer keyboard has a length of about 22 inches.

Area is how much space an object takes up in **two dimensions**. In other words, the size of the surface of a two-dimensional object. For example: If a table top is 2 feet long and 3 feet wide, the area is 2 ft * 3 ft = 6 feet squared.

Volume is how much space an object takes up in **three dimensions**. In other words, it's the amount of space inside a three-dimensional object. For example, if a storage box is 5 feet long, 3 feet wide, and 2 feet deep, then its volume is 5 ft * 3 ft * 2 ft = 30 feet cubed

Imagine a house and the small piece of land that surrounds the house. A small piece of land that can be purchased or sold is often called a lot.

Figure 1 (House and Lot): This scale drawing shows the rectangular lot (dark border), the house (dark shade near the middle), and the driveway (near the bottom left). The shaded area to the back of the house (right side of the figure) represents the backyard that is to be reseeded.

Scale drawing showing the House and lot, indicating the following scale: 1 square = 10 feet 2 squares = 20 feet 3 squares = 30 feet

In this collaboration, you are going to be thinking about and making sense of the scale drawings. With reference to the scale drawing, consider the questions below in your group:

(a) What are the dimensions of the lot?

(b) How many square feet are in the house?

(c) What is the length of the driveway?

(d) What is the area of the lot?

(e) What percentage of the lot is covered by the house?

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- formulas can be found by searching the Internet and reference books.
- a variable can be used to represent an unknown.
- using a formula requires knowing what each variable represents.

By the end of this collaboration, you should be able to

- use formulas from geometry and perform calculations that involve rates and measures to support financial decisions.
- evaluate an expression.
- use the appropriate units for length, area, and volume.

PROBLEM SITUATION: HOME IMPROVEMENTS

Bob and Carol Mazursky have purchased a house and the small piece of land that surrounds the house. (A small piece of land that can be purchased or sold is often called a **lot**.) Bob and Carol want to make some improvements to their new property. In the following few problems, you will calculate the costs of these improvements. Scale drawings of the house and the lot are displayed after the problems. You will need these drawings to calculate improvement costs.

(1) Review the drawing of the house and lot (Figure 1). What does the scale mean?

(2) Bob and Carol want to get the backyard (Figure 1) fertilized and reseeded. They found an ad for Gerry’s Green Team lawn service, which can be seen here:

Gerry's Green Team flyer. Grass seed = 4 pounds per 1000 sg ft at !1.25 per pound. Fertiziler: 50 pounds per 12000 sq ft at $0.50 per pound. Labor: 4 hours at $45 per hour.

Gerry came to their house and said that the job would take about half a day and would cost about $600. Is Gerry’s estimate consistent with his advertisement? Why or why not?

(3) There is a brick grill in the backyard. Bob and Carol are going to make a concrete patio in the shape of a trapezoid next to the grill (Figure 2). The concrete slab needs to be at least 2 inches thick. They will use 40-pound bags of premixed concrete. Each 40-pound bag makes 0.30 cubic feet of concrete and costs $6.50. How much will the materials cost, including the 7.5% tax?

Outdoor Grill: Bob and Carol are going to add a trapezoidal patio adjacent to the outdoor grill in the backyard. The shaded area is made of concrete and two inches deep.

Figure 2

Scale drawing showing the patio and grill, indicating the following scale: 1 square = 1 foot

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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3.3.1: Preparation 3.3

The next collaboration explores calculations needed for repairs and improvements to a house and lot. The problems will require that you understand concepts of length, area, and volume and appropriate units of measure, based on customary U.S. units.

Length

Length is one-dimensional. In the context of remodeling a house, an example would be the total length of baseboard needed to trim the walls of a room. Examples of units of measure for length are inches, feet, yards, or miles. A number line can be used to model lengths. The thicker segment on each number line is 3 units long. If the scale is in inches, each line segment is 3 inches long. If the scale is in feet, each line segment is 3 feet long.

Number line showing the total length of baseboard needed to trim the walls of a room. The x-axis represents the measurement, ranging from 1 to 10, in increments of 1. 2 to 5 are shown highlighted in red. 5.5 to 8.5 are shown highlighted in red.

A Number Line

Area

Area is two-dimensional and is measured in square units. The total number of one-foot square tiles needed to cover the floor of a room is an example of area measured in square feet. A rectangle is one shape that can be used to model area. Recall the formula for the area of a rectangle: $A = L \times W$. The area of a rectangle is the product of the length and the width, which is a shortcut for counting the number of square units needed to cover the rectangle.

Graph showing square units. Y-axis represents square units, ranging from 0 to 8. X-axis represents the square units, ranging from 0 to 11. Highlighted: - Between 2 and 8 on the x-axis and between 2 to 4 on the y-axis. - Between 6 and 10 on the x-axis and between 5 to 8 on the y-axis.

A Coordinate Axis

Each of the two shaded areas on the coordinate axis has an area of 12 square units. If the horizontal and vertical scales are in inches, each area is 12 square inches. If the scales are in feet, each area is 12 square feet. Notice that the regions measured do not have to be squares, yet the area is measured in square units.

Notice how the units in the calculation determine the units in the result:

$A = (2 \text{ inches}) \times (6 \text{ inches})$

$(2 \times 6)(\text{inches} \times \text{inches})$

12 square inches or inches^2

If the units are feet, the area of the rectangle on the top is $A = (3 \text{ feet}) \times (4 \text{ feet}) = 12$ square feet or feet^2.

Note 1: It is common to abbreviate the units of measure using exponents. Since the area might be 12 feet × feet, write $A = 12 \text{ ft}^2$. Notice the connection to algebra here! Multiplying (3 feet) by (4 feet) is similar to multiplying $(3x)$ by $(4x)$. You multiply the numbers in front of the variables (*coefficients*), and then multiply the variables: $(3 \cdot 4)(x \cdot x) = 12x^2$.

Note 2: It is common to confuse length and area formulas. To calculate the length of the line surrounding the rectangle, which is called the **perimeter**, simply add the total number of units as if traveling around the area. For example, if the units are in feet, then the length of the line around the bottom rectangle is 2 feet + 6 feet + 2 feet + 6 feet = 16 feet. The arithmetic operation is **addition**, and the unit of measure is feet. By comparison, the arithmetic operation to compute area is **multiplication** and the unit of measure is square feet. Again, this connects to algebra. To add algebraic terms, you must have **like terms**, meaning terms with the same variables: $2x + 6x + 2x + 6x = 16x$. You cannot add $2x + 3y$ just as you cannot add 2 feet + 3 inches.

The area of the circle is given by the formula, $A = \pi r^2$.

- π is a *constant* that is approximately 3.14159 (you probably learned 3.14, but that can lead to rounding errors).
- r is the **radius** of the circle, which varies. (A radius of a circle is the distance from the center of a circle to its outside edge).
- A is the area of the circle, which varies.

(1) Write each of the following products (the result to a multiplication problem) using exponents to express the results in a simpler form.

(a) (3a)(5a)

(b) (5p)(2p)

(c) (3 inches)(5 inches)

(d) (5 feet)(2 feet)

(2) How many square inches are in 1 square foot?

(3) The formulas for finding the area of two-dimensional geometric figures that occur in everyday use are published in reference books or available online. Use the Internet or some other reliable source to find a formula for the area of each figure. Define each variable in the formula, and label the figure with the variables to indicate the correct meaning of the variable. You may have to add to the figure to indicate all variables.

Example: Rectangle

Diagram showing a rectangle.

Variables: L = length; W = width

Area of a rectangle = L × W

(a) Parallelogram	(b) Triangle	(c) Trapezoid
Diagram showing a parallelogram. Variables: Area of parallelogram =	Diagram showing a triangle. Variables: Area of triangle =	Diagram showing a trapezoid. Variables: Area of trapezoid =

(4) Find the unknown quantity in each situation.

(a) A rectangle with an area of 72 square inches has a length of 6 inches. What is the width of the rectangle?

(b) A rectangle has a length of 9 inches and a width of 1.5 feet.

  (i) What is the area of the rectangle in square inches?

  (ii) What is the area of the rectangle in square feet?

(c) A rectangle with an area of 1 square foot has a width of 6 inches. What is the length of the rectangle?

(5) What is the area of a circle with a diameter of 5 feet? Round to the nearest tenth.

(6) If the radius of a circle is doubled, will the area also double? Explain your reasoning.

Hint: Compare the areas of two circles to see if it supports your reasoning: one circle with a radius of 10 inches and the other with a radius of 20 inches.

Volume

Volume is three-dimensional and is measured in cubic units. The formula to calculate the volume of a box is $V = L \cdot W \cdot H$. In the image below, the shaded volume is 3 units wide, 4 units tall, and 5 units long, a volume of 60 **cubic** units. A cubic unit is a unit that is cubed, or raised to the power of 3. If the scales are measured in inches, then the volume is V = (5 inches) · (3 inches) · (4 inches) = $5 \cdot 3 \cdot 4$ cubic inches, or 60 in^3. If the scales are in feet, the volume is 60 ft^3 or 60 cubic feet.

Diagram showing a highlighted rectangle indicating the volume is 3 units wide, 4 units tall, and 5 units long.

Note: The power of 3 is often called the *cube of a number* just as the power of 2 is called the *square of a number*. So 5^3 can be called 5 cubed.

(7) A cubic foot is a cube that is one foot on each side. How many cubic inches are in one cubic foot?

Roots

Sometimes we know the *area* of a square, and need to find the *length* of the sides. The formula for a square's area is $A=L\times L$ or $A=L^2$. We have two choices for finding L: we can guess and check, revising our guesses to make them increasingly accurate, or we can use the square root function on a calculator. For example, a square with an area of 36 square inches has the equation 36 $\text{in}^2 = L^2$. The square root function ($\sqrt{36}$) tells us the exact value of L. In this case, L = 6 in. Generally, square roots are not whole numbers, but have decimals that continue forever without repeating. It is also true that (-6) × (-6) = 36, so there are actually two answers for every square. However, the negative result rarely applies to the context of the problem, and it is usually discarded.

It is also possible to find the length of the sides of a cube using the cube root function. For example, a cube with a volume of 8 ft^3 would have a side that is 2 ft long because $8 = 2^3$. Finding cube roots is typically beyond the scope of this course.

After Preparation 3.3 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 3.3, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
use basic formulas for area and volume (area of a rectangle and circle; volume of a box).	
use appropriate units in calculations for length, area, and volume.	

3.3.2: Exercise 3.3

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Formulas are useful because they allow us to generalize a rule to many different situations.

(ii) Formulas use variables.

(iii) Geometry can be useful in home improvement projects.

(iv) When using a formula, you do not really need to know what the variables mean.

DEVELOPING SKILLS AND UNDERSTANDING

(2) The formulas for finding the **volume** (a three dimensional measure that uses cubic units) of three-dimensional geometric figures that occur in everyday use are published in reference books or available online. Use the Internet or some other reliable source to find a formula for the volume of each figure. Define each variable in the formula, and label the figure with the variables to indicate the correct meaning of the variable. **Note**: The volume of an object that is the same on the top and the bottom is typically found by determining the area of the base figure (two-dimensional) and "stretching" that base to the desired height.

Example: Box

Volume: $V = L \times W \times H$

Diagram showing a box.

Variables: V = volume; L = length; W = width; H = height
The base figure is a rectangle with area = $L \times W$, which is multiplied by the height (H) to get the volume of the figure.

(a) Cylinder Volume of the cylinder: Variables: Diagram showing a cylinder.	(b) Pyramid with a square base Volume of the pyramid: Variables: Diagram showing a pyramid.

(3) Refer to the figure of the box in Question 3. Which of the following would be appropriate units of measurement for the different parts of the figure?

(i) Bottom edge (L), the area of the top, and the volume are all measured in inches.

(ii) Bottom edge (L) is measured in square inches; the area of the top is measured in inches and the volume is measured in cubic inches.

(iii) Bottom edge (L) is measured in inches; the area of the top is measured in square inches, and the volume is measured in cubic inches.

(iv) Bottom edge (L), the area of the top and the volume are all measured in square inches.

MAKING CONNECTIONS ACROSS THE COURSE

(4) Bob and Carol want to hire Able Refinishing to sand and refinish the dining room floor to match the floor in the living room. Able charges $2.89 per square foot to sand and refinish a hardwood floor (rounded to the nearest square foot). The dining room is rectangular and measures 17 feet 9 inches by 11 feet 9 inches.

(a) Find the area of the dining room floor rounded to the nearest square foot and the cost of the work.

(b) Find the cost of the work.

(5) After doing some work in the house, Bob and Carol want to put a concrete patio on the side of the house to keep people from tracking mud inside. They decide to hire someone to do the work. The dimensions of the rectangular patio are 23 feet 9 inches by 10 feet 1 inch. The patio will need to be at least 4 inches deep.

(a) Calculate the volume of concrete needed, in cubic yards, adding 5% to allow for spillage and an uneven base, and round up to the nearest 1/4 cubic yard.

(b) The delivered cost of the concrete is “$150 per cubic yard (in increments of 1/4 cubic yard) plus a $50 surcharge for orders less than four cubic yards.” What is the total cost of the job?

- (i) Order 3.00 yd^3; total cost is $450.00
- (ii) Order 3.00 yd^3; total cost is $500.00
- (iii) Order 3.25 yd^3; total cost is $237.50
- (iv) Order 3.25 yd^3; total cost is $537.50

3.4: Braking Down the Variables

INTRODUCTION

In this collaboration, you will be exploring the effects of speed on braking distance of a car. Discuss the following questions in your group:

- Do you stop more quickly when the tires lock and skid or when they do not?
- What is meant by the “grade” of a road?

The formula used in this collaboration uses a subscript for a variable. Subscripts are used in math when there is a variable that we want to represent different values or to give extra information about the variable. For example, if we want to use the variable v to represent the velocity or speed at different times we could use v_1 to be the velocity after 1 minute and v_2 to be the velocity after 2 minutes. Frequently the subscript zero is used to represent a starting value. So, v_0 would be the velocity when the timer is started, or the starting velocity, which is also called the initial velocity.

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- a variable is a symbol that is used in algebra to represent a quantity that can change.
- many variables can be present in a scenario or experiment, but some can be held fixed in order to analyze the effect that the change in one variable has on another.

By the end of this collaboration, you should be able to

- evaluate an expression.
- informally describe the change in one variable as another variable changes.

PROBLEM SITUATION: CALCULATING THE BRAKING DISTANCE OF A CAR

Experts agree that driving defensively saves lives. Knowing how far it takes your vehicle to come to a complete stop is one aspect of safe driving. For example, when you are going only 45 miles per hour (mph), you are traveling about 66 feet every second. This means that to be a safe driver, you need to drive defensively (i.e., you need to know what is going on ahead of you so that you can react accordingly). In this lesson, you will learn more about what it takes to drive defensively by examining the braking distance of a vehicle. **Braking distance** is the distance a car travels in the time between when the brake is applied and when it comes to a full stop.

(1) What are some variables that might affect the braking distance of a car? Take a moment to think individually before sharing your ideas in your group. Record some of your answers in the space below.

(2) In building formulas, the factors or conditions are quantified; that is, we find a way to represent the different factors using numbers. The factors or conditions then become the variable in the formula. For example, the steepness of a road, or grade of road, is represented by a percentage of the vertical change divided by the horizontal change. (A 1% grade means you go up 1 foot for every 100 feet of horizontal travel.) For this collaboration, you will examine a variable that affects braking distance: speed. (In Exercise 3.4, you will consider the effects of other variables.)

Discuss with your group how you think speed affects braking distance. Think of some specific questions you might ask. For example, what happens to the braking distance if you double the speed? Would the answer be different for very low speeds or very high speeds? Think about questions individually before sharing in your group. Record your questions in the space below. You will return to these questions later in the collaboration.

Braking Distance: The Formula

The formula for the braking distance of a car is

$$d=\frac{V_0^2}{2g(f+G)}$$

V_0 = initial velocity of the car (feet per second). That is, the velocity of the car when the brakes were applied. The **subscript** (symbol written in small type below a variable) zero is used customarily to represent time equaling zero. So, V_0 is the velocity when t = 0.

d = braking distance (feet)

G = roadway grade. The **grade** can be defined as the quantified measure of the change in height of the road (percent written in decimal form). (**Note**: There are no units for this variable, as explained in the Preparation 3.4.)

f = coefficient of friction between the tires and the roadway (0 < f < 1). (**Note**: Good tires on good pavement provide a coefficient of friction of about 0.8 to 0.85.)

g = acceleration due to gravity (32.2 ft/sec^2)

The "g" in this formula is a constant value; therefore, the formula for braking distance has four variables.

While we normally think about the speed of cars as being measured in miles per hour (mph), in the description for the variables used in the formula above you might have noticed that the velocity (speed at which the car is traveling) is supposed to be measured in feet per second. This is because when we are thinking about the time it takes for a car to stop after hitting the brakes, and how far it travels, we are dealing with small distances and times, and it is more convenient to measure distance and time using feet and seconds rather than miles and hours. For example, when you are going 45 miles per hour (mph), you are traveling about 66 feet every second. You can use dimensional analysis to make this conversion:

$$\frac{45\ miles}{hour} \times \frac{hour}{60\ minutes} \times \frac{minute}{60\ seconds} \times \frac{5280\ feet}{mile} = \frac{237600\ feet}{3600\ seconds} = 66\ feet\ per\ second$$

In this problem situation, you will investigate the relationship between speed when the brakes are applied which is referred to as initial velocity (V_0) and braking distance (d). To understand the relationship between these two variables, you will need to hold the other two variables in the formula–friction (f) and roadway grade (G)–fixed. This means that the only variable we are changing is the speed of the car when the brakes are applied. The steepness of the road and the condition of the tires and the road will remain the same. So, we will use the same values for f and G in all of the calculations.

Here is an example of how to use the formula. If a car is traveling at 45 miles per hour and the grade of the road is 1%, and the road and tires are in good condition so coefficient of friction is 0.8, then the formula gives:

$$d = \frac{V_0^2}{2g(f+G)} = \frac{66^2}{2(32.2)(0.8+0.01)} = \frac{4356}{52.164} = 83.5\ feet$$

(3) Let f = **0.85** (good tires on good roads) and G = **0.06** (slight uphill road grade). We also have g = acceleration due to gravity is **32.2** ft/sec^2. Use these values in the braking distance formula to calculate the braking distance for a car traveling at **75 mph**. Take a moment to think on your own first before sharing in your group. Write your answer below rounded to one decimal place. **Note**: remember to convert miles per hour in feet per second!

(4) Let f = **0.8** (good tires on good roads) and G = **0.05** (slight uphill road grade). Use these values for f and G to simplify the braking distance formula. Take a moment to think on your own first before sharing in your group. **Note**: You also know the value for g, because the acceleration due to gravity is constant: g = acceleration due to gravity.

(5) How can you verify your predictions about the relationship between velocity and braking distance?

(6) Now you will explore the question(s) developed by the class in Question 2.

(a) Record the question(s) here:

(b) Create a strategy for exploring the question(s) with your group. Record your strategies here:

(c) Use your strategies to answer the question(s). Write a complete statement about your results here:

The three Skills You Need In Any Field

You have now used several different formulas in this course. In Lesson 3.3, you used common geometric formulas for area and volume (you probably have seen these formulas before). In this lesson, you used a formula that was more complex and probably less familiar to you. Almost every field has specialized formulas, but they all depend on three basic skills:

- Understanding and knowing how to use variables, including the use of subscripts.
- Understanding and knowing how to use the order of operations.
- Understanding and knowing how to use units, including dimensional analysis.

With these three skills, you will be able to use formulas in any field.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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3.4.1: Preparation 3.4

Use the formulas you found earlier to answer Questions 1 and 2.

(1) A courtyard shaped like a trapezoid is shown below. Find the area of the courtyard in ft^2.

Diagram showing a trapezoid where the summit measures 50ft, the base measures 90ft, and the height measures 60ft.

(2) Use the formula for the volume of a cylinder to find the volume of a water tank (in ft^3) with a radius of 10 feet and a height of 40 feet. Round to two decimal places.

Subscripts

The following information will be used in Collaboration 3.4.

A **subscript** is a symbol that is written in small type below a variable in regular type. For example: P_0 is read as *P sub-zero*.

Subscripts are used to distinguish between variables that represent similar quantities. For example, if you were working with a problem in which there were different prices over time, you might want to use P to represent all of those prices, but you also want to be able to tell the difference between them. So you could use P_0 for the initial price, then P_1 (*P sub-one*) for the second price. The subscript is only a label. It is not an operation.

The Grade of a Road

The **grade** of a road quantifies the rate of increase (or decrease) in height of the road over some horizontal span. The *grade* of a road is important information for drivers of large trucks in mountainous terrain. If a trucker begins to travel too fast going downhill, then it is possible for brakes to fail. Of course, as the driver of a car, you might be frustrated with a truck that is traveling up a hill with a steep grade, especially if you cannot pass. Runners and bicyclists who compete in hilly terrain also consider the grade of the hill in predicting their stamina and speed.

The grade of a road is written as a fraction with the numerator being the change in height (vertically) and the denominator being the change in distance (horizontally). In equations, the grade must be used as a positive value for cars moving uphill and as a negative value for cars moving downhill. (Note: when the grade of a road is reported, a sign is often not included because if one car is traveling uphill on a road, then oncoming cars are traveling downhill, so it could be confusing to report the grade as + or –.)

Notice that the units divide out when calculating grade, leaving a number that is dimensionless. That number is then written as a percent.

Example: Determine the grade of a road that decreases 72 feet in height over a horizontal distance of 600 feet.

Diagram showing a right-angle triangle indicating the following: - Height = 72 feet - Length = 600 feet

Answer: $\frac{-72\ feet}{600\ feet}=\frac{-72}{600}=-0.12=-12\%$

After Preparation 3.4 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 3.4, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can…	**Rating from 1 to 5**
evaluate expressions containing parentheses and exponents of two.	
understand dimensional analysis and how to use units—including squared units—in calculations.	
understand the use of subscripts.	

3.4.2: Exercise 3.4

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) When using variables, it is important to know what they represent and what units should be used with them.

(ii) When using variables, it is only important to know what numbers to substitute in for them.

(iii) A subscript is a label on a variable.

(iv) Braking distance is affected by many factors.

DEVELOPING SKILLS AND UNDERSTANDING

Mathematical Terminology

Formulas are a type of algebraic equation. You have probably seen algebraic equations such as “y = x + 3” in previous math classes. Each side of the equation is called an **algebraic expression**. So “x + 3” is an expression and “y” is an expression. An equation is a statement that two expressions are equal. The purpose of such an equation is to define a sequence of calculations using a shortcut language. In this example, the equation “y = x + 3” means:

(1) Start with x.

(2) Add three to x.

(3) The result is y.

The word *formula* is usually used to express important and non-changing relationships, especially in contexts such as science, business, medicine, sports, or statistics. For example the area of a rectangle, A = L ⋅ W, is a *formula* because the relationship between area and the length and width of a rectangle is always the same. It is also an equation, but that wording is less common.

Suppose you had a situation in which you make $12 per hour. This relationship could be written algebraically as P = 12h where P is your pay in dollars, and h is the number of hours you work. This would be called an *equation* instead of a *formula* because if you got a raise, the relationship would change. You also might call the equation a *model* because it models a situation mathematically.

In other math classes, you might see problems like the one shown below. Each line represents a simplification of the line above.

Example: Evaluate the expression $3x^2$+ 2y if x = 3 and y = 5.

$3(3)^2 +2(5)$

3(9) + 10

27 + 10

37

These types of problems are not included in this course because the focus is on using mathematics in a more every-day context. However, you should recognize that this type of expression uses the same skills that you used when working with formulas. Units are not involved, so the first step is to recognize that the values can replace the variables and the order of operations is applied to simplify the form of the problem.

(2) In the collaboration, you investigated the relationship between velocity and braking distance. You will now investigate the relationship between the coefficient of friction and the braking distance.

Recall that the formula for the braking distance of a car is $d=\frac{V_0^2}{2g(f+G)}$

(a) Define each variable:

(i) V_0

(ii) d

(iii) G

(iv) f

(v) g

(b) Which of the variables have no units?

(c) Which of the variables represent a constant?

(d) To investigate the relationship between the coefficient of friction and the braking distance, you need to hold the other variables fixed. Let G = 0.02. Which of the following is a correct interpretation of the value G = 0.02?

(i) The grade of a road is 0.02%, which is a vertical increase of 0.02 feet for every 1 foot of horizontal increase.

(ii) The grade of a road is 0.02%, which is a vertical increase of 0.02 feet for every 100 feet of horizontal increase.

(iii) The grade of a road is 2%, which is a vertical increase of 2 feet for every 1 foot of horizontal increase.

(iv) The grade of a road is 2%, which is a vertical increase of 2 feet for every 100 feet of horizontal increase.

(e) Let V_0 = 72 mph and use the value of G = 0.02. Which of the following expressions represents the simplified form of the formula using these values? (**Hint**: Convert V_0 and g into ft/s.)

(i) $d = \frac{11,151.36}{(64.4f + 0.02)} ft$

(ii) $d = \frac{11,151.36}{64.4f} + 8,657.89 ft$

(iii) $d = \frac{11,151.36}{(64.4f + 1.288)} ft$

(iv) $d = \frac{0.02}{(64.4f + 1.288)} ft$

(3) (a) Use the formula you found in Question 2(e). Complete the table of values for f and d (in feet). Use the values of f given in the table. Perform one of the calculations on paper showing the units. You may then use technology to complete the table. Round solutions to 3 decimal places.

f	d (feet)
0.30	(i)
0.50	(ii)
0.70	(ii)
0.90	(iv)

(b) The four values of f correspond to the coefficient of friction for four road and tire conditions: an icy road with fair tires, a very good road with great tires, an asphalt road with fair tires, and a wet road with fair tires. Match the coefficients of friction to the appropriate conditions by looking at the braking distance required.

(i) Icy road with fair tires, f =

(ii) Very good road with great tires, f =

(iii) Medium quality road with fair tires, f =

(iv) Wet road with fair tires, f =

(c) The coefficient of friction (f) is increasing at a constant rate, since each value is 0.2 more than the previous value. How is d changing as f increases at a constant rate?

(i) The stopping distance is decreasing.

(ii) The stopping distance is constant.

(iii) The stopping distance is increasing.

(4) Lenders such as banks, credit unions, and mortgage companies make loans. The person receiving the loan usually pays the loan off in small payments over a long period of time. The lender earns money by charging **interest**, which is based on a percentage of the amount that is borrowed. There are different types of interest. Car loans are usually calculated using the formula for **simple**

interest. The total amount repaid is based on the *interest* and the value of the original loan, called the **principal**. The formula for the total dollars needed to repay the loan, with interest, is found using the formula

$$A = P + P \cdot r \cdot t$$

where

- A is the *amount* (total principal plus interest) required to repay the loan.
- P is the amount borrowed, the *principal*.
- r is the *annual interest rate*, quoted as a percent, but used as a decimal in the formula.
- t is the *time*, in years, taken to repay the loan (six months would be 1/2 year).

Suppose you get a loan of $10,000 at an annual interest rate of 6.25%.

(a) Use the given information to write the formula for the total amount to be repaid in t years.

(b) Complete the table of values below that shows the payoff amount after certain amounts of time.

	t (years)	**A ($)**
0	(i)	(vi)
4 months	$\frac{1}{3}$	(vii)
6 months	(ii)	(viii)
1 year	(iii)	(ix)
3 years	(iv)	(x)
6 years	(v)	(xi)

(c) Estimate the maximum time allowed to repay the loan if you want the total payoff to be less than $16,000.

MAKING CONNECTIONS ACROSS THE COURSE

(5) The tuition at a daycare center is based on family income. A reduced tuition is offered to families with lower incomes, which is a form of financial aid called a **subsidy**. There are three levels of tuition:

- Full subsidy—the family does not pay any tuition
- Partial subsidy—the family pays part of the tuition
- No subsidy—the family pays the full tuition

The data for the daycare center for each age level is given below. Answer the questions below. Round to the nearest whole number.

	Full Subsidy	**Partial Subsidy**	**No Subsidy**	**Total**
3-year-olds	17	13	8	(i)
4-year-olds	22	14	15	(ii)
5-year-olds	15	16	11	(iii)
Total	(iv)	(v)	(vi)	(vii)

(a) Complete the last column and last row of the table of the table above.

(b) What percentage of 3-year-olds received a full or partial subsidy? Round to the nearest whole percent.

(c) What percentage of those who receive no subsidy are 5 years old? Round to the nearest whole percent.

(d) What percentage of the children are 3 years old?

(e) The daycare center receives federal funding based on the number of children receiving a subsidy. The amount of federal funding for the current term can be calculated using the formula below. Calculate the center's federal funding for the current term.

Funding = **1,530F + 1,750P + 1,875N**

where:

F = number of children receiving a full subsidy

P = number of children receiving a partial subsidy

N = number of children receiving no subsidy

(6) In Collaboration 1.7, you used a formula that was written as steps in a form to calculate self-employment taxes for different people. Formulas are often written in this way. One example is the Expected Family Contribution (EFC) Formula, which is used to determine if a college student is eligible for financial aid. The EFC has many different sections that each use different calculations. One section of the 2022–23 form is shown below.[14] This section includes lines 45–50.

Student’s Contribution from Assets

45. Cash, savings, and checking		
46. Net worth of investments If negative, enter zero	+	
47. Net worth of business and/or investment farm If negative, enter zero	+	
48. Net worth (sum of lines 45 through 47)	=	
49. Assessment rate	×	0.20
50. STUDENT’S CONTRIBUTION FROM ASSETS	=	

(a) Calculate the “Student’s Contribution from Assets” given the following information.

Cash: \$500 Investments: loss of \$2,000

Savings: \$3,240 Business: \$0

Checking: \$732

(b) Assuming that the “Net Worth of Investment” (N_I) and the “Net Worth of Business or Farm” (N_B) are positive, write a formula that summarizes the calculation in this form using the following variables:

C = Cash including savings and checking

N_I = Net worth of investment

N_B = Net worth of business or farm

S = Student’s contribution from assets

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[14] https://fsapartners.ed.gov/sites/default/files/2021-08/2223EFCFormulaGuide.pdf

3.5: Comparing Apples to Apples

INTRODUCTION

In this collaboration, you will take a critical look at pictographs, which are a type of data display that uses a picture of an object instead of a bar to represent a quantity. When we see an object on paper it is hard to know how each person will think about the object. Below are two illustrations of a basketball. Have each person in your group describe what they see in the pictures and compare the sizes.

Graphic showing two basketballs of different sizes. One is small, about 1.5 cm in height; the other is large, about 3 cm in height

Did anyone in your group talk about the diameter (or height) of the balls? If so, how do the sizes compare? Did anyone in your group talk about the area or how much of the paper is covered by each ball? If so, how do the sizes compare? How many little balls do you think you need to cover the big ball? Did anyone in your group talk about how it would be to hold the two balls, and how the sizes would compare?

It is hard to know what will come to mind for each person who views the pictures. And, it turns out that each of these ways of viewing the object will give a different impression of the “size” of the objects, which can lead to misunderstanding or confusion when we compare pictures of objects of different sizes.

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- pictographs can be misleading because areas and heights of figures do not increase proportionally.

By the end of this collaboration, you should be able to

- solve dimensional analysis scenarios involving multiple conversion factors.
- analyze misrepresentations in graphs related to area and volume.
- evaluate formulas and use the results to make a decision.

PROBLEM SITUATION: ANALYZING DATA ON APPLE JUICE IMPORTS

In the United States, in recent years, there has been an increase in consumption of apple juice from foreign countries. Since imported apple juice is also potentially cheaper, and therefore available to a larger percentage of the population, importing it can be seen as a positive thing. However, importing food from other countries also causes some concerns. These include a loss of control over the quality of imported food, an increased reliance on food from other countries, and a reduction in business for U.S. farmers. You will examine some of these issues below.

(1) The following graph is similar to ones commonly used in media reports. It is called a **pictograph** because it uses pictures (instead of bars) to represent quantitative changes. This pictograph was created to show the changes in apple juice imports over the 20-year time period from 2001 to 2021.[15]

Graph showing the Apple Juice Imports to the United States. 2001 = 318.3 millions of gallons of apple juice 2021 = 565.3 millions of gallons of apple juice

(a) Based on the graph, would you say that apple juice imports grew a little, some, or a lot over this time period? Can you quantify? What are you looking at when you make this comparison?

(b) People who study how to make visual displays of data (like graphs) are called **data scientists**. Data scientists urge caution when constructing pictographs because if they are not carefully constructed, they can be misleading. For example, in the graph above it is unclear if you should compare the height of the apples, the area of the apples, or the volume of the apples.

To better understand how different interpretations of the data (in this instance the apple’s height, area, and volume) can significantly impact one’s understanding of the change in apple juice imports between 2001 and 2021, fill in the table below.

Assume the area of an apple is approximately the area of a circle and the volume of an apple is approximately the volume of a sphere. The area of a circle is given by the equation $A = \pi r^2$ and the volume of a sphere is given by $V = \frac{4}{3}\pi r^3$ (where r is the radius). Remember that the radius of a circle is the distance from the center of the circle to its outside edge. For this question, you may assume the following:

- 2001 (small apple) has a height of 32mm
- 2021 (big apple) has a height of 55mm

Work in your group to complete the table. When necessary, round figures to one decimal place.

	Height of Apple	Approximate Area of Apple Graphic	Approximate Volume of Apple Graphic
2001 Little Apple	32mm	(ii)	(v)
2021 Big Apple	55mm	(iii)	(vi)
Relative change (in percent)	(i)	(iv)	(vii)

(c) What percentage did apple juice imports increase by between 2001 and 2021? Round to one decimal place.

(d) Using your answer for part (c), which interpretation of the pictograph (height, area, or volume) most accurately represents the actual change in apple juice imports over this 20-year period? Explain.

(e) How could you make a graph that would portray the data more accurately?

(2) An American company has been criticized for using imported apple juice. The company's public relations department is asked to prepare a statement defending the use of imports. You find the following information:

- *The New York Times* reported, "China's advantage is its cheap labor. A picker makes about 28 cents an hour, or $2 a day, according to the U.S. Apple Association. At that time, workers in Pennsylvania made about $9 to $10 per hour, and those in Washington State about $14 per hour."[16]
- It takes about 36 apples to create one gallon of juice.
- One bushel of apples contains about 126 medium apples.
- An experienced apple picker can harvest about 12.5 bushels of apples per hour.

As members of this department, you are asked to calculate how much it costs to pick enough apples to make one gallon of apple juice in China and in the United States.

(3) Select one of the two prompts below. Write a paragraph presenting your argument and supporting it with specific quantitative information.

- (a) Use the information in Question 1 to make the argument that the United States is importing too much apple juice. Include your reasons explaining why this is a problem.
- (b) Use the information in Question 2 to make the argument that it is good that the United States is importing apple juice. Include your reasons explaining how this benefits the country.

FURTHER APPLICATIONS

(4) Total U.S. apple juice consumption for the year 2001 was 505.6 million gallons, and for the year 2021 it was 656.1 million gallons.[17] Use this information and the data from this lesson to create a new graph of the import data, by changing the vertical axis to "percentage of total apple juice consumption." Because pictographs can be eye-catching and can make the data memorable, you can use a pictograph in your graph, but choose one that preserves the integrity of the data.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[15] https://www.ers.usda.gov/webdocs/DataFiles/54499/FruitYearbookSupplyandUtilization_GTables.xlsx?v=7527.9

[16] www.nytimes.com/2007/06/25/business/worldbusiness/25iht-apples.1.6312540.html

[17] https://www.ers.usda.gov/webdocs/DataFiles/54499/FruitYearbookSupplyandUtilization_GTables.xlsx?v=7527.9

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3.5.1: Preparation 3.5

It is often necessary to change money into different currencies when traveling or doing business in different countries. Exchange rates, which change constantly, are used to make these conversions. Consider the following situation. Sonia is traveling to Mexico. Answer the following questions about her trip.

(1) Sonia starts with \$100 in cash and changes it into pesos at a rate of 1 peso = \$0.055. Which of the following would be a correct way to use dimensional analysis to make this conversion?

(i) $\$100 \cdot \frac{\$0.055}{1\ peso}$

(ii) $\$100 \cdot \frac{1\ peso}{\$0.055}$

(iii) $\frac{1}{\$100} \cdot \frac{1\ peso}{\$0.055}$

(iv) $\frac{1}{\$100} \cdot \frac{\$0.055}{1\ peso}$

(2) After converting her \$100 to pesos, Sonia spends 984 of her pesos while in Mexico. She converts her remaining pesos back into dollars. How many dollars does she have? Round to the nearest dollar.

After Preparation 3.5 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 3.5, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
use dimensional analysis to make conversions.	
understand the use of variables in formulas.	
evaluate expressions and formulas.	

3.5.2: Exercise 3.5

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Pictographs are a type of graph in which pictures are used to represent quantities.

(ii) Units are important in dimensional analysis, but do not matter in using formulas except when giving the answer to a problem.

(iii) Units are important in many mathematical skills including graphs, dimensional analysis, and formulas.

(iv) Imports of apple juice have increased due to the high cost of labor in the United States.

DEVELOPING SKILLS AND UNDERSTANDING

(2) Over the last decade, the use of bottled water has increased dramatically in the United States and around the world. In 2010, the annual U.S. consumption of bottled water was 28.3 gallons per capita. In 2020, this had increased to 45.2 gallons.[18] The graphic below is designed to illustrate this increase.

Diagram showing the U.S. bottled water consumption. 2010 = 128.3 gallons per capita 2020 = 45.2 gallons per capita

The table below gives the dimensions of the figures. Complete the table as instructed.

(a) Approximate the area of each figure. The bottles shown are similar in shape to rectangles. So, to compare the areas of the figures, think of the rectangle that contains the figure with the given dimensions and calculate the area of that rectangle.

(b) The figures give the illusion of three dimensions. Using the given dimensions, calculate the volume of the cylinder represented by each figure to the nearest tenth. (Use an Internet search or reliable source to look up the formula, if needed.)

(c) Calculate the relative change from the small bottle to the large bottle. Round to the nearest whole percent.

	Actual data	Height	Width	Area of rectangle	Volume of cylinder
Small bottle	28.3 gallons/ capita	4.0 cm	1.3 cm	(a)(i)	(b)(i)
Large bottle	45.2 gallons/ capita	6.5 cm	2.2 cm	(a)(ii)	(b)(ii)
Relative change (in percent)	(c)(i)	(c)(ii)	(c)(iii)	(c)(iv)	(c)(v)

(d) Which of the following aspects of the graphic are accurate depictions of the date? There may be more than one correct answer.

(i) Area

(ii) Volume

(iii) Height

(iv) Width

(e) Overall, is the graph accurate or misleading? Explain.

(3) In addition to paying attention to distortions of data, data scientists also rely on artistic ideas to design visually appealing graphs. One theory they rely on is the **golden ratio** for rectangles. The ratio is given by the formula

$$g = \frac{l}{w}$$

where l is the length of the longest side of the rectangle and w is the length of the shortest side of the rectangle. When g is close to 1.6, the rectangle is thought to be visually appealing.[19]

Identify if each item listed below has dimensions that match the golden ratio. You will have to look up the dimensions of some of the items.

Write the exact ratio. Then write if it is “close” or “not close” to the golden ratio.

(a) Small water bottle graphic from Question 2

(i) Ratio:________ (ii) Does this match the golden ratio?__________________

(b) Large water bottle graphic from Question 2

(i) Ratio:________ (ii) Does this match the golden ratio?__________________

(c) Credit card

(i) Ratio:________ (ii) Does this match the golden ratio?__________________

(d) Football field

(i) Ratio:________ (ii) Does this match the golden ratio?__________________

MAKING CONNECTIONS ACROSS THE COURSE

(4) The total U.S. apple juice consumption for the marketing years 2001 and 2021 was 505.6 million gallons and 656.1 million gallons, respectively. Calculate the average consumption of apple juice in fluid ounces per capita per week for 2001 and 2021. Use the following U.S. population figures: 284.97 million people in 2001 and 331.89 million people in 2021. Round answers to the nearest hundredth.

(a) In 2001:

(b) In 2021:

When comparing quantities that change over time, you usually compute the difference in values according to:

New value – Old value

If the average price of a gallon of gasoline in 2020 was \$2.19 and the average price in 2022 was \$3.98, one might compare these two prices by computing \$3.98 – \$2.19 = \$1.79. Since this number is positive, you can say that the average price of gasoline has increased \$1.79 in two years, which is an annual change of 89.5 cents (\$1.79 divided by two years).

If the average price for a loaf of bread was \$3.90 in 2020 and \$3.50 in 2022, then the change in this basic item of food would be –\$0.40. The negative sign tells us that the price of bread has decreased over this two-year period.

(5) How much did the average price of a loaf of bread decrease by per year?

(6) The total change in the price of a basic Mac Mini from 2018 to 2023 was –\$200.[20]

(a) The price in 2018 was \$799. What was the price in 2023?

(b) Write a sentence describing the average *annual* change in the price of a Mac Mini during this period.

(7) The following pie charts represent how a typical household budget might be broken into categories.

Circle graph showing Chart A: The typical household budget broken into categories, indicating the largest to smallest portions: - Housing - Food - Other - Paying debt - Medical.

Circle graph showing Chart B: The typical household budget broken into categories, indicating the largest to smallest portions: - Housing - Other - Food - Paying debt - Medical

(a) For Chart A, which of the following statements best describes the comparison of the categories "Food" and "Other?"

(i) "Food" is a smaller percentage than "Other."

(ii) "Food" is a greater percentage than "Other."

(iii) "Food" is about the same percentage as "Other".

(b) For Chart B, which of the following statements best describes the comparison of the categories "Food" and "Other?"

(i) "Food" is a smaller percentage than "Other."

(ii) "Food" is a greater percentage than "Other."

(iii) "Food" is about the same percentage as "Other."

(c) The two charts are actually created from the same data (shown below). Do they both accurately represent these data? Why or why not?

Housing	40%

Food	20%
Medical	5%
Paying debt	10%
Other	25%

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[18] https://www.statista.com/statistics/183377/per-capita-consumption-of-bottled-water-in-the-us-since-1999

[19] https://en.Wikipedia.org/wiki/Golden_rectangle

[20] https://en.Wikipedia.org/wiki/Mac_Mini

3.6: Balancing Blood Alcohol

INTRODUCTION

As you keep taking more advanced math courses in the future, you will keep seeing more operations, procedures, or functions that cancel each other out. Many operations, procedures, or functions come in pairs. Whatever one operation does, the other one will undo.

In this collaboration, we will be focusing on two pairs of such operations:

- Addition and subtraction
- Multiplication and division

Let’s illustrate the operation of addition and subtraction with a simple example:

1. Pick a number. You can pick any number you want: positive, negative, a fraction, or a decimal. In this example, we’ll use 25.
2. Next, add 10 to the number that you picked. In the example, that would be 25+10=35.
3. Next, subtract 10 from the number you found in the previous step. In this example, that would be 35-10=25.
4. From this example, and your own work, you can see we just got the same number that we started with! This shows how addition and subtraction cancel each other out.

This is not a coincidence. It works for any numbers and variables. For example,

- 50 + 2 - 2 = 50
- -20 - 76 + 76 = -20
- $x + 2 - 2 = x$
- -60R - 91 + 91 = -60R

The same applies to multiplication and division. To illustrate, let’s look at another example, but instead of adding and subtracting, we will be multiplying and dividing.

1. Pick a number. You can pick any number you want - positive, negative, a fraction, or a decimal. This example will use 80.
2. Next, multiply the number that you picked by 20. In this example, $80 \times 20 = 1{,}600$.
3. Next, divide the number that you found in the previous step by 20. In this example, $1{,}600 \div 20 = 80$.
4. From this example, and your own work, you can see we just got the same number that we started with! This shows how multiplication and division cancel each other out.

This is not a coincidence. It works for any numbers and variables. For example,

- 50 × 10 ÷ 10 = 50
- T × 25.81 ÷ 25.81 = T
- -7 ÷ 8 × 8 = -7
- 25 × y ÷ 25 = y

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- addition and subtraction are inverse operations.
- multiplication and division are inverse operations.
- solving for a variable includes isolating it by “undoing” the actions to it.

By the end of this collaboration, you should be able to

- solve for a variable in a linear equation.
- explicitly write out the order of operations to evaluate a given equation.

PROBLEM SITUATION: CALCULATING BLOOD ALCOHOL CONTENT

Blood alcohol content (BAC) is a measurement of how much alcohol is in someone’s blood. It is usually measured as a percentage. So, a BAC of 0.3% is three-tenths of 1%. That is, there are 3 grams of alcohol for every 1,000 grams of blood. A BAC of 0.05% impairs reasoning and the ability to concentrate. A BAC of 0.30% can lead to shortness of breath, a blackout, and loss of bladder control. In most states, the legal limit for driving is a BAC of 0.08%.[21]

BAC is usually determined by a Breathalyzer, urinalysis, or blood test. However, the Swedish physician E.M.P. Widmark developed the following equation for *estimating* an individual's BAC.[22] This formula is widely used by forensic scientists:

$$B = -0.015t + \left(\frac{2.84 \cdot N}{W \cdot r}\right)$$

B = percentage of BAC

N = number of "standard drinks" (A standard drink is one 12-ounce beer, one 5-ounce glass of wine, or one 1.5-ounce shot of liquor.) N should be at least one.

W = weight in pounds

r = the distribution rate for alcohol through the body (this value is a constant), 0.68 for males

and 0.55 for females

t = number of hours since the first drink

(1) Looking at the equation, discuss why the items (t, N, W, and r) on the right of the equation make sense in calculating BAC.

(2) Consider the case of a male student who has three beers and weighs 120 pounds. Simplify the equation as much as possible for this case. What variables are still unknown in the equation? Round values where necessary to the nearest thousandth.

(3) (a) Using your simplified equation from Question 2, find the estimated BAC for this student

one hour, three hours, and five hours after his first drink.

1 hour:

3 hours:

5 hours:

(b) What patterns do you notice in the data from 3(a)?

(4) Discuss with your group how you arrived at the BAC values mathematically. For example, did you multiply, add, subtract, etc., and what did you do first? Outline the steps that you took to get the answers for Question 3(a).

(5) How long will it take for this student's BAC to be 0.08, the legal limit? How long will it take for the alcohol to be completely metabolized resulting in a BAC of 0.0? Round to one decimal place.

When BAC = 0.08, t =

When BAC = 0.0, t =

(6) A female student, weighing 110 pounds, plans on going home in two hours. The simplified Widmark formula is:

$$B = -0.03 + \frac{2.84 \cdot N}{60.5}$$

(a) Compare the student's BAC for one glass of wine versus three glasses of wine at the time she will leave. Round to three decimal places.

When N = 1, B =

When N = 3, B =

(b) In this scenario, determine how many drinks she can have so that her BAC remains less than 0.08. Round to the nearest drink.

FURTHER APPLICATIONS

(7) Solve the following equation for the values given in (a) and (b). In each case, outline the steps you took as you did in Question 4.

$$y = -4x - 2$$

(a) Solve for y if $x = -3$. Outline your steps.

(b) Solve for x if $y = -3$. Outline your steps.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[21] http://en.Wikipedia.org/wiki/Blood_alcohol_content

[22] http://www.avvo.com/legal-guides/ugc/explaining-blood-alcohol-levels-bac-what-does-the-08-bac-mean-how-many-drinks-is-that-1

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3.6.1: Preparation 3.6

This preparation will help you prepare for both Collaboration 3.6 and Module 4. In the next module, you will make graphs on a **coordinate plane** like the one shown below. Over the next few assignments, you will learn about using a coordinate plane and practice how to graph points. This material is spread over several Exercises to give you time to be fully prepared for Module 4.

Unmarked graph with x and y axes from −5 to 5, indicating the following graph points: A = (2, 3) B = (-3, 1) C = (-1, -1) D = (-1, 3) E = (0, 1) F = (4, 0) G = (0, -3) H = (2, 1) I = (1, -1) J = (-4, 0)

You will begin with some vocabulary. A coordinate plane has two *axes* that measure distance in two dimensions. The **horizontal axis** goes from left to right. In previous classes, you may have called this the **x-axis**. The vertical axis goes up and down. This is sometimes called the **y-axis**. The axes are two number lines that create a grid on the coordinate plane. (**Note**: *Axis* is singular and *axes* is plural.)

The point at which the two axes intersect or cross is called the **origin**. This point represents 0 for both axes. To the left of this point, the horizontal axis is negative; to the right it is positive. Below the origin, the vertical axis is negative; above the origin it is positive. You can see this in the numbers along each axis above. These numbers are called the **scale**.

Each location or *point* on the coordinate plane is defined by an **ordered pair**. You can think of this as the address of a point. An ordered pair must contain two numbers. Ordered pairs are written in a set of parentheses ( ) with a comma separating the numbers, such as (2, 4). The first number is the distance and direction going left or right from the origin and the second number is the distance and direction going up or down. The ordered pair for the origin is (0, 0).

Follow these steps to find the point represented by the ordered pair (2, 3):

1. First, think about the "address" of the point. If this were a street address, the ordered pair tells you to walk 2 blocks horizontally in the positive direction (right) and then walk 3 units vertically in the positive direction (up).
2. Start at the origin. Go 2 units to the right because this is the positive side of the horizontal axis.
3. Go 3 units up.

Point A on the graph above is the point (2, 3). A few other examples from the graph are given below:

Point B: (–3, 1)

Point E: (0, 1)

Point F: (4, 0)

(1) Write the ordered pairs for the following points on the graph.

Point C:

Point D:

Point G:

Point H:

Point I:

Point J:

Statement of Equality

Remember that an **equation** is a statement of equality, meaning that it tells you that two expressions are equal to each other. An equation can be as simple as 3 = 3 or it can have complicated expressions with multiple terms on one or both sides of the equal sign. One of the most important things to remember is that if the *value* of one side of an equation is changed, then it is no longer an equation because the two sides are no longer equal. If you need to change the value of one side and you want to keep the equation true, you must change the value of the other side in the same way.

(2) Each of the following examples starts with the equation 3 = 3. Then a new operation is performed to one or both expressions. The new operations are shown in bold. If the new operations maintain the statement of equality, put an equal sign (=) in the blank. If the new operations do not maintain the statement of equality, put a not-equal sign (≠) in the blank.

(a) 3 = 3 3 + **2** ____ 3 + **2**	(b) 3 = 3 **2** + 3 ____ 3 + **2**

(c) 3 = 3 **5** + 3 ____ **4** + 3	(d) 3 = 3 **0.5** × 3 ____ 3 × **1/2**
(e) 3 = 3 3 ÷ **2** ____ 3 × **2**	(f) 3 = 3 **6** – 3 ____ 3 – **6**

It is important to note that you are talking about changes to the value of an expression. Remember that there are multiple ways to write expressions without changing their value. For example, if you change 1/2 to 2/4, you have not changed the value because you have changed the fraction into an equivalent form.

(3) The following examples start with the equation 2x + 3x + 1 = 11. Then an operation, shown in bold, is performed on one or both expressions. If the new expressions maintain the statement of equality, put an equal sign (=) in the blank. If the new operations do not maintain the statement of equality, put a not-equal sign (≠) in the blank.

(a) 2x + 3x + 1 = 11 **5x** + 1 ____ 11	(b) 2x + 3x + 1 = 11 **x + x + 3x** + 1 ____ 11
(c) 2x + 3x + 1 = 11 **6x** ____ 11	(d) 2x + 3x + 1 = 11 2x + 3x + 1 **– 1** ____ 11
(e) 2x + 3x + 1 = 11 2x + 3x + 1 **– 1** ____ 11 **– 1**	

The equation in Question 3 contains a variable that represents an unknown value. In an equation like this, there may be one or more values that can be substituted in for the variable to make a true equation. This is called a **solution** to an equation.

(4) Determine if each of the following is a solution to the equation 2x + 3x + 1 = 11. Write “solution” or “not a solution” for each.

(a) x = –2

(b) x = 2

(c) x = 0

After Preparation 3.6 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 3.6, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
understand the use of variables in mathematical equations.	
substitute a value for a variable in a mathematical equation and simplify the equation.	
understand that an equation is a statement of equality.	

3.6.2: Exercise 3.6

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Blood Alcohol Content (BAC) is affected by many different variables.

(ii) Multiplication undoes division.

(iii) The way you solve an equation that contains addition has nothing to do with the way to solve a different equation that contains subtraction.

(iv) An equation is a statement saying that two expressions are equal, so an operation that changes the value of one side must also be done to the other side of the equation.

DEVELOPING SKILLS AND UNDERSTANDING

(2) Find the solution to each of the following:

(a) $3x + 5 = 14$

(b) $6x – 5 = 10$

(c) $2x – 1 = –7$

(d) $\frac{x}{4} + 3 = 8$

(3) Recall that Blood Alcohol Content (BAC) is a measurement of how much alcohol is in someone's blood as a percentage. However, police and the public typically omit the language for percent (%) when quoting the BAC and simply say, "BAC is 0.04."

Write an interpretation of what each of the following BAC values means in terms of the amount of alcohol per 1,000 grams of blood. You may want to refer back to the example in the collaboration.

(a) BAC = 0.1

(b) BAC = 0.02

(4) Use information from the Wikipedia article "Short-term effects of alcohol consumption" (https://en.Wikipedia.org/wiki/Short-term_effects_of_alcohol_consumption) to list effects on an individual having the following BAC. Give at least three effects for each.

(a) BAC = 0.05

(b) BAC = 0.1

(c) BAC = 0.5

(5) Use the Widmark Equation, $B=-0.015t+\left(\frac{2.84\cdot N}{W\cdot r}\right)$, to solve this question. Recall that r = 0.68 for men and r = 0.55 for women.

A male student had five glasses of wine at a party. He weighs 160 pounds. How long will it take before his BAC is 0.08?

(i) 1.31 hours

(ii) 2.00 hours

(iii) 2.84 hours

(iv) 3.37 hours

(6) The percentage of Americans who are retired has been increasing over the last several decades. This is causing some concern because health care, Social Security, and other costs will be the responsibility of a smaller group of people. That is, as the percentage of retired people increases, the percentage of working-age people decreases. The following model predicts the percentage of retired people based on demographic data:[23]

$$R=\frac{t}{588.24}-3.24$$

where R is the percentage (as a decimal) of Americans who are retired in the year t (for example, in 2016, t = 2016). Use this model to complete the table below.

Year	% of Americans who are Retired
(i)	18%
(ii)	21%
(iii)	24%

MAKING CONNECTIONS ACROSS THE COURSE

(7) Crown molding is a decorative trim installed over the joint between the walls of a room and the ceiling. (If you are not sure that you understand the idea, do an Internet search to find an example picture of crown molding). Andy intends to install crown molding around the four sides of the dining room. There will be no gaps. The dining room ceiling is a rectangle with dimensions 14 feet 9 inches by 13 feet. The crown molding is sold in 8-foot lengths that cost $24 each. He decides to purchase enough pieces to allow for 10% waste due to possible mistakes.

(a) What is the *perimeter* of the dining room? Perimeter is the distance around the room.

(b) How many 8-foot boards are needed?

(c) If sales tax is 8.25%, what is the total cost?

(8) To answer the following question, use the formula for the perimeter of a rectangle:

$$P = 2L + 2W$$

where P = perimeter;

L = Length; and

W = Width

Andy's house is on a large lot. He bought 100 yards of chain-link fence on sale. He wants to use all of the material to fence in an area in his backyard. He can only make the fenced area 60 feet wide and he wants it to be as long as possible. What is the longest length possible for the sides?

(9) Now, you will return to graphing on a coordinate plane in preparation for Module 4. Label the following items on the coordinate plane given below. For the points, place a dot at the location of the point and label it with the ordered pair.

(a) (–2, 3)

(b) (2, –4)

(c) (–3, 2)

(d) (0, 2)

(e) (3, 0)

Blank coordinate plane graph with x-axis from -4 to 4 and y-axis from 4 to -4.

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[23] https://www.census.gov/content/dam/Census/library/publications/2020/demo/p25-1144.pdf

3.7: A Return to Proportional Reasoning

INTRODUCTION

Lucas Fuel Treatment is formulated for both gasoline and diesel engines. It gives your fuel system what it really needs – a blend of super slick oils and additives with a high detergent action that allows the engine to operate at maximum efficiency. Also, it cleans and lubricates the carburetor and injectors and causes the fuel to burn more thoroughly for increased power and less fuel consumption.

Image of Lucas Oil website. Key Benefits: A great tune-up in a bottle Cleans and lubricates fuel system Neutralizes low sulfur fuel problems Increases power and miles per gallon by burning excess exhaust emissions Directions: Pour Lucas Oil directly into fuel tank Recommended dosage is 2-3 ounces of fuel treatment per 10 gallons of fuel	**KEY BENEFITS** • A great tune-up in a bottle • Cleans and lubricates the fuel system • Neutralizes low sulfur fuel problems • Increases power and mile per gallon by burning excess exhaust emissions • Increases the life of pumps and injectors **DIRECTIONS FOR USE** The recommended dosage is **2-3 ounces** of Fuel Treatment for every **10 gallons** of gasoline or diesel fuel. Pour Lucas Fuel treatment directly into your fuel tank.

In your group, use the information from the image to answer the following two questions:

- What is the recommended dosage of Fuel Treatment per 50 gallons of fuel?
- What is the recommended dosage of Fuel Treatment per 37 gallons of fuel?

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- proportional relationships are based on a constant ratio.
- rules for solving equations can be applied in unfamiliar situations.

By the end of this collaboration, you should be able to

- set up a proportion based on a contextual situation.
- solve a proportion using algebraic methods.

PROBLEM SITUATION: PROPORTIONS IN ARTWORK

Many professionals such as graphic artists, architects, and engineers work with objects that are enlarged or shrunk. It is usually important that the objects have the same appearance despite the change in size. For example, a business logo on a billboard needs to look the same as a logo on a coffee mug.

In this collaboration, you will explore the mathematics behind these changes in size. First, review the following example of the impact on a graphic's appearance when we change the dimensions.

Imagine that this graphic is our original image. It has the following dimensions: Width: 1.9 inches Height: 1 inch	Image of sun and cloud. Width: 1.9 inches Height: 1 inch
Let's say we make a change to the width only. This new graphic has the following dimensions: Width: 2.45 inches Height: 1 inch	Image of sun and cloud. Width: 2.45 inches Height: 1 inch

Now, let’s say we take the original image and we make changes to both the width and the height, but we do not maintain proportionality. This new graphic has the following dimensions: Width: 2.2 inches Height: 1.19 inches	Image of sun and cloud. Width: 2.2 inches Height: 1.19 inches
And lastly, let’s imagine we take the original image and make changes to both the width and the height, maintaining proportionality. This new graphic has the following dimensions: Width: 2.55 inches Height: 1.29 inches	Image of sun and cloud. Width: 2.55 inches Height: 1.29 inches

Notice that it can be difficult to spot just by looking at the new image if it is proportional to the original image. In Question 1, you’re going to explore what can be done to check for proportionality.

(1) Suppose you were given three different kinds of dimensional changes done to the original graphic without actually seeing the new graphics. The dimensional changes are shown in the table below.

Original graphic dimensions
Width: 1.9 inches
Height: 1 inch

Three Different Kinds of Changes					
Change One		Change Two		Change Three	
Width	Height	Width	Height	Width	Height
7.45	1	4	1.19	0.95	0.5

How could you tell which changes were proportional and which were not? Think individually about this before discussing in your group. **Hint**: Remember that in a proportional relationship, the image is not distorted.

(2) Imagine you are graphic artists hired to make a billboard for a college. The original logo is 2-1/4 inches (width) by 3-3/8 inches (length). You need to enlarge the logo so it has a length of six feet. How wide will the enlarged version be?

(3) In Question 2, you could have used the following proportion to represent the relationship between the original and enlarged objects:

$$\frac{2.25}{3.375} = \frac{4}{6}$$

Could this proportion be written in other ways?

(4) Suppose you had set up the following proportion to solve the original problem in Question 2. What steps would you use to solve the equation?

$$\frac{3.375}{2.25} = \frac{6}{x}$$

(5) Solve the equation. Round to the nearest tenth.

$$\frac{12.7}{x} = \frac{0.2}{3}$$

(6) Solve the equation. Round to the nearest tenth.

$$\frac{8,500}{4,200} = \frac{x}{5}$$

(7) Many small engines for saws, motorcycles, and utility tractors require a mixture of oil and gas. If an engine requires 20 ounces of oil for five gallons of gas, how much oil would be needed for eight gallons of gas? Take a minute to think about this individually before sharing in your group.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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3.7.1: Preparation 3.7

(1) Ben has \$75 in his savings account. He plans to deposit \$35 per week to build his account balance.

(a) Complete the following equation to represent the amount of money (A) Ben will have in his account after any number of weeks. Use x as your input variable.

A =

(b) What does the input variable (x) represent in this problem?

(c) Which of the following values could be the value of the input variable (x) in this context? There may be more than one correct answer.

(i) -5

(ii) 3

(iii) 4.2

(iv) 18

(d) Ben wants to use his savings to buy a computer for \$740. Use the equation from part (a) to determine the number of weeks it will take him to save enough money to buy the computer.

Proportionality

Length-to-width and width-to-length ratios

The dimensions of a figure can be written as a ratio. Imagine a rectangle with a length of ten inches and a width of three inches.

Diagram showing a rectangle, indicating the following: Width = 3 inches Length = 10 inches

You can say that the ratio of the length to width is 10:3 or 10/3. It is also correct to say that the ratio of the width to length is 3:10 or 3/10.

In Collaboration 3.7, when we compare multiple figures using their respective ratios, it will be important to be consistent.

Equivalent Fractions (review)

As you recall from Unit 3.1, equivalent fractions (ratios) can be written in many equivalent forms. For example, all of the following fractions simplify to 10/3 in lowest terms: 10/3 = 20/6 = 30/9 = 100/30

Proportional Figures

Now, we can define what it means for two figures to be proportional. Consider the following figures and compare them using their length-to-width ratios.

Small rectangle: Width = 3 inches Length = 10 inches Large rectangle: Width = 6 inches Length = 20 inches

The length-to-width ratio of the figure on the left is 10/3.

The length-to-width ratio of the figure on the right is 20/6 = 10/3 in lowest terms. Note that 10/3 and 20/6 are equivalent fractions.

While these figures are not *equivalent*, they are *proportional* to each other because their dimensions have the same ratio.

(2) Use the figure below to answer the following questions.

Diagram showing the cylinder, indicating the following: Height = 9 feet Width = 5 feet

(a) Write the ratio of the dimensions of the cylinder shown above in the form of diameter to height.

(b) Give the dimensions of a cylinder that would be proportional to the one shown.

Diameter:

Height:

(3) Which of the following fractions has a ratio of 4:3? There may be more than one correct answer.

(i) $\frac{24}{18}$

(ii) $\frac{16}{9}$

(iii) $\frac{9}{12}$

(iv) $\frac{20}{15}$

(v) $\frac{8.8}{6.6}$

After Preparation 3.7 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 3.7, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
interpret the meaning of ratios, including when written as fractions.	
understand the use of a variable to represent an unknown.	
solve a two-step equation such as 2x + 9 = 13.	

3.7.2: Exercise 3.7

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Graphic artists have to be very aware of proportionality and know how to solve proportions.

(ii) The rules for solving equations are the same for all types of equations.

(iii) The rules for solving equations depend on the type of equation.

(iv) To solve for a variable in the denominator of a fraction, multiply both sides of the equation by the variable.

(2) In Module 1, you learned that a statement such as “30% of voters support Candidate A” can be interpreted as 30 out of 100.

(a) How many voters out of 1,000 support Candidate A?

(b) How many voters out of 1,500 support Candidate A?

(c) Do the ratios from parts (a) and (b) represent a proportional relationship?

DEVELOPING SKILLS AND UNDERSTANDING

(3) The tables below each give the dimensions of four different rectangles. Write in the blank if the four rectangles in each set are “Proportional” or “Not Proportional”.

(a) ____________

	Width	**Length**
Rectangle 1	17	110.5
Rectangle 2	23.4	152.1
Rectangle 3	33	214.5
Rectangle 4	52.2	339.3

(b) ____________

	Width	**Length**
Rectangle 1	7.4	15.5
Rectangle 2	17	40.8
Rectangle 3	23.4	68.5
Rectangle 4	33	72.6

(4) A marine biologist would like to feed some dolphins a mix of fish that consists of 9 parts cod to 4 parts mackerel. List a combination that would be an acceptable mixture of these fish.

(5) Identify the proportions that have the same solution as the one below. There may be more than one correct answer.

$$\frac{3}{x}=\frac{24}{56}$$

(i) $\frac{x}{3}=\frac{24}{56}$

(ii) $\frac{3}{24}=\frac{x}{56}$

(iii) $\frac{24}{3}=\frac{56}{x}$

(iv) $\frac{x}{24}=\frac{3}{56}$

(v) $\frac{x}{3} = \frac{56}{24}$

(6) Solve the following proportions (round to one decimal place):

(a) $\frac{x}{5} = \frac{10}{13}$

(b) $\frac{3}{4} = \frac{15}{2x}$

(7) Erica would like to bake an 8-pound roast for a family gathering. The cookbook tells her to bake a 5-pound roast for 135 minutes. Create and solve a proportion that would allow Erica to cook her 8-pound roast.

(8) Cefaclor is a medication used for infections. It is often given in liquid form by mixing the powdered medication with a fluid. A pharmacist is mixing a dosage for a child. The instructions indicate that 125 mg of the medication should be mixed with 5 mL of fluid. If the child only requires a dosage of 100 mg of Cefaclor, how much fluid should the pharmacist use?

MAKING CONNECTIONS ACROSS THE COURSE

(9) A company is making pennants or flags for a sports team. The team wants small versions for fans and large versions that will fly over the stadium. The dimensions of the small version are shown below.

Diagram showing the small version indicating the following: Height = 8 inches Width = 20 inches

(a) The large version needs to be 12.5 feet across the base (the short side of the triangle). How long should it be?

(b) How much material will be used for the large version of the flag? Round up to the nearest tenth of a square foot to ensure the company has enough material.

(10) A staircase is made up of individual steps that should be consistent in height and width. The height of each step is called the *rise*, and the width of the step is called the *run*.

(a) The staircase below is made up of four steps each with a rise of 6.5" and a run of 8.25". Find the (i) height and (ii) depth of the entire staircase.

Staircase diagram highlighting H and D

(b) Builders have to follow guidelines on the rise and run of stairs when building a staircase. One acceptable ratio is a rise of 7-3/4 inches for a run of 9-3/4 inches. If a builder is using this ratio to build a staircase that is 15 feet high, how deep will the staircase need to be (d in the drawing below)? Note that the drawing does not show the correct number of steps. Round to the nearest tenth of a foot.

Staircase diagram highlighting 15' H and d

Graphing on a Coordinate Plane

Now, you will once again return to graphing on a coordinate plane. You may have noticed that the two axes split the coordinate plane into four sections. These are called **quadrants** and are numbered using Roman numerals as shown below.

For practical reasons, only a small part of the coordinate plane can be shown, but understand that the axes can go on infinitely in all four directions. The **scale** of the grid tells you which numbers are included in the portion of the plane that is shown. You can change the scale to make graphs with very large or very small numbers. The scale on a single axis must be consistent. In other words, if the distance between the gridlines represents five units on one part of the horizontal axis, then that same distance must always represent five units on that axis. However, the vertical and horizontal axes can have different scales as in the example below. As you have seen with other types of graphs, it is important to pay close attention to the scale.

Image showing a blank graph indicating the following: Top-right corner = Quadrant I Top-left corner = Quadrant II Bottom-left corner = Quadrant III Bottom-right corner = Quadrant IV

Blank coordinate plane graph. Axes go from -14 to 14.

(11) Place the following points on the graph. Label each with its ordered pair.

(a) (–14, 7)

(b) (0, 9)

(c) (13, –6)

(12) Indicate if each statement is true or false.

(a) The point (–7, –5) is in Quadrant II.

i) True ii) False

(b) The point (0, 5) lies on the vertical axis.

i) True ii) False

(c) All the points in Quadrant IV have a positive horizontal coordinate and a negative vertical coordinate.

i) True ii) False

(d) The points (20, 12) and (20, 200) lie on the same horizontal line.

i) True ii) False

Many applications use only positive numbers. In these cases, only Quadrant I of the graph is usually shown because that is the only quadrant that is used. An example of this is given below.

(13) You learned about the *golden ratio* in Exercise 3.5. A rectangle whose dimensions match the golden ratio is called a *golden* rectangle. The graph below shows the widths and lengths of golden rectangles.

Image showing the three terms, as described in the following table.

(a) Based on the graph, is a rectangle with a width of 17 inches and a length of 30 inches a golden rectangle?

i) Yes ii) No

(b) Use the graph to complete the table of values below. Estimate to the nearest whole number.

Width	**Length**
5	(i)
8	(ii)
(iii)	20
(iv)	31

3.8: Solving more Equations

INTRODUCTION

As you keep taking more advanced math courses in the future, you will keep seeing more operations, procedures, or functions that cancel each other out. Many operations, procedures, or functions come in pairs. Whatever one operation does, the other one will undo.

By now, you are fairly familiar with two pairs of such operations:

- Addition and subtraction
- Multiplication and division

In this collaboration, we will be discussing two more pairs of operations that cancel each other out:

- Squaring a number (or simply multiplying it by itself two times) and finding a square root.
- Cubing a number (or simply multiplying it by itself three times) and finding a cube root.

Similar to what we did in Collaboration 3.6, let's illustrate it with an example:

1. Pick any non-negative number. This example will use 11.
2. Next, square the number that you picked. In this example, 11^2 = 121.
3. Next, square root the number for the previous step. You can do it by pressing the √ key on your calculator. In my case, $\sqrt{121}$ = 11. **Note**: In this collaboration, all examples involve positive numbers, which is why we disregard the negative value.
4. 11 is the same number that I started with! This shows how squaring a number and taking a square root of a number cancel each other out.

This is not a coincidence. It works for any numbers and variables. For example,

- $\sqrt{71^2}$ = 71
- $\sqrt{x^2}$ = x

The same applies to cubes and cube roots. Let's illustrate with an example:

1. Pick any number. This example will use 8.
2. Next, cube the number that you picked. In my case, 8^3 = 512.
3. Next, cube root the number for the previous step. You can do it by pressing the $\sqrt[3]{x}$ key on your calculator. In my case, $\sqrt[3]{512}$ = 8.
4. 8 is the same number that I started with and so did you! This shows how cubing numbers and taking a cube root of a number cancel each other out.

This is not a coincidence. It works for any numbers and variables. For example,

- $\sqrt[3]{87.5^3}$ = 87.5
- $\sqrt[3]{T^3}$ = T

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- To solve any equation, one must follow the basic rules of undoing and keeping the equation balanced.

By the end of this collaboration, you should be able to

- Solve linear equations that require simplification before solving.
- Solve for a variable in a linear equation in terms of another variable.
- Solve for a variable in a single-term quadratic equation.

PROBLEM SITUATION: SOLVING EQUATIONS OF DIFFERENT FORMS

Solving equations such as the Widmark equation for blood alcohol content (BAC) and proportional equations for resizing graphics is an important skill. Mathematical models are often constructed to represent real-life situations. Being able to use these equations fully includes being able to solve for various unknown variables in the equation. Below are three scenarios to help you practice and enhance your equation-solving skills. With each answer, check that it is reasonable given the context and check that you have included the correct units with your solution.

(1) Paula has two options for going to school. She can carpool with a friend or take the bus. Her friend estimates that driving will cost 22 cents per mile for gas and 8.2 cents per mile for maintenance of the car. Additionally, there is a $25 parking fee per week at the college. If Paula carpools, she would pay half of these costs. The cost of the carpool can be modeled by the following equation where C is the cost of carpooling per week and m is the total miles driven to school each week:

$$C = \frac{1}{2}(0.082m + 0.22m + 25)$$

(a) On your own, take a minute to think about what the 12 represents in the equation above.

(b) In your group, discuss what each term inside the parentheses represents.

(c) In the given formula, 0.082m is the total maintenance costs for m miles and 0.22m is the total gas cost for m miles. Since both terms include the same variable with the same exponent, they can be combined into one term by adding (or subtracting) their coefficients. Discuss in your group how to rewrite the given formula by combining like-terms. In other words, rewrite the given formula to only include one term that represents the cost per mile.

(d) Find the total weekly carpooling cost if the commute to school is seven miles each way and Paula goes to school three times a week. First try this on your own, then discuss as a group when everyone is ready. Round to the nearest cent.

(e) A weekly bus pass costs $22.00 dollars. How many total miles must Paula commute to school each week for the carpool cost to be equal to the bus pass? First try this on your own, then discuss as a group when everyone is ready. Round to the nearest hundredth.

(f) How many trips to school each week must Paula make for the bus pass to be less expensive than carpooling? First try this on your own, then discuss as a group when everyone is ready.

(2) Recall Widmark's equation for BAC. In the case of the average male who weighs 190 pounds,[24] you can simplify Widmark's formula to get

$$B = -0.015t + 0.022N$$

Forensic scientists often use this equation at the time of an accident to determine how many drinks someone had. In these cases, time (t) and BAC (B) are known from the police report. The crime lab uses this equation to estimate the number of drinks (N).

(a) Find the number of drinks if the BAC is 0.09 and the time is two hours. First try this on your own, then discuss as a group when everyone is ready. Round to the nearest drink.

(b) Since forensic scientists use the formula to solve for N, it is easier if the formula is rewritten so that it actually solves for N. In the space below, solve for N in terms of t and B. In other words, rewrite the equation so N is isolated on one side of the equation and all other terms are on the other side. First try this on your own, then discuss as a group when everyone is ready.

(c) Use the new formula to find the number of drinks if the BAC is 0.17 and the time is 1.5 hours. First try this on your own, then discuss as a group when everyone is ready. Round to the nearest drink.

(3) You volunteer for a nonprofit organization interested in women's issues. The logo for your nonprofit organization is three identical squares arranged as follows:

Image showing the three identical squares, one next to the other, staggered. Image showing the three identical squares, one next to the other, staggered.

Image showing the three identical squares, one next to the other, staggered.

(a) The organization wants to make banners of different sizes. Find an equation that can be used to find the total area of the logo based on the length of the side of one of the squares. First try this on your own, then discuss as a group when everyone is ready.

Assume that the 3 squares have the same dimensions.

(b) The organization is sponsoring a walk-a-thon to raise funds for breast cancer research. You want to recreate this logo (forming the three squares) in the middle of the racetrack with bras that have been collected at multiple drop-off sites around the city. You estimate that approximately 1,500 square feet of bras have been donated. How long should you make each side of the square? Round to two decimal places.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[24] http://www.cdc.gov/nchs/fastats/body-measurements.htm

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3.8.1: Preparation 3.8

(1) Which expressions are equivalent to the expression below? There may be more than one correct answer.

$$2(5x + 4) – 3x + 1$$

(i) $10x + 8 – 3x + 1$

(ii) $18x – 3x + 1$

(iii) $15x + 1$

(iv) $7x + 9$

Working with Negatives

It is important to pay careful attention to notation in working with negatives. For example, $–5^2$ is not the same as $(–5)^2$. You can verify this by evaluating each expression on a calculator. The reason has to do with order of operations. The negative in each expression can be thought of as multiplication by –1. Look at the expressions rewritten with a –1 and think about what operation you would perform first.

$$-1 \cdot 5^2 \ (-1 \cdot 5)^2$$

In the first expression, the exponent is done first: $–1 \cdot 5^2 \rightarrow –1 \cdot 25 \rightarrow –25$.

In the second expression, the multiplication is done first: $(–1 \cdot 5)^2 \rightarrow (–5)^2 \rightarrow 25$.

(2) Simplify each of the following:

(a) 5^2

(b) $–3^2$

(c) $(–4)^2$

(d) $–4^2$

(e) $–(–6)^2$

Algebraic Expressions

In algebra, a **term** is an individual part of an algebraic expression. Terms are separated by addition (+) or subtraction (–) signs, and can consist of numbers, variables (letters), or the product of numbers and one or more variables. In instances where a number and variables are being multiplied, the number is called a **coefficient**. For example, the expression below has three terms as shown in boxes. Notice that the last term is (–5). In the original expression, this was written as *minus* 5, but this can be rewritten as *adding negative 5* as shown below. When breaking an expression into terms, you ask, *what is being added?*

Equation. Minus a cubed plus two b minus five equals - one a cubed plus two b plus minus five.

Terms	Coefficients
$–a^3$	–1
2b	2
–5	Does not have a coefficient because there is no variable. This is called a constant term because it never changes.

(3) State the number of terms in each expression:

(a) $3x + 4$

(b) $5x – 4x^2 + 2$

(c) 5

(4) What is the coefficient of the x^2 term in the expression $5x – 4x^2 + 2$?

(5) Recall the formula from Collaboration 3.2 used to find Jenna's cost to drive her own car for work. In this formula, J = Cost of driving Jenna's car in $/mile and g = Cost of gas in $/gallon.

$$J = \frac{g}{18} + 0.0626$$

(a) Find the cost of driving Jenna's car (J) when the price of gas is $3.56/gallon. Round to the nearest cent.

(b) Find the cost of gas (g) when the cost of driving Jenna's car is $0.25/mile. Round to the nearest cent.

After Preparation 3.8 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 3.8, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
understand order of operations when simplifying an expression.	
substitute a value for a variable in a mathematical model and simplify the model.	
square a number.	
solve a two-step linear equation such as 2 = x/3 + 5.	
understand the meaning of the word *term* as in a term in an algebraic expression.	

3.8.2: Exercise 3.8

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

- (i) The order of operations is used in determining the order of steps in solving an equation.
- (ii) The order of operations is not related to solving equations.
- (iii) Weight, gender, and time are all important factors in Blood Alcohol Content (BAC).
- (iv) You can undo addition by subtracting.

DEVELOPING SKILLS AND UNDERSTANDING

(2) Solve the following equations for the unknown variable:

(a) 12.4 = 3w + 8 – 0.5w

(b) 3(r + 1.6) – 8r = 9.5

(c) 53 = 8 + $\frac{6}{x}$

(d) 3 = $\frac{18+n}{44}$

(3) Solve for the specified variable in each equation.

(a) A group of French students plan to visit the United States for two weeks. They are trying to pack appropriate clothing, but are not familiar with Fahrenheit. One student remembers this formula:

$$F=\frac{9}{5}C+32$$

where F is the temperature in Fahrenheit and C is the temperature in Celsius. Solve the equation for C.

(b) Recall using the simple interest formula, A = P + Prt, from Exercise 3.4. In the formula:

A = the full amount paid for the loan

P = the principal or the amount borrowed

r = the interest rate as a decimal

t = time in years

A car dealership wants to use the formula to find the rate needed for certain values of the other variables. Solve the formula for r.

(4) Akiko earns $2335 per month at her full time job. She also works part-time on weekends and evenings for $10.70/hour.

(a) Write an equation for Akiko’s monthly income, M. Use h for the number of hours she works part-time.

(b) Akiko would like to set up a spreadsheet that will calculate how many hours she has to work on weekends and evenings to earn different amounts. Her spreadsheet is shown below.

Screenshot of spreadsheet with two columns, A and B. Cell A1 = Monthly income. Cell B1 = Hours. A2 = 2400. A3 = 2500. A4 = 2600. A5 = 2700. A6 = 2800. B cells are blank.

Write a formula that Akiko can use in cell B2 to calculate the hours. Test your formula on a calculator or spreadsheet to make sure it is correct.

(c) Assume there are 4 weeks in a month. Assume that she will be working approximately the same number of hours each week. How many hours does Akiko need to work on weekends and evenings each week to earn $2,600 per month?

(5) Recall the simplified formula for the braking distance of a car:

$$d=0.018268\cdot V_0^2$$

where V0 is the initial velocity of the car (feet per second) and d is the braking distance (ft). In this model, the roadway grade is kept constant at 5% and the coefficient of friction at 0.8. In a school zone, you want the maximum braking distance to be 10 feet since this seems like a reasonable distance to see a child who might be in the way of a driver.

(a) What should you set as the speed limit (i.e., initial velocity of the car) so that the braking distance is 10 feet or less? Round to the nearest foot per second.

(b) The equation $x^2 = 9$ has two solutions: 3 and -3.

(i) With that in mind, what is the other solution to the equation $10 = 0.018268 \cdot V_0^2$, besides what you answered in (a)?

(ii) Is this a valid solution in the context of this problem?

MAKING CONNECTIONS ACROSS THE COURSE

(6) Refer to Collaboration 1.6 in which you compared the water footprint of different countries. The following information was given. Use the table to answer the questions below.

Country	**Population (in thousands)**	**Total Water Footprint[25] (in 10^9 cubic meters per year)**
China	1,277,208	1,368
India	1,051,290	1,145

(a) What does 10^9 cubic meters mean?

(i) One trillion cubic meters

(ii) One billion cubic meters

(iii) One million cubic meters

(iv) One hundred thousand cubic meters

(b) The following equation is based on information from the table.

$$\frac{1,368}{1,277,208} = \frac{x}{1,051,290}$$

What does x represent?

(i) The water footprint of China if it used water at the same rate as India.

(ii) The population of China if it used water at the same rate as India.

(iii) The water footprint of India if it used water at the same rate as China.

(iv) The population of India if it used water at the same rate as China.

(c) Calculate China’s water footprint if it used water at the same rate as India. Round to the nearest million.

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[25] http://www.waterfootprint.org/Reports/Report50-NationalWaterFootprints-Vol1.pdf

3.9: Module 3 Review Survey

As with Modules 1 and 2, you should assess your understanding of Module 3 to prepare for the Module 3 test. Your instructor may give you specific assignments for your review in addition to this self-assessment.

Assessing Your Understanding

The table on the following page lists the Module 3 concepts and skills you should understand. After completing it, you will be able to prioritize your review time more effectively.

Assess your understanding:

- Go through the topics list and locate each concept or skill in the Module 3 student workbook or in the online activities.
- If you recently used the skill and feel confident that you did it correctly, rate your understanding a 4 or 5.
- If you remember the topic but could use more practice, rate your understanding a 3.
- If you cannot remember that skill or concept, or still find it difficult, rate your understanding a 1 or 2.

Now that you have done an initial rating of your understanding, it is time to begin reviewing. The goal is to have a confidence rating of 4 or 5 on all the topics in the table when you have finished your review of Module 3.

Things to consider for reviewing:

1. Start at the beginning of the module and review the material in the Preparations, Collaborations, and Exercises, and your notes on the skills and concepts you rated 3 or below.
2. Select a few questions to do.
3. Once you have finished, remember to check the solutions. If you are not sure if you have done the problems correctly, check with your instructor, other classmates, your previous work or work with a tutor in the learning center or online.
4. Rate your confidence on this skill again. If you understand the concept better, rate yourself higher. Begin a list of topics that you want to review more thoroughly.

For topics that you need to review more thoroughly, make a plan for getting additional assistance by studying with classmates, visiting your instructor during office or virtual office hours, working with a tutor in the learning center or online, or looking up additional information on the Internet.

Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Module 3 Concept or Skill	Rating
Making Conversions	
Understand use of units in making conversions (3.1, 3.2)	
Use dimensional analysis to make a conversion involving multiple conversion factors (3.1, 3.2)	
Geometric Reasoning	
Understand concepts of and units for linear measurement, area, and volume (3.3)	
Identify and use the appropriate geometric formula to apply in a given situation (3.3)	
Using Formulas and Algebraic Expressions	
Understand the use of variables in formulas and algebraic expressions, including the appropriate way to define a variable (3.4)	
Understand the role of a constant in a formula (3.4)	
Use a formula to solve for a value (3.4, 3.6, 3.8)	
Using Graphical Displays	

Read and interpret a pictograph (3.5)	
Understand the limitations and potential for distortion in pictographs (3.5)	
Creating and Solving Equations	
Solve a linear equation in one variable (3.6, 3.8)	
Interpret the solution to an equation (3.6, 3.7, 3.8)	
Solve an equation or formula for a variable (3.8)	
Write and solve proportions (3.7)	
Solve complex equations with multiple variable terms and variables in the denominator (3.8)	
Solve or estimate the solution to equations with a variable raised to the power of 2 (3.8)	

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CHAPTER OVERVIEW

4: Student Workbook

- 4.1: Lining Up
  - 4.1.1: Preparation 4.1
  - 4.1.2: Exercise 4.1
- 4.2: Comparing Change
  - 4.2.1: Preparation 4.2
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- 4.3: That Is Close Enough
  - 4.3.1: Preparation 4.3
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- 4.9: Saving Green
- 4.10: Module 4 Review Survey

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4.1: Lining Up

INTRODUCTION

Quick Check-in:

Please discuss the following questions with your group:

- What specific concept from Preparation 4.1 do you feel fairly confident about?
- Are there any concepts from Preparation 4.1 that you find challenging?

SPECIFIC OBJECTIVES

In this collaboration, you will learn about how linear models (linear equations in context) can be useful in examining some situations encountered in real life. A model is a mathematical description of an authentic situation. You can also say that the mathematical description "models" the situation.

You will practice using the four representations you read about in Preparation 4.1 to express the linear models for two situations.

PROBLEM SITUATION 1: DATA PLANS

Data plans for cell phone plans can be purchased in two different ways. One is an unlimited data plan where the customer pays a set monthly fee for unlimited data. The other is a per-gigabyte (GB) pricing service where the customer pays a set monthly fee plus a specified amount for each GB. The GBs are not prorated (that means it doesn't matter if the customer uses 0.001 GB or 1 GB, they must still pay for the entire 1 GB). The cost of the phone itself and other fees are the same amount under each plan.

You are shopping for a data plan and need to decide which option costs less. (**Note**: The descriptions of these options are examples of verbal representations of mathematical relationships.)

- Per-GB Pricing: There is a monthly fee of $20 plus $10 per GB.
- Unlimited Data: The unlimited plan costs $60 per month.

(1) Which plan do you think is less expensive and why? Take a minute to consider this on your own first before sharing your ideas in your group.

(2) (a) Use the table below to build a linear model for the **Per-GB Pricing** data plan. First, create equations for 3, 4, 5, 10 GBs used, and then create the linear model for g GBs used.

Per-GB Pricing (P)

GBs Used (g)	Cost	Equation
0	20	P = 20
1	20 + 1(10)	P = 20 + 1(10)
2	20 + 1(10) + 1(10)	P = 20 + 2(10)
3		P =
4		P =
5		P =
…		
10		P =
…		
g	20 + g times	P =

(b) Use the table below to build a linear model for the Unlimited data plan. First, create equations for 1, 2, and 10 GBs used, and then create the linear model for g GBs used.

Unlimited Data (U)

GBs Used (g)	Cost	Equation
0	60	U = 60
1	60	U =
2		U =
…		
10		U =
…		
g		U =

(c) Use your models to fill in the following table.

GBs Used (g)	Monthly Cost of Per-GB Plan (P)	Monthly Cost of Unlimited Data (U)
0		
1		
2		
3		
4		
5		
6		
7		
8		

(d) Using the table from part (c), under what conditions is P less expensive?

(i) When less than 4GBs are used.

(ii) When less than 3GBs are used.

(iii) When more than 4GBs are used.

(e) Using the table from part (c), under what conditions is U less expensive?

(i) When less than 5GBs are used.

(ii) When 5GBs or more are used.

(iii) When more than 4GBs are used.

PROBLEM SITUATION 2: DAILY LATTE

A local coffee shop offers a coffee card that you can preload with any amount of money and use like a debit card each day to purchase coffee. At the beginning of the month (when you get your paycheck), you load it with $60. Your favorite small soy latte costs $3.28.

(3) Pretend you purchase a latte every weekday. Estimate whether the amount of money you loaded onto your coffee card will last until the end of the month. Take a minute to think about this on your own before discussing in your group.

(4) Fill in the graph and table below to help you answer the question: Will your $60 coffee card last until the end of the month? (**Note**: Any given month has about 22 weekdays; there could be as few as 20 weekdays in February, or as many as 23 in some other months.)

(a) Fill in the table.

Lattes (n)	Amount Remaining on Card (A)
0	60
1	56.72
2	
3	
4	
…	
10	
…	
15	
…	
20	

(b) Graph the data from the table in (a). Be sure to include scales and labels on the axes. Let the horizontal axis be the number of lattes purchased and the vertical axis be the amount of money remaining on the card. **Note**: If completing this problem online, follow the instructions given online to create your graph.

Blank coordinate plane graph.

(c) Build a linear model. Use A to represent the remaining money on the coffee card and n to represent the number of lattes purchased. Take a minute to consider this on your own first before sharing your ideas in your group.

(d) Answer the following questions:

(i) Will the card last the entire month?

(ii) If so, how much money is left at the end of the month?

(iii) If not, how many days will it last?

The two mathematical relationships in the cell phone plans are *linear*. They have certain important characteristics. The following terms are vocabulary words you need to know to talk about linear models and other types of mathematical relationships. You will discuss these in your group. Make sure you take good notes about what each means.

Constant rate of change: This is the characteristic that defines a linear relationship. A constant change can be seen both with the table and the graph of both plans.

Slope: This is a ratio that describes the rate of change. Be sure you understand the slope and its relation to the graph as the rise (vertical movement) of the line over the run (horizontal movement). The concept of slope is the rate of change of the quantity represented. The slope is multiplied by the variable because you use it as a rate to find a new value.

Intercepts: The points at which a graph touches one of the axes.

Vertical intercept: The vertical intercept is sometimes called the *starting value*. It is the point at which the graph touches the vertical axis. Note that this often has important meanings in context.

Horizontal intercept: The point at which the graph intersects the horizontal axis, where the output's value is zero.

Linear Model: **A = mt + b**. Where A is the amount of the quantity at any time t; m is the constant rate of change (the slope); and b is the initial amount (when t = 0).

(5) Use the information above to answer the following questions.

(a) What is the rate of change for the model in “Problem Situation 2: Daily Latte” above? What does it mean in the context of the problem situation? Take a minute to try this on your own first before sharing your ideas in your group. Remember to Include units.

(b) What is the vertical intercept for the model in “Problem Situation 2: Daily Latte” above? What does it mean in the context of the problem situation?

(c) Does the model in “Problem Situation 2: Daily Latte” have a horizontal intercept? If so, what is it and what does it mean in the context of the problem situation?

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

4.1.1: Preparation 4.1

Recall a problem you examined in Collaboration 3.2 when you tried to see if it cost more for Jenna to drive her own car or a rental car. As a part of that lesson, you looked at the relationship between the cost of gas and the cost for Jenna to drive her car in $/mile. You also learned in previous collaborations how algebra can be used to express relationships between variables. Now you are going to expand this idea to talk about four representations of a mathematical relationship.

Model or Equation

In Collaboration 3.2, your class wrote a mathematical equation for the relationship. An equation is useful because it can be used to calculate the cost values. As you saw with the formula for braking distance in Collaboration 3.4, equations are also useful for communicating complex relationships. In writing equations, it is always important to define what the variables represent, including units. For example, in Collaboration 3.2, the variables were defined as shown below. Note that each definition includes what the variable represents, such as *cost of Jenna's car*, and the units in which this quantity is measured, such as *$/mile*.

J = Cost of Jenna's car in $/mile

g = Price of gas ($/gal)

These variables were used in the mathematical equation, $J = \frac{g}{18} + 0.0626$.

Table

Another way that you could have represented this relationship between the price of gas and the cost of driving the car is in a table that shows values of g and D as **ordered pairs**. An ordered pair is two values that are matched together in a given relationship. You used this representation in Collaboration 3.4 when you explored how one variable affected another. Tables are helpful for recognizing patterns and general relationships or for giving information about specific values. A table should always have labels for each column. The labels should include units when appropriate.

Price of Gas ($/gal)	Cost of Driving Jenna's Car ($/mile)
3.00	0.23
3.50	0.26
4.00	0.28
4.50	0.31

Graph

A graph provides a visual representation of the situation. It helps you see how the variables are related to each other, and it helps you to make predictions about future values or values in between those in your table. The horizontal and vertical axis of the graph should be labeled, including units.

Line graph showing the cost of driving Jenna's car. The y-axis represents the cost driving (dollar per mile), ranging from 0.00 to 0.40, and the x-axis represents the cost of gas (dollar per gallon), ranging from 0.00 to 6.00, as described in the table on the following page.

Verbal Description

A verbal description explains the relationship in words. As discussed in Collaboration 3.4, some relationships are difficult to put into words, while in other cases a verbal description can help you make sense of what the relationship means in the context. The verbal description for Jenna's car is too complex to discuss here. You will see examples of verbal descriptions in the next lesson.

Summary

Throughout this course, you have learned that having the skill to move between different forms and tools is important in problem solving. Alternating among the four representations of mathematical relationships is another example of this. In some cases, you may struggle with writing an equation, but find it helpful to start with a table . You might want a graph for a visual representation, but also need to express a relationship in words. It is important that you can translate one form into another and also that you can choose which form is most useful in a specific situation.

You will practice these skills with the following questions using the above situation of finding the cost of driving Jenna's car.

(1) Complete the three missing entries in the table:

Price of Gas ($/gal)	Cost of Driving Jenna’s Car ($/mile)
2.00	
3.00	0.23
3.20	
3.50	0.26
4.00	0.28
4.50	0.31
	0.35

(2) Use the graph to estimate the cost of driving if gas is $2.50/gallon.

Line graph showing the cost of driving Jenna’s car. The y-axis represents the cost driving (dollar per mile), ranging from 0.00 to 0.44, and the x-axis represents the cost of gas (dollar per gallon), ranging from 0.00 to 6.00,. Line goes from (2, 0.17) to (5.17, 0.35). Points plotted on line are: Price of Gas ($/gal) 2.00 3.00 3.20 3.50 4.00 4.50 5.59, corresponding to Cost of Driving Jenna's Car ($/mile) 0.217 0.23 0.24 0.26 0.28 0.31 0.35.

After Preparation 4.1 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 4.1, you should understand the concepts and demonstrate the skills listed below:

Skill or Concept: I can …	Rating from 1 to 5
understand the basic meaning and use of variables.	
solve for an unknown variable in a one-variable equation.	
graph points on a coordinate plane.	

4.1.2: Exercise 4.1

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Quantitative literacy is useful in comparing cell phone plans.

(ii) A horizontal line looks “flat.”

(iii) A linear model has a constant rate of change.

(iv) The only way to tell if a model is linear is to look at the graph.

DEVELOPING SKILLS AND UNDERSTANDING

The models you explored in Collaboration 4.1 are linear models. Linear models can represent a wide assortment of realistic problems, including problems that you may encounter in your personal life. Fortunately, linear models are simple to identify. Look for the following characteristics:

- Linear models are applicable when there is a constant rate of change between two variables.
- The graph of a linear model is a line.
- A linear model can be written in a format you may have seen in a previous course:
  y = mx + b where m is the slope and b is the vertical intercept or y-intercept. But note that linear models can also be written in other, equivalent formats such as Ax + By = C. One important characteristic is that the variables in the equation have the power of 1.

In Preparation 4.1, you learned about four ways to represent a linear model using the example of the model for the cost of driving Jenna’s car from Collaboration 3.2. The equation is shown below.

$J=\frac{g}{18}+0.0626$ where J is the cost per mile of driving and g is the price of gas per mile.

This is a linear model and can be written in the form y = mx + b. As you learned in Collaboration 1.4, a fraction is a way to write division. As you learned in Preparation 3.1, division can be written as *multiplying by the reciprocal*: $\frac{g}{18}=\frac{1}{18}\cdot g$. The equation can be written as shown below.

$$J=\frac{1}{18}g+0.0626$$

The slope of this model is $\frac{1}{18}$, or approximately 0.055, meaning the cost per mile of driving the car increases by $0.055 for each dollar that a gallon of gasoline costs. The vertical intercept (0.0626) represents the fixed cost of owning the car averaged out per mile. As you may recall from Collaboration 3.2, this was the cost of maintenance, insurance, and registration.

(2) Do the following representations depict a linear model or nonlinear model? Use the graph, equation, or table provided to determine your answer. **Justify** each decision based on one of the three characteristics listed above.

(a) From Exercise 3.6: $R=\frac{t}{588.25}-3.24$ where R is the percent of Americans who are retired in the year t. The graph is shown below.

Line graph showing the percentage of Americans retired in a given year from 1960 to 2080. The y-axis represents the percentage of Americans retired, ranging from 0% to 030%, and the x-axis represents the years, ranging from 1960 to 2080, as described in previous paragraphs.

(b) From Exercise 3.6: P = 120 + 2L, where P = the perimeter of a rectangular area (in feet), and
L = the length of the rectangle (in feet).

(c) From Collaboration 3.4: The graph for braking distance versus speed for the coefficient of friction of 0.8 and a roadway grade of 5% is shown below.

Line graph showing the braking distance. Y-axis represents the distance in feet, ranging from 0 to 50. X-axis represents the speed in mph, ranging from 0 to 60. 0 = 0 10 = just over 0 ft 20 = just under 10 ft 30 = in between 10 and 20 ft 40 = just under 30 ft 50 = in between 40 and 50 ft

(d)

Time (min)	Distance (ft)

0	3.5
2	13.5
4	23.5
6	33.5

(e) The formula for the area of a circle: $A = \pi r^2$.

(3) Indicate if each of the following statements is true or false.

(a) The slope is a ratio that describes how one quantity changes (increases or decreases) when another quantity increases by 1 unit.

(i) True

(ii) False

(b) The slope of a linear model changes depending on the values substituted into the model.

(i) True

(ii) False

(c) The horizontal intercept is represented by an ordered pair in which the first value is zero: (0, __).

(i) True

(ii) False

(d) The vertical intercept is sometimes called a *starting* or *initial* value.

(i) True

(ii) False

(4) Sheila wants to lose weight to improve her health. She currently weighs 186 pounds and her goal is to weigh 140 pounds. After consulting with her doctor, she feels she can safely lose two pounds per week. The graph tracks the projected weight loss over time.

Line graph showing the weight loss trend. Y-axis represents the weight, ranging from 0 to 200. X-axis represents the number of weeks, ranging from 0 to 28. 0 = in between 150 and 200 4 = in between 150 and 200 8 = in between 150 and 200 12 = in between 150 and 200 16 = just over 150 20 = just under 150 24 = in between 100 and 150 28 = in between 100 and 150

(a) Use the graph to estimate how many weeks it will take Sheila to achieve her desired weight goal.

(b) If Sheila's next health screening is in four months, will Sheila have achieved her goal? Justify your answer.

(c) Write an equation for the weight loss trend. Use W = weight (lbs) and t = time (weeks).

(d) What is the slope of the equation you wrote in (c)? (Include the units in your response.)

(5) During the summer, Tamir likes to swim at his local pool a few times a week. He is debating if he should buy a "season pass" that allows him to swim as many times as he wants or just pay each time he goes swimming. The pool charges $4.50 each day if he pays each time. The "season pass" is $207, which is good for the entire summer (91 days).

(a) Fill in the following table to compare the two options.

Number of Swim Days	Total Cost of Paying Per Day ($)	Cost of Season Pass ($)
0	(i)	$207
10	(ii)	$207
20	(iii)	$207
30	(iv)	$207
40	(v)	$207
50	(vi)	$207
60	(vii)	$207

70	(viii)	\$207
80	(ix)	\$207
90	(x)	\$207

(b) How many times would Tamir have to swim to make the “season pass” less expensive than paying for each visit individually?

At least ______ times

(c) Which of the graphs below models each situation? Assume each graph depicts Quadrant I only and that the origin is in the lower left hand corner of each graph. Assume number of days is on the horizontal axis and total cost is on the vertical axis.

Season pass: _______________ Paying per day: _______________

Graph showing a straight-line decreasing diagonally.	Graph showing a straight-line increasing diagonally starting from above 0 in the y-axis.
Graph showing a straight-line remaining the same along the y-axis.	Graph showing a straight-line increasing diagonally.

(6) Doctors use intravenous (IV) drips to deliver fluids and medications to patients. Suppose an IV drip starts with 1,000 mL of saline and dispenses fluid at a rate of 2.5 mL per minute.

(a) Write an equation for this situation using F to represent the amount of fluid (saline solution) left in the IV drip in milliliters and t to represent the time in minutes. (**Hint**: If you have trouble writing a model, try making a table first.)

(b) Is this a linear model? Explain your answer.

(c) How long will it take to use all of the saline solution? Give your answer in hours and minutes.

(d) Which of the following can be used to describe the point you found in (c)?

(i) Horizontal intercept

(ii) Vertical intercept

(iii) Slope

(iv) Origin

MAKING CONNECTIONS ACROSS THE COURSE

(7) The Bureau of Labor and Statistics published the following table on its website.[1]

Employment in Healthcare by Industry Segment,
2021 and Projected Change, 2021–31 (employment in thousands)

Industry Segment	2021 Employment	2021–31 Percent Change
Healthcare diagnosing or treating practitioners, total	6,118.5	9.2
Registered nurses	3,130.6	6.2
Nurse anesthetists	45.2	11.8
Nurse midwives	8.1	7.5
Nurse practitioners	246.7	45.7
Physicians	701.3	2.8

(a) Based on this prediction, how many total healthcare diagnosing or treating practitioners will be employed in the United States in 2031?

(b) What is this number in words?

(c) On average, how many healthcare diagnosing or treating practitioners will be added each year during this period?

(d) Which of the following can be used to describe the value you found in (c)?

- (i) Horizontal intercept
- (ii) Vertical intercept
- (iii) Slope
- (iv) Origin

(e) Write a linear equation for this relationship. Let W = the number of workers, and t = the number of years after 2021 (for example, t = 1 for 2022).

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[1] https://www.bls.gov/emp/tables/occupational-projections-and-characteristics.htm

4.2: Comparing Change

INTRODUCTION

Quick Check-in:

Please discuss the following questions with your group:

- What specific concept from Unit 4.1 do you feel fairly confident about?
- Are there any concepts from Unit 4.1 that you find challenging?

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand

- that linear models are appropriate when the situation has a constant rate of increase/decrease or can be approximated by a constant rate.
- that the rate of change (slope) has units in context.
- the difference between a positive slope and a negative slope.
- that the linear models for authentic situations have limitations in using them to make predictions.

By the end of this collaboration, you should be able to

- make a linear model when given data or information in context.
- calculate a slope given data or information in context.
- estimate the value that makes two linear models equivalent.

LINEAR MODELS AND ESTIMATION

In this collaboration, we will build linear models to estimate soda and coffee consumption over time. While our models will not be perfectly accurate for each year, they will provide rough estimates that allow us to see how soda and coffee consumption have changed over time.

PROBLEM SITUATION: COFFEE AND SOFT DRINK CONSUMPTION

Over the last 70 years, consumption per person of coffee and soft drinks has changed drastically in the U.S. In 1950, each person in the U.S. consumed an average of 38 gallons of coffee during the year. In 2020, that number had decreased to 27.6 gallons of coffee per person. Meanwhile, the average number of gallons of soft drinks consumed per person in 1950 was 10.8. By 2020, this number increased to 43 gallons of soft drinks per person.[2]

In the following questions, you will find out when the consumption per person of coffee equaled (or will equal) the consumption per person of soft drinks. For this problem, we will build linear models for coffee and soft drink consumption. Assume that the change in consumption is linear.

Note: The slope is the “rise over the run,” which is the vertical change (rise) of a line divided by the horizontal change (run) of a line.

$$Slope = \frac{rise}{run} = \frac{Absolute\ change\ in\ the\ quantity}{Absolute\ change\ in\ theinput\ (time\ here)}$$

In mathematical terms, if you have two points (x_1 , y_1) and (x_2 , y_2), then the slope between them is:

$$\frac{y_2 - y_1}{x_2 - x_1}$$

In the formula above, y is the quantity and x is the input value. In Collaboration 4.1, the quantity was cost and the input value was GBs. In this problem situation, the quantity is gallons and the input value is time in years.

(1) Find the slopes (rates of change) for both the U.S. consumption of coffee per person and the U.S. consumption of soft drinks per person. Take a minute to try this on your own first before sharing in your group. Round to two decimal places.

(2) Let the time (t) be zero in 1950. What would time (t) be in 2020?

(3) Let the time (t) be zero in 1950 as in Question 2. Use the slope you found for the U.S. consumption of coffee per person to find the quantity of coffee consumed in 1985. Take a minute to consider this on your own first before sharing your ideas in your group.

Round to one decimal place. Include units.

(4) Let time (t) be zero in 1950 as in Question 2, and use the ideas from Collaboration 4.1 to build linear models for both U.S. consumption of coffee per person and U.S. consumption of soft drinks per person.

C =

S =

(5) In your group, use your linear models to complete the table below. Enter answers rounded to one decimal place (where necessary).

Calendar Year	Years Since 1950 (t)	Coffee (in gallons)	Soft Drinks (in gallons)
1950	0		
1960			
1970			
1980			
1990			
2000			
2010			
2020			

(6) Plot the data for U.S. consumption of coffee and U.S. consumption of soft drinks on the graph. **Note**: If completing this problem online, follow the instructions given online to create your graph.

Blank coordinate plane graph.

(7) Can you use the table and graphs to approximate when the consumption of coffee per person equaled or will equal the consumption of soft drinks per person? Explain.

(8) (a) Referring to the graph in Question 6, what is the significance of the point in each of the models when time (t) is zero in the context of the situation?

(b) What do each of these points from (a) represent on the graph in mathematical terms?

(9) Use the equation models in Question 4 to find when the consumption of coffee and soda was or will be equal. What is the number of gallons for each at that time? Consider this on your own first before discussing your ideas in your group.

FURTHER APPLICATIONS

(10) In Collaboration 3.4, you probably created a table of values like the one below for the relationship between the velocity of a car in miles per hour (mph) and the braking distance in feet.

(a) Calculate the slope between each of the two points in the table. The first answer is shown as an example.

Velocity (mph)	Braking Distance (ft)	Slope Between Two Points
5	0.98	
15	8.84	The slope between (5, 0.98) and (15, 8.84) is 0.786.
20	15.72	The slope between (15, 8.84) and (20, 15.72) is
30	35.37	The slope between (20, 15.72) and (30, 35.37) is

50	98.24	The slope between (30, 35.37) and (50, 98.24) is

(b) Is this a linear relationship? Explain your answer.

(c) Which of the following is the best explanation for the meaning of the first slope in the table?

(i) At speeds between 5 and 15 miles per hour, the braking distance increases *exactly* 0.786 feet for every increase of 1 mile per hour in speed.

(ii) On average, at speeds between 5 and 15 miles per hour, the braking distance increases 0.786 feet for every increase of 1 mile per hour in speed.

(iii) At speeds between 5 and 15 miles per hour, the braking distance decreases *exactly* 0.786 feet for every increase of 1 mile per hour in speed.

(iv) On average, at speeds between 5 and 15 miles per hour, the braking distance decreases 0.786 feet for every increase of 1 mile per hour in speed.

(d) Use the trend of the data in the table to make predictions about the braking distance for speeds between 50 and 70 mph. Which of the following is a correct statement?

(i) The braking distance between 50 and 70 mph will increase by about 3.144 feet per mile.

(ii) The braking distance between 50 and 70 mph will increase by exactly 3.144 feet per mile.

(iii) The braking distance between 50 and 70 mph will increase by more than 3.144 feet per mile.

(iv) The braking distance between 50 and 70 mph will increase by less than 3.144 feet per mile.

(e) Explain your answer to (d).

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[2] https://www.ers.usda.gov/data-products/food-consumption-and-nutrient-intakes/

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4.2.1: Preparation 4.2

(1) In Exercise 3.4, you used a model to determine the amount owed on a loan of $5,000 with simple interest of 4.25%. The model was A = 5000 + 5000(0.0425t), where A is the amount (total principal plus interest) required to repay a loan, and t is the time, in years.

(a) What is the slope of this model? Include units in your answer. (**Hint**: Think about simplifying the equation or use the equation to find a few values and see how much the amount increases each year.)

(b) In Exercise 3.4, you estimated how long it would take for the loan amount to reach $7,000. Now, instead of estimating it, use the given model to find it. Round your answer to the nearest tenth. Be sure to show your work.

(2) Ali wants to save money to buy a new video game. Ali's mother agrees to help him by paying him for each "A" he receives on a test in school. Ali's mother creates a spreadsheet to show Ali how much money he will have based on the number of tests on which he earns an "A." The spreadsheet with the formula is shown below.

Screen capture of data in an excel sheet, indicating the following in cells: Column B formula = 5 plus 1.5 multiplied by A2 A1 = Number of Tests A2 = 0 A3 = 1 A4 = 2 A5 = 3 A6 = 4 A7 = 5 A8 = 6 A9 = 7 A10 = 8 A11 = 9 A12 = 10 B1 = Amount Saved B2 = $5.00 B3 = $6.50 B4 = $8.00 B5 = $9.50 B6 = $11.00 B7 = $12.50 B8 = $14.00 B9 = $15.50 B10 = $17.00 B11 = $18.50 B12 = $20.00

Indicate if each statement is true or false.

(a) Ali did not have any money saved to begin with.

(i) True

(ii) False

(b) Ali will receive $1.50 for each "A" he earns.

(i) True

(ii) False

(c) The slope of this model is $1.50/test with a grade of "A."

(i) True

(ii) False

(d) The vertical intercept of this model is $1.50.

(i) True

(ii) False

(e) The horizontal intercept of this model is $1.50.

(i) True

(ii) False

(f) If Ali's grandfather offers to add $0.50 to the amount Ali gets for each test, the slope of the model would increase.

(i) True

(ii) False

After Preparation 4.2 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 4.2, you should understand the concepts and demonstrate the skills listed below:

Skill or Concept: I can …	**Rating from 1 to 5**
understand a rate of change as a ratio or proportion.	
solve for an unknown variable in a linear relationship.	
create a linear model from an authentic context with units.	

4.2.2: Exercise 4.2

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Coffee consumption is decreasing as soft drink consumption increases.

(ii) The slope is a ratio that describes how one quantity changes (increases or decreases) when another quantity increases by 1 unit.

(iii) A slope tells you how much one variable has changed.

(iv) A slope can be written as a fraction or a decimal.

(2) You have learned about absolute change, relative change, and average rate of change. Identify which type of change best describes each of the following values and explain your answer.

(i) 30%

(ii) $30/year

(iii) $30

DEVELOPING SKILLS AND UNDERSTANDING

(3) The three graphs below model the relationship between time spent in class and time spent studying for three students. Use the graphs to answer the following questions.

Line graph showing the relationship between time spent in class and time spent studying for three students. Y-axis represents the number of hours spent studying, ranging from 0 to 24. X-axis represents the number of hours spent in class, ranging from 0 to 12. 0 = 0 2 = 4 4 = 8 6 = 12 8 = 16 10 = 20

Line graph showing the relationship between time spent in class and time spent studying for three students. Y-axis represents the number of hours spent studying, ranging from 0 to 6. X-axis represents the number of hours spent in class, ranging from 0 to 12. 0 = 0 2 = 1 4 = 2 6 = 3 8 = 4 10 = 5

Line graph showing the relationship between time spent in class and time spent studying for three students. Y-axis represents the number of hours spent studying, ranging from 0 to 12. X-axis represents the number of hours spent in class, ranging from 0 to 12. 0 = 0 2 = 2 4 = 4 6 = 6 8 = 8 10 = 10

(a) Which of the graphs has a slope of 2?

(b) Which graph shows a relationship of one hour studying for each hour in class?

(c) Which graph matches the following table?

Hours in class	1	4	7	9
Hours studying	2	8	14	18

(d) What is the equation for Graph B? Use C for hours in class and s for hours studying.

(e) Which graph would contain the point (20, 10) if it were extended?

(4) A parents' group is writing a grant to support an afterschool program for their children. They want to make the point that government funding for afterschool programs in their school has decreased over the last decade while the cost of offering programs has increased. Their argument is based on the data below.

	2000	**2010**
Government funding for afterschool programs ($/child)	$3,000	$1,800
Cost of services ($/child)	$3,800	$5,100

(a) Which of the following is the best interpretation of the average rate of change in the government funding?

(i) Government funding has changed by an average of $120 each year.

(ii) Government funding has decreased by an average of $120 per child each year.

(iii) Government funding has decreased by an average of $1,200 per child each year.

(iv) Government funding has decreased by an average of $1,800 per child each year.

(b) Which of the following is the best interpretation of the average rate of change in the cost of services?

(i) The cost of afterschool programs has increased by an average of $5,100 per child each year.

(ii) The cost of afterschool programs has changed by an average of $130 each year.

(iii) The cost of afterschool programs has increased by an average of $1,300 per child each year.

(iv) The cost of afterschool programs has increased by an average of $130 per child each year.

(c) Sketch a graph showing linear models of the two sets of data. Be sure to include a title, scales, and labels on the axes. Label each of the models on the graph.

(5) The Recording Industry Association of America (RIAA) tracks revenue data for the music industry in the United States. In 2018, the sales of music CDs were approximately $700 million and just $480 million in 2022.[3]

The association reported that during that same time period, vinyl records (EP/LP) increased from $420 million in 2018 to $1,200 million in 2022.

(a) What is the slope, including units, of the model for CD (compact disc) sales?

(b) Fill in the blank to complete the following statement:

Sales of CDs are ________________ at a rate of $44 million per year

(c) Write an equation that represents the linear model for CD sales. Let C = the sales of CDs in millions of dollars, and t = the number of years after 2018.

(d) What is the slope, including units, of the model for vinyl records?

(e) Fill in the blank to complete the following statement:

Sales from vinyl records are ________________ at a rate of $195 million per year.

(f) Write an equation that represents the linear model for vinyl records. Let V = the total sales of vinyl records in millions of dollars, and t = the number of years after 2018.

(g) In what year do you expect vinyl records sales to exceed CD sales?

(6) Suppose you work for a shuttle service. You see the following spreadsheet that gives information about what your company will charge for rides of different lengths. The formula shown is for cell B2. Select which statement tells how the fare is calculated based on the number of miles.

Screen capture of data in an excel sheet, indicating the following in cells: Column B formula = A2 multiplied by 0.25 plus 3 A1 = Number of Miles A2 = 10 A3 = 15 A4 = 20 B1 = Total Cost of Ride B2 = $5.50 B3 = $6.75 B4 = $8.00

(i) The fare is $3.25 per mile.

(ii) The fare is a $3.25 flat fee.

(iii) The fare is $3 per mile plus a $0.25 or 25 cents flat fee.

(iv) The fare is $0.25 or 25 cents per mile plus a $3 flat fee.

(7) Delilah wants to join a gym, so she shops around to find the one with the lowest overall price (she is not sure how long she will be a member). She finds the Harbor Square Athletic Club is running a special, and only charges a $25 initiation fee plus $87 a month to be a member. The local YMCA charges $100 to join, and has a monthly fee of $72.

(a) Find the equations for the linear models for the costs to join the Harbor Square Athletic Club and the YMCA. Use C = cost in dollars and m = months.

(b) Calculate how many months Delilah would have to be a gym member for it to be **less expensive** to join the YMCA. Then fill in the blank.

The YMCA will be less expensive if she remains a member for more than ________ months.

MAKING CONNECTIONS ACROSS THE COURSE

(8) In Collaboration 2.3, you used the following data about population changes in the United States. Suppose state planners in Illinois and Nevada think that the average rate of change will continue through 2025.

State	2010 Population	2020 Population
Nevada	2,700,551	3,108,462
Illinois	12,864,380	12,822,739

(a) Write equations to model the populations for each state. Use N for the population of Nevada and I for the population of Illinois and t for time in years after 2010. Round answers to the nearest person.

(b) Use your equations to find the projected populations for Nevada and Illinois in 2025.

---

[3] https://www.riaa.com/u-s-sales-database/

4.3: That Is Close Enough

INTRODUCTION

Recall the information in Preparation 4.3 about Social Security Reform. Refresh your memory by reading the information below and note any questions you have before discussing the prompts below in your group.

There is concern about the Social Security program because it is projected to eventually run out of money. Based on the current program, the Social Security program is projected to be able to pay all benefits through the year 2033. After that, it will only take in enough money to pay three-fourths of benefits. There is broad agreement that the program should be reformed now to avoid this future budget problem, but there is not an agreement about how that should be done.

The following are possible solutions that have been proposed:

- Increase the tax rate for paying into Social Security.
- Increase the limit on wages so that people pay Social Security on wages above $160,200.
- Decrease future benefits.
- Increase the retirement age.
- Change the program into a system of private retirement accounts in which individuals invest their own money.

Projections about Social Security are based on many variables. One of these is life expectancy. Life expectancy is a prediction about how long people live on average. It is important to understand that this is a mean. When you studied mean in Module 2, you learned that a data set can have values far above or below the mean. If a large group of people has a life expectancy of 63 years, some people will die very young, even as infants, and some will live to be over 100.

This might lead you to ask if using a mean to measure life expectancy is very accurate. In Collaboration 2.7, you saw data sets with home prices in which the mean was not a good representation of the data because there were a few very high home prices that made the mean much higher than most of the data. Prices of luxury homes can be 10 or 20 times higher than the price of “average” homes. This makes the mean a poor representation of the data. The range of life expectancy is different because it has more defined limits. As of 2023, the person known to have lived the longest was Jeanne Calment, who died at the age of 122 in 1997. This is an extremely high age, but it is rare and represents the maximum possible age known. So, although it is twice an average age in the range of 60–70 years, it would not have much impact on the mean. This indicates that the mean is a fairly accurate way to summarize the data. This is useful when making projections about a large population. This is why life expectancy statistics are used for projecting costs for Social Security. But you should remember that these types of statistics are not always good predictors for individuals.

- Discuss in your group any questions you have.
- Do you think the Social Security retirement age should be raised or whether other methods should be used to reform the system? Justify your reasons.
- Do you think life expectancy is different for certain populations in the United States?

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- linear equations can approximate nearly linear data.

By the end of this collaboration, you should be able to

- find the equation of a line that estimates nearly linear data by calculating the rate of change and estimating the vertical intercept of the line.
- use approximate linear models to interpolate and extrapolate.

PROBLEM SITUATION: LIFE EXPECTANCY AND SOCIAL SECURITY

As you read in Preparation 4.3, one proposal to reform Social Security is to increase the retirement age. This, however, raises concerns about fairness. No matter what the retirement age is, some people will pay into Social Security but die before retirement and never receive a benefit. This happens to more people when the retirement age is increased. In this collaboration, you will examine the effects of raising the retirement age to 75. Specifically, you will answer the question of whether this change would have a greater impact on some groups than others.

To explore this question, you will use life expectancy data from the Centers for Disease Control. Real data rarely fall on a straight line, but sometimes data show a definite trend. If the trend is close to linear, the data can be *approximated by a linear model*. This means that a linear model gives good estimates of what the data will be if the trend continues. A model can also be used to estimate values between data points. In this collaboration, you will learn to create linear models from data.

Note: The graphs in this collaboration use the rescaled number of years on the horizontal axis rather than the calendar years. Published graphs are more likely to show the calendar years (you will see data in Exercise 4.3 given in terms of calendar years).

(1) The following data show the life expectancy of African American males in the United States at birth.

Line graph showing the life expectancy of African American males in the United States at birth. Y-axis represents the life expectancy at birth, ranging from 0 to 80. X-axis represents the number of years since 1900, ranging from 0 to 120. 20 = in between 40 and 50 30 = in between 40 and 50 40 = in between 50 and 60 50 = 60 60 = just over 60 70 = 60 80 = in between 60 and 70 90 = in between 60 and 70 100 = in between 60 and 70 110 = 70

In your group, find a linear model to approximate these data, letting L be the life expectancy at birth and y the year of birth measured in years after 1900. Take a minute to do this individually before discussing your ideas in your group.

(2) In what year will African American male babies have a life expectancy of 75 years? In what year will these male babies be eligible to begin drawing Social Security if the retirement age is raised to 75?

(3) The last part of this collaboration uses data about the life expectancy of a different population of Americans: white females.

Line graph showing the life expectancy of white females in the United States at birth. Y-axis represents the life expectancy at birth, ranging from 0 to 90. X-axis represents the number of years since 1900, ranging from 0 to 120. 20 = in between 50 and 60 30 = in between 60 and 70 40 = in between 60 and 70 50 = in between 70 and 80 60 = in between 70 and 80 70 = in between 70 and 80 80 = in between 70 and 80 90 = in between 70 and 80 100 = 80 110 = 80

(a) Working with your group, find a linear model to approximate your dataset, letting L be the life expectancy at birth and y the year of birth measured in years after 1900. Be prepared to develop the line you feel best represents the data graphically and how you found the equation of this line.

(b) When does your model predict the population will first have a life expectancy of 75 years at birth? In what year will this group begin collecting Social Security if the retirement age is raised to 75? Be prepared to share your results with the class.

(c) When does your model predict the population will first have a life expectancy of 90 years at birth?

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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4.3.1: Preparation 4.3

Your next collaboration will focus on the Social Security program. Social Security provides income for people who are elderly or disabled.

The following information is from the Social Security Administration website:[4]

> Social Security is financed through a dedicated payroll tax. Employers and employees each pay 6.2 percent of wages up to the taxable maximum of $160,200 (in 2023), while the self-employed pay 12.4 percent.

When a person reaches “full retirement age,” he or she can claim a full retirement benefit. If someone retires before full retirement age, he or she receives a lower benefit.

(1) Use the following information from the Social Security Administration website to answer the questions below.[5]

Full retirement age is the age at which a person may first become entitled to full or unreduced retirement benefits.

> No matter what your full retirement age (also called "normal retirement age") is, you may start receiving benefits as early as age 62 or as late as age 70.
>
> **If You Retire Early**
>
> You can retire at any time between age 62 and full retirement age. However, if you start benefits early, your benefits are reduced a fraction of a percent for each month before your full retirement age.
>
> The chart below lists age 62 reduction amounts and includes examples based on an estimated monthly benefit of $1000 at full retirement age.

Full Retirement and Age 62 Benefit By Year Of Birth

Full Retirement At Age 62

Year of Birth	**Full (normal) Retirement Age**	**Months between age 62 and full retirement age**		**A $1000 retirement benefit would be reduced to**	**The retirement benefit is reduced by**	**A $500 spouse's benefit would be reduced to**	**The spouse's benefit is reduced by**
1937 or earlier	65	36		$800	20.00%	$375	25.00%
1938	65 and 2 months	38		$791	20.83%	$370	25.83%
1939	65 and 4 months	40		$783	21.67%	$366	26.67%
1940	65 and 6 months	42		$775	22.50%	$362	27.50%
1941	65 and 8 months	44		$766	23.33%	$358	28.33%
1942	65 and 10 months	46		$758	24.17%	$354	29.17%
1943-1954	66	48		$750	25.00%	$350	30.00%
1955	66 and 2 months	50		$741	25.83%	$345	30.83%
1956	66 and 4 months	52		$733	26.67%	$341	31.67%

1957	66 and 6 months	54		$725	27.50%	$337	32.50%
1958	66 and 8 months	56		$716	28.33%	$333	33.33%
1959	66 and 10 months	58		$708	29.17%	$329	34.17%
1960 and later	67	60		$700	30.00%	$325	35.00%

(a) Who can collect full retirement benefits at age 65?

(b) At what age can someone born in 1950 collect full benefits?

(c) What is the youngest age at which you can begin collecting reduced benefits?

SOCIAL SECURITY REFORM

There is concern about the Social Security program because it is projected to eventually run out of money. Based on the current program, the Social Security program is projected to be able to pay all benefits through the year 2033. After that, it will only take in enough money to pay three-fourths of benefits.[6] There is broad agreement that the program should be reformed now to avoid this future budget problem, but there is not an agreement about how that should be done. The following are possible solutions that have been proposed:

- Increase the tax rate for paying into Social Security.
- Increase the limit on wages so that people pay Social Security on wages above $160,200.
- Decrease future benefits.
- Increase the retirement age.
- Change the program into a system of private retirement accounts in which individuals invest their own money.

Projections about Social Security are based on many variables. One of these is life expectancy. Life expectancy is a prediction about how long people live on average. It is important to understand that this is a mean. When you studied mean in Module 2, you learned that a data set can have values far above or below the mean. If a large group of people has a life expectancy of 63 years, some people will die very young, even as infants, and some will live to be over 100.

This might lead you to ask if using a mean to measure life expectancy is very accurate. In Collaboration 2.7, you saw data sets with home prices in which the mean was not a good representation of the data because there were a few very high home prices that made the mean much higher than most of the data. Prices of luxury homes can be 10 or 20 times higher than the price of “average” homes. This makes the mean a poor representation of the data. The range of life expectancy is different because it has more defined limits. As of 2023, the person known to have lived the longest was Jeanne Calment, who died at the age of 122 in 1997.[7] This is an extremely high age, but it is rare and represents the maximum possible age known. So, although it is twice an average age in the range of 60–70 years, it would not have much impact on the mean. This indicates that the mean is a fairly accurate way to summarize the data. This is useful when making projections about a large population. This is why life expectancy statistics are used for projecting costs for Social Security. But you should remember that these types of statistics are not always good predictors for individuals.

After Preparation 4.3 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 4.3, you should understand the concepts and demonstrate the skills listed below:

Skill or Concept: I can …	**Rating from 1 to 5**
understand the meaning and use of variables.	
solve for an unknown variable in a linear relationship.	
calculate a rate of change (slope) from data.	

make a linear model given linear data in context.	

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[4] https://www.ssa.gov/news/press/factsheets/HowAreSocialSecurity.htm

[5] http://www.socialsecurity.gov/retire2/agereduction.htm#chart

[6] http://socialsecurity.gov/OACT/TRSUM/index.html

[7] http://en.Wikipedia.org/wiki/Oldest_people

4.3.2: Exercise 4.3

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Keeping the Social Security program going is an important and complex issue. Data and quantitative skills can be useful in addressing the issue.

(ii) A linear model has a vertical intercept.

(iii) If a model based on data is created correctly, it will accurately predict values beyond the data.

(iv) Models created from data are based on assumptions. It is important to ask if the assumptions are correct or will remain correct over time.

DEVELOPING SKILLS AND UNDERSTANDING

The flight times and distances from Memphis, Tennessee to various cities are given in the table and graph below. Use this information to answer **Questions 2–4**.

Time to Fly from Memphis to Various Cities

City	Distance (miles)	Time (minutes)
Atlanta	332	82
Baltimore	787	162
Boston	1139	191
Cincinnati	403	100
Detroit	611	121

Line graph showing the flight times and distances from Memphis, Tennessee to various cities. The y-axis represents the number of minutes, ranging from 0 to 250, and the x-axis represents the number of miles, ranging from 0 to 1,200, as described in the table on the previous page.

(2) Use what you know about the relationship of the equation to the graph to select the model from the list below that is most representative of the data. Do not actually find a linear model to make your choice. In each option, d represents the distance in miles and t represents the time in minutes.

(i) $t = -0.13d + 40$

(ii) $t = 0.13d + 40$

(iii) $t = 0.13d + 110$

(iv) $t = -0.13d + 110$

(3) Using the model from Question 2, calculate the flight time from Memphis to Philadelphia if the distance between the two cities is 874 miles. Round your answer to the nearest tenth of an hour.

(4) Based on the model from Question 2, select the response that best tells how long it will take a Delta plane flying at full speed to travel 1 mile.

(i) The plane goes 0.13 miles in one minute so it takes between 7 and 8 minutes to go 1 mile.

(ii) It takes 40 minutes to go 1 mile.

(iii) It takes 0.13 minutes or around 8 seconds to go 1 mile.

(iv) It takes 40.13 minutes to go 1 mile.

WORLD RECORD SPEEDS OVER TIME

The table below shows the world record times in the men's 100-meter dash from 1912 to 2009 (as of 2023, this time continues to be the record). Use this to answer Questions 5–8.

Men’s 100-Meter Dash[8]

Year Record Time was Run	World Record Time (nearest tenth of a second)
1912	10.6
1921	10.4
1930	10.3
1936	10.2
1956	10.1
1960	10
1968	9.9
1999	9.8
2008	9.7
2009	9.6

(5) (a) Create a scatterplot using the data relating “years since 1900” and “World Record Time” for the Men’s 100-meter dash.

(b) Which of the following is a suitable linear model for representing the data, when T = time in seconds and y = number of years after 1900?

(i) $T = 10.6y – 0.009$

(ii) $T = 0.009y – 10.6$

(iii) $T = –10.6y + 0.009$

(iv) $T = –0.009y + 10.6$

(6) Use the model to predict in what year the world record will fall below 9.0 seconds.

(7) Identify which of the following statements are correct based on the model. Note that the values given are approximations. There may be more than one correct answer.

(i) On average, the men’s world record in the 100-meter dash has decreased about one-hundredth of a minute each year.

(ii) On average, the men’s world record in the 100-meter dash has decreased about one-tenth of a minute every 10 years.

(iii) On average, the men’s world record in the 100-meter dash has decreased about one-hundredth of a second each year.

(iv) On average, the men’s world record in the 100-meter dash has decreased about one-tenth of a second every 10 years.

(8) Models often only work well for a limited range of input values. Outside that range, the model is said to break down. Which of these explains why the world record model might break down? There may be more than one correct answer.

(i) The data do not form a perfect line.

(ii) Eventually the model will predict negative times for the race.

(iii) The times are not really accurate.

(9) The following chart gives sunrise times for New York City in hours after midnight as measured in Eastern Standard Time (EST) on the fifteenth day of each month. The first month is August, the second month is September, and so on. Would it be appropriate to use a linear model to represent these data? Explain.

Line graph showing the sunrise times for New York City in hours after midnight as measured in Eastern Standard Time (EST) on the fifteenth day of each month. Y-axis represents the hours after midnight, ranging from 0 to 9. X-axis represents the months. Aug = 6 Sept = in between 6 and 7 Oct = in between 7 and 8 Nov = in between 7 and 8 Dec = in between 8 and 9 Jan = in between 8 and 9 Feb = in between 7 and 8 Mar = in between 7 and 8 Apr = in between 6 and 7 May = in between 5 and 6 Jun = in between 5 and 6 Jul = in between 5 and 6

MAKING CONNECTIONS ACROSS THE COURSE

In baseball, a team must score more runs than its opponent to win the game. Players have to reach base for a team to score runs. How often a player reaches base is measured by the *on-base percentage*. However, a player who reaches a base may or may not score. The scatter plot gives data relating the number of runs and the on-base percentage. Use the graph to answer Questions 10 and 11.

Line graph showing the relationship between the number of runs and the on-base percentage. Y-axis represents the number of runs, ranging from 0 to 900. X-axis represents the on-base percentage, ranging from 27% to 34% 28% = in between 500 to 600 runs (1 dot) 29% = in between 500 to 600 runs (1 dot) 30% = in between 600 to 700 runs (2 dot) 31% = around 700 runs (3 dots) 32% = in between 600 to 700 runs (1 dot) 33% = around 800 runs (2 dots)

(10) Katy used the graph to create a linear model that she could use to predict the number of runs scored using the on-base percentage. Her model was r = 30b + 500, where r = number of runs and b = on-base percentage (i.e., for 32%, b = 32). What is wrong with Katy's model?

(i) The data are not nearly linear so a linear model should not be used.

(ii) Katy put the rate and the vertical intercepts in the wrong places in the equation.

(iii) Katy used the vertical intercept in her equation where she should have used the horizontal intercept.

(iv) The vertical intercept of 500 runs is incorrect because the scale on the horizontal axis does not start at zero.

(11) Based on the slope in Katy's model, if on-base percentage for a team goes up by 1% then by about how much does the number of runs scored go up?

(i) 480 runs

(ii) 450 runs

(iii) 30 runs

(iv) 0.3 runs

(12) The graph below shows the average wholesale price for a 10mL vial of insulin from 2012 to 2022.

Vertical bar graph showing the cost of basic a 10mL vial of insulin from 2012 to 2022. Y-axis represents the cost in dollars, ranging from 0 to 400. X-axis represents the years, ranging from 2012 to 2022. 2012 = $136 2013 = $137 2014 = $149 2015 = $173 2016 = $217 2017 = $265 2018 = $289 2019 = $316 2020 = $330 2021 = $341 2022 = $353

(a) An employee of the pharmaceutical company made a linear equation of the data using the 2012 cost as a starting value and the 2015 cost as a second data point. What is the **company's equation**? Use C for the cost per vial and t for time in years since 2012.

(b) A consumer advocate made a linear equation of the data using the 2012 cost as a starting value and the 2022 cost as a second data point. What is the **advocate's equation**? Use C for the cost per vial and t for time in years since 2022.

(c) What will the cost per vial be in 2027 based on the company's model found in (a)? Based on the consumer advocate's model found in (b)? (**Hint**: Remember that t is the number of years since 2012.)

(i) Company's projection:

(ii) Consumer advocate's projection:

(d) Which of the following is the most accurate estimate of the relative increase in the vial cost of insulin from 2012 to 2022?

(i) 150–175%

(ii) 100–125%

(iii) 80–100%

(iv) 50–75%

---

[8] http://en.Wikipedia.org/wiki/Men's_100_metres_world_record_progression

4.4: The Cost of Business

INTRODUCTION

In this collaboration, we will be discussing price markups and markdowns using both spreadsheet software (Google Sheets or Excel) and algebra to calculate prices before or after a markup or markdown.

- **Markup**: the amount added to the original price of an item, usually calculated as a percentage of the original price
- **Markdown**: the amount subtracted from the original price of an item, usually calculated as a percentage of the original price

Both markups and markdowns are defined as the amount added to or subtracted from the original cost. However, to calculate the cost after a markup or markdown, we will be using multiplication, as shown in Preparation 4.4.

According to fastcompany.com[9], "Amazon has finally implemented what was long considered a given after it bought Whole Foods for $13.7 billion last year. Now Amazon Prime members get 10% off hundreds of sales items in Whole Foods stores. In addition, they'll receive steep discounts on other items each week.

The discounted items will be labeled in-store with yellow "10% off" sale signs and "Prime Member Deal" signs. In order to get the savings, customers will just need to scan the Whole Foods mobile app's "Prime Code" barcode at checkout or give the cashier their phone number. One caveat: Right now, the Prime Member discounts only apply to stores in Florida, but Amazon says the new program will quickly roll out to all its 463 U.S. stores this summer."

If a Whole Foods customer who is not an Amazon Prime member spent $250 on groceries labeled with "Prime Member Deal," how much would an Amazon Prime member pay for the same groceries? Discuss in your group, and try to come up with two ways of calculating your answer.

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- using algebra to generalize calculations is a valuable tool.

By the end of this collaboration, you should be able to

- write or interpret an expression that models a percentage increase or decrease.
- use expressions and equations to solve contextual problems about percentage increases or decreases.

PROBLEM SITUATION: WRITING FORMULAS WITH PERCENTAGES

In this collaboration, you will find ways in which Eduardo and Paula can use spreadsheet formulas and percentages to calculate data for their business.

(1) The information from Preparation 4.4 told you that Eduardo and Paula were writing a formula to calculate prices for their merchandise. Their formulas are given below (B2 = the amount they pay to buy the item in dollars).

Paula: B2 + 0.65*B2

Eduardo: B2*1.65

(a) In your group, discuss if both of these formulas are equivalent to one another. Be prepared to justify your decision.

(b) Calculate the price for each item shown in the spreadsheet below. Round your answers to two decimal places.

Screen capture of data in an excel sheet, indicating the following in cells: Column C = B2 plus 0.65 multiplied by B2 A1 = Item A2 = Women's t-shirt A3 = Men's t-shirt A4 = Youth's t-shirt A5 = Skirt A6 = Dress B1 = Cost of Item B2 = $8.79 B3 = $10.13 B4 = $6.59 B5 = $23.45 B6 = $35.67 C1 = Price

ADJUSTING PRICING: SIX MONTHS LATER

Six months later, Eduardo and Paula are doing very well with their business. They learned at the first event that prices to the nearest cent are not very practical because it takes so much time to make change for customers. One advantage of a spreadsheet is that you can set it up to give the results of calculations in different forms. Paula revised the spreadsheet to round the prices to the nearest dollar. The rounded results and the formula she used are shown below.

(2) Although their products have sold very well, Paula and Eduardo have some items that have not sold. They decide to put these items on sale for 15% off.

(a) Estimate the sale price of the items. Try not to use calculators or paper or pencil! Discuss your estimation strategies with your group.

Screen capture of data in an excel sheet, indicating the following in cells: Column C formula = ROUND(B2 plus 0.65 multiplied by B2,0) A1 = Item A2 = Women's t-shirt A3 = Men's t-shirt A4 = Youth's t-shirt A5 = Skirt A6 = Dress B1 = Cost of Item B2 = $8.79 B3 = $10.13 B4 = $6.59 B5 = $23.45 B6 = $35.67 C1 = Price C2 = $15.00 C3 = $17.00 C4 = $11.00 C5 = $39.00 C6 = $59.00 D1 = 15% Discount

(b) Write a formula that Paula can use in Cell D2 to find the sale price of a women's t-shirt. Try to find more than one formula that works. You may choose whether to include the Round function.

(c) Paula and Eduardo have a few skirts that do not sell even after being discounted. They decide to take another 15% off. Eduardo says this is the same as a 30% discount, but Paula disagrees. Who is right? What is the sale price?

(3) At some events, Eduardo and Paula have to charge sales tax. This means that the total charge is no longer rounded to the nearest dollar, and they have to take the time to make change again. For example, for a women's t-shirt they need to charge about $15 to make a profit. With 7.5% tax, the total cost to the customer is $16.13. Eduardo and Paula decide to set their prices so that the price comes out to an even dollar amount after the tax is added. So, for the women's t-shirt, they want the total after tax to be $17. (Eduardo and Paula decide to round the amount up to the next dollar so they do not lose any money.)

(a) The following are the amounts Eduardo and Paula want to charge after tax. Calculate the price before tax that gives these amounts. The tax rate is 7.5%.

Screen capture of data in an excel sheet, indicating the following in cells: A1 = Item A2 = Women's t-shirt A3 = Men's t-shirt A4 = Youth's t-shirt A5 = Skirt A6 = Dress B1 = Price Before Tax C1 = Price after Tax C2 = $17.00 C3 = $19.00 C4 = $12.00 C5 = $42.00 C6 = $64.00

(b) What formula could Paula use in Cell B2 to calculate the price before tax for a women's t-shirt?

(4) The following question gives you more practice with using percentages in expressions and equations. Column A gives expressions that either increase or decrease the quantity x by a given percentage. Column B gives verbal descriptions of the changes. Write the letter of the description next to the correct expression. A verbal description may be used more than once or not at all.

Answer	Column A		Column B
	x + 0.45x		a) increase x by 45%
	0.45x		b) increase x by 55%
	x – 0.55x		c) increase x by 145%
	1.45x		d) increase x by 245%
	3.45x		e) increase x by 345%
	2.45x		f) decrease x by 55%
	x + 0.55x		g) decrease x by 45%
	x – 0.45x		h) decrease x by 145%
	0.55x		

FURTHER APPLICATIONS

(5) Organizations and groups often receive grants that help pay for activities over several years. When planning a budget over several years, people often increase the amounts budgeted each year by a small percentage to account for an increase in prices or inflation. A neighborhood activity center is applying for a three-year grant to provide an after-school program. The amounts budgeted for different items for the first year of a grant are shown below. The grant writer wants to increase all the amounts by 2.5% for Year 2 and by another 2.5% for Year 3.

- Salaries: $120,000
- Benefits: $21,600
- Insurance: $10,000
- Materials and supplies: $6,000

Make a spreadsheet that calculates the amounts. You will either make an actual spreadsheet or complete the table below as if it were a spreadsheet and record your formulas as indicated. Your instructor will tell you how to complete the assignment.

Screen capture of data in an excel sheet, indicating the following in cells: A1 = Item B1 = Year 1 C1 = Year 2 D1 = Year 3

(a) Formula for Year 2 salaries:

(b) Formula for Year 3 salaries:

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[9] https://www.fastcompany.com/40573252/amazon-prime-members-now-get-10-off-at-whole-foods

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4.4.1: Preparation 4.4

(1) In Module 1 you developed skills in estimating and calculating percentages. Use these skills to do the following.

(a) **Estimate**: 37 is what percentage of 105?

(b) **Estimate**: What is 72% of 300?

(c) **Calculate**: 16 is what percentage of 39? Round to the nearest one percent.

(d) **Calculate**: What is 52% of 280?

(e) **Calculate**: What is 125% of 9?

In Preparation 3.3, you were given information about adding algebraic terms. This was compared to adding measurements, as in the examples below.

Adding measurements	2 feet + 6 feet = 8 feet	2 ft^2 + 6 ft^2 = 8 ft^2
Adding algebraic terms	$2x + 6x = 8x$	$2x^2 + 6x^2 = 8x^2$

In the first row, the *quantity* or *number* changes, but the units of measure stay the same. The same concept is true with the algebraic terms in the second row. The quantity or number changes but the variable stays the same.

Another similarity between adding measurements and adding algebraic terms is that you must have like terms to combine terms together. In terms of measurement, this means you must have the same units. You can state the operation 3 feet + 4 inches, but you cannot actually combine the values into one unless you convert one of the units. In the same way, you can use the expression $3y + 4x$, but you cannot combine the two terms into one because they do not represent the same types of things.

Some important points to remember:

- *Like terms* have the same variables. This means the variables also have to have the same exponent.
- Only like terms can be combined by addition or subtraction.
- When adding or subtracting like terms, add or subtract the coefficients. The variables do not change.
- Terms do *not* have to be like terms to be combined with multiplication or division.

(2) Simplify the following expressions by adding or subtracting like terms or simply rewrite the expression if it cannot be simplified.

(a) $2x + 3.2x - 1x$

(b) $5a - 7b + 2b$

(c) $3a + 5ab$

(d) $y + 0.2y$

(3) Marissa wants to create a spreadsheet to track her monthly budget. She always puts 5% of her take home pay into savings before paying any other expenses. The spreadsheet format is shown below. She will enter each of the values for the items listed in Column A into the corresponding cell in Column B. The cells in gray will have formulas for the calculations. Write the formulas for each of these cells. Be sure to start out the formulas with an = sign.

Screen capture of data in an excel sheet, indicating the following in cells: A1 = Take home pay: A2 = Amount put into savings: A3 = Amount for expenses: A4 = N/A A5 = Utilities A6 = Food: A7 = Child Care: A8 = Bus Fare: A9 = Entertainment: A10 = Total Expenses:

(a) Cell B2:

(b) Cell B3:

(c) Cell B10:

(4) Eduardo and Paula are starting a small business making and selling tie-dyed clothing. They buy the clothing ready-made and then dye the items in colorful patterns. Eduardo and Paula plan to sell their products at events such as street fairs and festivals. They decide to set their prices by adding 65% to the cost of the clothing. This covers their expenses and gives them a small profit. Paula creates the following spreadsheet to calculate their prices.

The *Cost of Item* is the amount Paula and Eduardo pay for the clothing. The *Price* is the amount they charge for the item. The formula bar at the top of the image shows the formula in Cell C2. This formula is used to calculate the price when the cost of the item in A2 is entered into Cell B2 the spreadsheet.

Screen capture of data in an excel sheet, indicating the following in cells: Column B formula = B2 plus 0.65 multiplied by B2 A1 = Item A2 = Women's t-shirt A3 = Men's t-shirt A4 = Youth's t-shirt A5 = Skirt A6 = Dress B1 = Cost of Item C1 = Price

Eduardo looks at the spreadsheet and says he does not think the formula is correct. He thinks it should be "= B2*1.65."

Who is correct? Justify your decision.

After Preparation 4.4 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 4.4, you should understand the concepts and demonstrate the skills listed below:

Skill or Concept: I can …	**Rating from 1 to 5**
estimate and calculate percentages.	
understand the use of variables in expressions and equations.	
enter a formula into a spreadsheet.	
add and subtract like terms.	

4.4.2: Exercise 4.4

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Algebra is based on rules that have no connection to thinking about numbers or context.

(ii) Spreadsheets are the only useful application of algebra.

(iii) An increase of 23% can be calculated by multiplying the reference value by 0.23.

(iv) Having a good understanding of how to work with numbers is helpful in algebra.

DEVELOPING SKILLS AND UNDERSTANDING

(2) The following is an excerpt from an article about tuition increases at Rutgers University.[10]

> After hearing the concerns of students, faculty and staff about rising tuition and salary freezes, the Board of Governors decided to raise in-state tuition for the upcoming fiscal year by 1.8 percent at its meeting yesterday in Winants Hall on the College Avenue campus …
>
> Aside from tuition, mandatory fees for next term will rise 0.6 percent, and room and board will see a 3-percent rise. The overall working budget, originally set at about $2 billion … will be around $12 million less. Out-of-state tuition will rise 5 percent.

The excerpt contains information about the relative change in four different quantities. For each one, identify the quantity and write an expression that could be used to calculate the new value. Carefully define any variables that you use, including units. One is done for you as an example.

Example:

Quantity: In-state tuition in dollars

Expression: 1.018t

Variable: t = old in-state tuition in dollars

(a) Quantity:

Expression: 1.006f

Variable: f = old mandatory fees (in dollars)

(b) Quantity: Cost of room and board (in dollars)

Expression: 1.03r

Variable: r =

(c) Quantity: Out-of-state tuition (in dollars)

Expression:

Variable: T = out-of-state tuition (in dollars)

(d) In addition to the relative changes you listed above, the excerpt also gives information about the absolute change in the working budget. Which of the following is the best estimate for the relative change in the budget?

(i) Decrease of less than 1 percent.

(ii) Decrease of between 1 and 5 percent.

(iii) Decrease of about 10 percent.

(iv) Decrease of more than 10 percent.

(3) Read the following excerpt from an article about changes in gas rates in Charleston, West Virginia.[11]

> Mountaineer Gas wants to lower rates for customers for the second consecutive year.
>
> Gas rates for residential customers will likely drop by 5.24 percent, starting Nov. 1.
>
> Mountaineer Gas filed for the rate reduction earlier this month, citing a decline in natural gas prices.
>
> If the state Public Service Commission approves the lower rate, the average residential customer's monthly gas bill would drop by about $7.50 a month.
>
> “Natural gas has been relatively cheaper for the last couple of years,” said Byron Harris, who heads the PSC’s Consumer Advocate Division. “Gas rates are definitely going in the right direction, while electricity rates keep going up.”
>
> Mountaineer’s business customers will see their gas rates decline by about 6 percent, while industrial customer rates will drop 9.7 percent.

Suppose you are creating the spreadsheet below to track the gas rates.

Screen capture of data in an excel sheet, indicating the following in cells: A1 = Type of Customer B1 = Original Rate C1 = New Rate

(a) Enter each type of customer in Column A.

(b) Assume the original rates would be in Column B (you do not need to enter anything in Column B). Write a spreadsheet formula for the new rates in Column C.

(c) Based on the information in the article, what was the average monthly gas bill for a residential customer before the rate change? Round to the nearest cent.

(4) One of your friends missed class, and you are helping her get caught up. Which of the following is a good explanation for the answer for 2.45x in Question 4 of Collaboration 4.4?

(i) If you convert 2.45 to a percent, you get 245%. So multiplying by 2.45 is an increase of 245%.

(ii) 2.45x increases x by 245% because x + 1.45x = 2.45x.

(iii) Think of 2.45x as adding 1x + 1.45x, where 1x is 100% of x and 1.45x is 145% of x. So you are adding 100% and 145%, which is an increase of 145% over the original 100%.

(iv) Think of 2.45x as a percent, so when you multiply x by 2.45%, you are actually decreasing x by 97.55%.

(5) The value of investments in the stock market changes daily. Suppose you buy a stock for $1,000. It increases in value by 4.5% and then falls 4.5%. What is the new value? Round to the nearest cent.

MAKING CONNECTIONS ACROSS THE COURSE

(6) In Preparation 4.4, you wrote formulas for Marissa’s budget spreadsheet. Her spreadsheet was set up to calculate the amount to be put into savings and then used to find the amount left for expenses. Write a new formula for Cell B2 in the spreadsheet shown below that will calculate the amount for expenses in one step. Marissa puts 5% of her take home pay into savings.

Screen capture of data in an excel sheet, indicating the following in cells: A1 = Take home pay: A2 = Amount for expenses:

(7) Historically, women in the United States have earned less than men.[12] The table and graph below show national data on women’s earnings as a percentage of men’s earnings from 1992 to 2021.

Line graph showing the women’s earnings as a percentage of men’s earnings from 1992 to 2021. The y-axis represents the percentage, ranging from 0% to 90%, and the x-axis represents the year, ranging from 1992 to 2022, as described in the following table.

Year	Percentage		Year	Percentage
1992	75.8		2007	80.2
1993	77.1		2008	79.9
1994	76.4		2009	80.2
1995	75.5		2010	81.2

1996	75		2011	82.2
1997	74.4		2012	80.9
1998	76.3		2013	82.1
1999	76.5		2014	82.5
2000	76.9		2015	81.1
2001	76.4		2016	81.9
2002	77.9		2017	81.8
2003	79.4		2018	81.1
2004	80.4		2019	81.5
2005	81		2020	82.3
2006	80.8		2021	83.1

Use the table and graph to create a linear equation that will model the data. Use the following variables:

E = Women's earnings as a percentage of men's

t = Time in years after 1992

Use the following two points:

Point 1: Year 1992 , Percentage 75.8

Point 2: Year 2021 , Percentage 83.1

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[10] https://dailytargum.com/article/2011/07/bog-agrees-on-1-8-percent-tuition-increase

[11] http://www.highbeam.com/doc/1P2-29398083.html

[12] https://www.bls.gov/opub/ted/2022/median-earnings-for-women-in-2021-were-83-1-percent-of-the-median-for-men.htm

4.5: Compounding Interest Makes Cents

INTRODUCTION

Recall the following information about CDs from Preparation 4.5. Take a minute to read through it again below and, if necessary, discuss any difficulties in your group.

A Certificate of Deposit (CD) is a type of investment used by many people because it is very safe and predictable. CDs can be purchased through banks and other financial institutions. The U.S. Security and Exchange Commission gives the following information about CDs.

The ABCs of CDs

A CD is a special type of deposit account with a bank or thrift institution that typically offers a higher rate of interest than a regular savings account. Unlike other investments, CDs feature federal deposit insurance up to $250,000.

When you purchase a CD, you invest a fixed sum of money for a fixed period of time—six months, one year, five years, or more—and, in exchange, the issuing bank pays you interest, typically at regular intervals. When you cash in or redeem your CD, you receive the money you originally invested, plus any accrued interest. If you redeem your CD before it matures, you may have to pay an “early withdrawal” penalty or forfeit a portion of the interest you earned …

At one time, most CDs paid a fixed interest rate until they reached maturity. But, like many other products in today’s markets, CDs have become more complicated. Investors may now choose among variable-rate CDs, long-term CDs, and CDs with other special features.

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- compounding is repeated multiplication by a compounding factor.
- compounding is best expressed in terms of exponential growth, using exponential notation.
- exponential growth models the compounding of interest on an initial investment.

By the end of this collaboration, you should be able to

- calculate the earnings on a principal investment with annual compound interest.
- write a formula for annual compound interest.
- compare and contrast linear and exponential models.

PROBLEM SITUATION: THE FIVE-YEAR CD

Suppose you invest $1,000 **principal** (the original deposit in an investment) into a certificate of deposit (CD) with a five-year **term** (the agreed upon period of time that money is in an investment) that pays a 5% annual interest rate. The **compounding period** is one year. Recall that in **compound interest**, interest is added to the starting principal and accrues after set periods of time through the duration of the whole term.

(1) How much money will you have in your account at the end of the five-year term? (Be ready to explain your calculations.) Use the table below to find a pattern and develop a formula to model the total amount accrued in a CD with annual compounding after t years, if the principal = $1,000 and the APR = 5%. Round figures in the Amount Accrued column to two decimal places.

Term	Calculation	Amount Accrued
1 year	$1,000 + $1,0000.05 = $1,000(1 + 0.05) = $1,000*1.05	$1,050.00
2 years	($1,000*1.05)(1.05) = $1,000(1.05)2	
3 years		
4 years		
5 years		
…		

t years		

(2) Is your formula from Question 1 linear? Explain your reasoning.

(3) Using the formula and patterns you developed when answering Question 1, work in your group to fill out the following table. Round figures in the Amount Accrued column to two decimal places.

Term	Calculation	Amount Accrued
10 years		
20 years		
30 years		
40 years		
50 years		

(4) (a) Use the values from the table above to create a graph of the model you developed. **Note**: If completing this problem online, follow the instructions given online to create your graph.

Blank graph showing the amount accrued in CDs. The y-axis represents the accrued value in dollars, ranging from 0 to 14,000, and the x-axis represents the number of years, ranging from 0 to 60.

Behavior of Exponential Growth/Decay

Note from the graph in Question 4a, that the growth is very small at first, but then it changes dramatically. Here are some other graphs that illustrate both exponential growth and exponential decay:

Exponential **growth** (increasing rate of change)

Desmos graph showing example of exponential growth. Lines graphed are: A=1000*1.05^t A=750*1.02^t A=2000*1.1^t These curved lines start at the bottom left and curve to the top right.

Exponential **decay** (decreasing rate of change)

Desmos graph showing example of exponential decay. Lines graphed are: A=1000*0.5^t A=750*06^t A=2000*.72^t These curved lines start at the top left and curve to the bottom right.

(b) Use the information in the table below to plot a graph of the model. **Note**: If completing this problem online, follow the instructions given online to create your graph.

- You have $1,000 in savings and add $20 each year.
- Equation: A = 1,000 + 20n where A = amount ($) and n = number of years

Term	Calculation	Amount Accrued
1 year	1,000 + 20	$1,020
2 years	(1,000 + 20) + 20 = $1,000 + 2*20	$1,040
3 years	(1,000 + 20 + 20) + 20 = $1,000 + 3*20	$1,060
4 years	(1,000 + 20 + 20 + 20) + 20 = $1,000 + 4*20	$1,080
5 years	(1,000 + 20 + 20 + 20 + 20) + 20 = $1,000 + 5*20	$1,100

(5) Write a general formula that could be used to find the accrued amount (A) for a CD with annual compounding. Let P = the principal, r = the annual interest rate as a decimal, and t = number of years of investment.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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4.5.1: Preparation 4.5

A **Certificate of Deposit (CD)** is a type of investment used by many people because it is very safe and predictable. CDs can be purchased through banks and other financial institutions. The U.S. Security and Exchange Commission gives the following information about CDs.[13]

> **The ABCs of CDs**
>
> A CD is a special type of deposit account with a bank or thrift institution that typically offers a higher rate of interest than a regular savings account. Unlike other investments, CDs feature federal deposit insurance up to $250,000.
>
> When you purchase a CD, you invest a fixed sum of money for a fixed period of time—six months, one year, five years, or more—and, in exchange, the issuing bank pays you interest, typically at regular intervals. When you cash in or redeem your CD, you receive the money you originally invested, plus any accrued interest. If you redeem your CD before it matures, you may have to pay an "early withdrawal" penalty or forfeit a portion of the interest you earned …
>
> At one time, most CDs paid a fixed interest rate until they reached maturity. But, like many other products in today's markets, CDs have become more complicated. Investors may now choose among variable-rate CDs, long-term CDs, and CDs with other special features.

In this course, you will only be looking at situations in which the CD has a fixed interest rate. The period of time that you agree to leave your money in the CD is called the **term**. Terms can range from three months to multiple years. Some CDs require a minimum deposit, but others do not. The deposit, or amount you put into the CD, is called the **principal**.

CDs pay **compound interest**. Recall in previous assignments you have worked with situations that use **simple interest**. In simple interest, the total interest is calculated from the original principal. In compound interest, the interest is added to the principal after a set period of time. Then interest for the next period of time is calculated based on the new, higher balance in the account.

Example:

You invest $100 at 5% interest compounded **annually** or each year. At the end of the year, you earn 5% of $100 or $5. This is added into the account so now the balance is $105. In the second year, you earn 5% of $105.

The following are some common compounding periods:

- Quarterly (4 times a year)
- Monthly (12 times a year)
- Daily (365 times a year)

(1) If you were to invest $200 at 4% interest compounded **annually** or each year, what would be your balance at the end of the first year?

(i) $192

(ii) $8

(iii) $208

(iv) $96

(2) If you were to invest $200 at 4% interest compounded **annually** or each year, what would be your balance at the end of the **second** year? Round to two decimal places.

Note: There is also a type of compounding called continuous, but that will not be discussed in this course.

After Preparation 4.5 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 4.5, you should understand the concepts and demonstrate the skills listed below:

Skill or Concept: I can …	Rating from 1 to 5
understand the connection between percents and decimals.	

convert percents to decimals and vice-versa.	
calculate a percent of a value.	
add, subtract, multiply, and divide decimals.	
understand how certificates of deposit and compound interest work, including the vocabulary terms principal, term, and compounding period.	

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[13] www.sec.gov/investor/pubs/certific.htm

4.5.2: Exercise 4.5

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Algebra gives us a way to write general rules that can be used in multiple situations.

(ii) Compound interest is an important concept in savings and investment.

(iii) A linear model never uses percentages but an exponential model does.

(iv) Annual compounding is done once a year.

DEVELOPING SKILLS AND UNDERSTANDING

(2) Write a statement about the rate of change for each model below:

A linear model:	An Exponential model:

(3) Recall the simple interest formula used earlier in this module:

$$A = P + P \cdot r \cdot t$$

Where A is the *Amount* (principal and interest in the account, also called the balance), P is the *Principal*, r is the *annual interest rate* (quoted as a percent, but used as a decimal), and t is the *time* in years.

(a) Complete the table below with the amounts you would have if you invested $1,000 at 5% interest with simple interest and with interest compounded annually.

	Simple Interest	**Compound Interest**
1 year	(i)	(vi)
2 years	(ii)	(vii)
3 years	(iii)	(viii)
4 years	(iv)	(ix)
5 years	(v)	(x)

(b) Is simple interest a linear model? How do you know?

(4) Which investment strategy will have a higher balance after 10 years: investing $500 at 10% APR or $1,000 at 2% APR? Both investments have annual compounding.

(a) After 10 years, the $500 at 10% APR would grow to $_____________ .

(b) After 10 years, the $1,000 at 2% APR would grow to $_____________ .

(c) Which investment strategy would have a higher balance after 10 years?

i. $500 at 10%

ii. $1000 at 2%

MUTUAL FUNDS

CDs are a very safe investment because they are usually insured by the U.S. government. (There are some CDs that are not insured so it is important to always check!) Because they are so safe, CDs earn low rates of interest. The amount earned on an investment is often called the **return**. In general, investments with higher risk also have the potential for higher rates of return. For example, if you invest in a stock, which is like buying a piece of a company, you might earn far more than you would with a CD. However, you also run the risk of losing all of your money.

Mutual funds are another type of investment used by many people. The U.S. Security and Exchange Commission defines a mutual fund as follows:[14]

> A mutual fund is a company that brings together money from many people and invests it in stocks, bonds, or other assets. The combined holdings of stocks, bonds, or other assets the fund owns are known as its portfolio. Each investor in the fund owns shares, which represent a part of these holdings.

Mutual funds are an attractive investment to many people for several reasons.

- You can invest small amounts of money at a time.
- The fund is managed by a company that does all the work of researching and choosing specific investments.
- In general, there is less risk than owning a single stock because the money is spread across many investments.
- In general, there are higher rates of returns than those available in insured investments like CDs and savings accounts.

However, it is important to understand that mutual funds do still have risks. It is possible to lose some or all of your investment. Also, funds charge fees to pay for the management. Sometimes these fees can be very high.

Selecting investments is an important decision and should be researched carefully. Through this course, you will be introduced to a few basic concepts related to investment, but you should not make any decisions based only on the information presented here.

(5) One of the most important concepts in investing is to take advantage of the power of compounding over time. The following example will help you explore this idea.

Lorenzo and Michael each decide to invest in a mutual fund to save for retirement. They each choose a mutual fund that has had an average return of 5.6% over the last decade. There is no guarantee that the mutual fund will continue to earn this same rate, but it can be used as an estimate of future returns. Use the information given below to estimate how much each man will have when he retires. Round to the nearest dollar. (**Hint**: The amount earned is how much they have without the initial investment.)

- Lorenzo invests $2,000 when he is 25 years old.
- Michael invests $5,000 when he is 45 years old.
- Both men plan to retire at age 65.

Name	Amount at Retirement	Amount Earned
Lorenzo	(i)	(ii)
Michael	(iii)	(iv)

MAKING CONNECTIONS ACROSS THE COURSE

As with linear models, exponential models can increase, which is called **exponential growth**, or decrease, which is called **exponential decay**. You will see an example of *exponential decay* in Collaboration 4.6.

Compound interest is one application of exponential growth. There are many others. The formula that you found for annual compound interest can be generalized to all exponential growth models:

$$New\ Value = Starting\ Value \times (1 + rate)^{variable\ power}$$

Populations often grow exponentially. Question 7 examines how you can use this to make predictions about the world population.

(6) In Collaboration 1.2, you looked at historical data of the world's population. Scientists use models to project population in the future. While these models have limitations, they help leaders plan ahead for the resources people will need. The U.S. Census Bureau estimated the 2023 world population to be about 7.94 billion people. They estimated the world population growth rate in 2023 to be about 0.94%.[15]

(a) This is an exponential situation because the population is increasing each year by a percentage of the previous year. Complete the table below based on the growth rate of 0.94%. Some of the entries are done for you. Round the populations to the nearest tenth of a billion. (**Hint**: 5% written as a decimal is 0.05. 4.5% written as a decimal is 0.045. How would 0.94% be written as a decimal?)

Calendar Year	Number of Years After 2023	Projected Population
2023	0	7.94 billion

2024	1	
2025		
2026		

(b) Write an exponential equation for the world population growth after 2023. Let P = the projected population in billions and t = the number of years after 2023. (**Hint**: Think about how you calculated the entries in the table. If you started with 7.94 billion each time, how could you calculate each population? Think back to your work in Collaboration 4.5.)

(c) Use your model to predict the population in 2033.

(7) The Center for Disease Control (CDC) published the following information about emergency department visits for respiratory illness from a meta-analysis of data collected in 2014.[16] Respiratory disease is any illness that affects a person's ability to breathe and use oxygen.

> In 2014, approximately 400,000 emergency department visits for respiratory illness. Of these, 7% were for pneumonia, 15% for asthma, 17% for bronchitis, 40% for acute pharyngitis, and 21% for other respiratory diseases, including influenza.

(a) Based on this information, how many people were diagnosed with pneumonia in emergency departments in 2014?

(b) Which of the following pie charts accurately represents the data?

Circle graph showing the following from highest to lowest: - pharyngitis, other, bronchitis, asthma, and pneumonia.	Circle graph showing the following from highest to lowest: - pharyngitis, bronchitis, asthma, pneumonia, and other
Circle graph showing the following from highest to lowest: - pharyngitis, bronchitis, asthma, other pneumonia	

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[14] http://www.sec.gov/investor/tools/mfcc/mutual-fund-help.htm

[15] https://www.census.gov/newsroom/press-releases/2022/new-years-day-population.html

[16] https://www.cdc.gov/nchs/data/nhsr/nhsr151-508.pdf

4.6: Compounding Makes More Cents

INTRODUCTION

Let’s think about different compounding periods. Consider two *extreme* examples of compound interest: **annual** compounding, and **daily** compounding. In your group, discuss which of the above should have a greater yield after one year, assuming equal principals and APRs (as shown in the table below).

Annual Compounding	Daily Compounding
Principal = \$10,000	Principal = \$10,000
APR = 5%	APR = 5%
Term = 1 year	Term = 1 year

At this point, you are not required to do any calculations.

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- there are differences and similarities between exponential growth and decay.

By the end of this collaboration, you should be able to

- use the compound interest formula for different compounding periods.
- write an exponential decay model.

PROBLEM SITUATION 1: THE VALUE OF A CD

In Collaboration 4.5, you used the following formula to learn how to find the accrued amount (A) of a certificate of deposit (CD) with a compounding period of one year:

$$A = P(1 + r)^t$$

In this problem situation, you will use the same formula to explore the accrued amount (A) of a CD with a shorter compounding period. Remember that the variable (P) represents the principal, r represents the interest rate per period (expressed as a decimal), and t represents the number of years of the investment. Now that we will be exploring the accrued amount (A) of a CD with multiple compounding periods in one year, we will have to develop the formula from Collaboration 4.5.

(1) (a) Suppose you invest \$1,000 in a two-year CD, advertised with an annual percentage rate (APR) of 12%, where compounding occurs monthly. If the APR is 12% per year, what is the monthly interest rate?

(b) Use your answer from (a) to help you complete the table below. Record how you found the results in the middle column. Round the figures in the Amount Accrued column to two decimal places. Think about the first 3 rows by yourself, and then check your answers with your group. Then, complete the table as a group.

- Principal = \$1,000
- APR = 12%
- Term = two years
- Compounding period = one month (i.e. interest will be compounded 12 times in one year)

Period	Calculation	Amount Accrued
1 month		
2 months		
3 months		
6 months		
12 months		

24 months		

(2) Write a general formula that can be used to calculate the value of any CD with any number of compounding periods in a year. Define your variables.

(3) How does the compounding period affect the accrued amount? Work in your group to investigate this question. (**Hint**: Use quarterly compounding to compute the accrued amount after eight quarters (n = 4, t = 2), using the same principal, APR, and the number of years as in question 1b. Then, compare your answer to the results in the above table.)

PROBLEM SITUATION 2: UNDERSTANDING DEPRECIATION

Depreciation is a process of losing value. For example, new automobiles lose 15 to 20% of their value each year for the first few years you own them.

(4) (a) Based on this fact, develop a depreciated-value (D) formula for a $26,000 automobile.

Assume that the depreciation is 15% per year. Discuss in your group before using the table below to guide your calculations.

Age of Automobile	Calculation	Value
Brand new		$26,000
1 year old		
2 years old		
5 years old		
t years old		

(b) Make a graph of the value of the vehicle over time. **Note**: If completing this problem online, follow the instructions given online to create your graph.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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4.6.1: Preparation 4.6

In Collaboration 4.5, you learned how to calculate the accumulated amount (A) of a certificate of deposit (CD), when *compounding occurs annually*. In this assignment, you will learn a more general method of compounding for which compounding periods may be of various durations. In addition, you will connect the exponential mathematics of compounding to related applications (e.g., automobile depreciation). More importantly, you will see the effect of different compounding factors.

(1) For the given compounding periods, fill in the “Periods per Year” column.

Compounding Period	Periods per Year
Annually	(i)
Semiannually	(ii)
Quarterly	(iii)
Monthly	(iv)
Weekly	(v)
Daily	(vi)
Hourly	(vii)

Example: Let’s say you invest \$1,000 in a one-year CD, advertised with an annual percentage rate (APR) of 6%, where compounding occurs monthly.

We have an APR of 6% per year, and because the compounding occurs monthly, we need to determine the monthly interest rate (also called the periodic rate).

We can do this by simply dividing the APR expressed as a decimal (6% expressed as a decimal is 0.06) by the number of compounding periods. In this example, the number of compounding periods is 12. So, the monthly interest rate is $\frac{0.06}{12} = 0.005$.

We can then use this monthly rate to calculate how much our initial investment will increase by month to month.

1 month: \$1,000 + \$1,000 × 0.005 = \$1,000(1 + 0.005) = \$1,000 × 1.005 = \$1,005

2 months: (\$1,000 × 1.005)1.005 = \$1,000(1.005)2 = \$1,010.03 (rounded up)

3 months: (\$1,000 × 1.0052)1.005 = \$1,000(1.005)3 = \$1,015.08

(2) If you were to invest \$2,000 over 1 year at 6% interest compounded **monthly**, what would your balance be after *one month*?

(3) If you were to invest \$4,000 at 6% interest compounded **monthly**, what would your balance be after *one month*?

Collaboration 4.6 builds on the concepts from Collaboration 4.5, so the prerequisite skills listed below were all covered in the previous sections of this assignment.

After Preparation 4.6 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 4.6, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	Rating from 1 to 5
write a basic exponential model.	
calculate values raised to powers using a calculator.	

4.6.2: Exercise 4.6

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) All exponential models share certain characteristics such as the general shape of the graph, increase or decrease by a percentage, and a vertical intercept.

(ii) Multiplying a number by a second number between 0 and 1 will give you a larger number than you started with.

(iii) Cars lose value quickly after purchase.

(iv) Quarterly compounding happens four times a year.

DEVELOPING SKILLS AND UNDERSTANDING

(2) Write the letter of each graph next to the equation that best describes it.

_________ $y = 1{,}000(0.95)^x$	_________ $y = 200 + 11x$	_________ $y = 5(1.1)^x$

Graph showing an increasing curb, indicating the following: Start: 0 = 0 End: 60 = 1200 Graph showing a decreasing curb, indicating the following: Start: 0 = 1,000 End: 60 = just above 0

Graph showing an increasing straight-line, indicating the following: Start: 0 = 200 End: 60 = just above 800

(3) In this problem, you will compare the effects of different compounding periods on the interest an investment earns. Complete the table below using the values indicated. Show the formula you used, with the correct values, in the second column. The equation used for an annual compounding period has been done for you.

- Principal: $1,000
- APR: 4.5%
- Time: 10 years

Compounding Period	**Equation Used for Calculation (with values inserted)**	**Amount Accrued After 10 Years**
Annual	$A = 1000\left(1 + \frac{0.045}{1}\right)^{10\times 1}$	(i)
Quarterly	(ii)	(iii)
Monthly	(iv)	(v)
Daily	(vi)	(vii)

(4) Certain drugs are eliminated from the bloodstream at an exponential rate. Doctors and pharmacists need to know how long it takes for a drug to reach a certain level to determine how often patients should take medications. Answer the following questions about this situation.

(a) Circle the number of the correct statement:

(i) The same amount of the drug will be eliminated in the first hour as in the second hour.

(ii) More of the drug will be eliminated in the first hour than in the second hour.

(iii) Less of the drug will be eliminated in the first hour than in the second hour.

(b) Write an exponential model for the following situation. The drug dosage is 500 mg. The drug is eliminated at a rate of 5.2% per hour. Use D = the amount of the drug in milligrams and t = time in hours. Simplify your answer.

(c) How much of the drug is left after six hours? Round to the nearest milligram.

MAKING CONNECTIONS ACROSS THE COURSE

(5) Police officers must investigate the scene of an accident in which a person dies or is severely injured. The police investigators use measurements of skid marks to determine the speed at which the vehicle was traveling at the instant when the driver hit the

brakes. The following formula, along with many other factors, is used to estimate the speed of the vehicle:

$$S=\sqrt{30D\cdot f\cdot n}$$

where

S = Speed (mph)

30 is a constant value with units of $\frac{miles^2}{feet\cdot hour^2}$

D = Skid distance (The skid distance is in feet)

f = Drag factor. The drag factor depends upon the road surface. Portland cement ranges from 0.55 to 1.20, asphalt ranges from 0.50 to 0.90, gravel from 0.40 to 0.80. Units are not used for f.

n = Braking efficiency is quoted as a percent, but used as a decimal. The maximum braking efficiency is 100% (n = 1.00), if all four tires produce skid marks. If the rear brakes were not functioning at all, then n = 0.60. Experts must study the skid marks to determine the appropriate value of n.

(a) Consider the case where a car skids to a stop, leaving four skid marks with an average length of 80 feet. The road is asphalt. Skid tests reveal a drag factor of 0.75. Since all four wheels were braking, the braking efficiency is 100%.

What values should be used for the variables D, f, and n? Fill in the blanks:

D = ________ f = __________ n = ________

Use the formula to estimate the speed of the vehicle (round to the nearest mile per hour).

S = ____________________

(b) Create a table of values for distances of 60, 80, 100, and 120 feet. Round to the nearest mile per hour.

D	S
60	(i)
80	(ii)
100	(iii)
120	(iv)

(c) Sketch a graph of the model. Include a point for 0 feet.

(d) Is this a linear model? Why or why not?

(e) Is this an exponential model? Why or why not?

4.7: Short-Term Loans

INTRODUCTION

Let’s start this collaboration by reviewing linear and exponential functions. As a group, try to come up with at least one example of each type:

- Exponential growth
- Exponential decay
- Linear growth
- Linear decay

To review, the following table summarizes exponential models. In this table, we assume that $r > 0$.

	Exponential Growth	**Exponential Decay**
Formula	$y = A(1 + r)^x$	$y = A(1 - r)^x$
Exponential growth/decay rate	r (percent of the previous value)	r (percent of the previous value)
Vertical intercept (starting value)	A	A
Applications	Any quantity (y) that increases by the same percent per unit increase in x	Any quantity (y) that decreases by the same percent per unit increase in x

To review, the following table summarizes linear models. In this table, we assume that $m > 0$.

	Linear Growth	**Linear Decay**
Formula	$y = A + mx$	$y = A - mx$
Linear growth/decay rate	m (constant)	m (constant)
Vertical intercept (starting value)	A	A
Applications	Any quantity (y) that increases by the same amount (m) per unit increase in x	Any quantity (y) that decreases by the same amount (m) per unit increase in x

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- compound interest can be applied to short-term loan scenarios.
- linear and exponential equations for growth differ significantly in the long run.

By the end of this collaboration, you should be able to

- build both exponential and linear models.
- compare an exponential and linear model, and make a decision based on the comparison.

PROBLEM SITUATION: WHICH OPTION?

Suppose you have an unexpected expense of $100 and do not have the money to pay for it. Usually banks do not make loans for this small of an amount of money. In this collaboration, you will consider two options for short-term loans: 1. Payday loan, and 2. Borrow from a friend.

You will investigate the two options by considering how the unpaid balance accumulates over time. You should find specific values of the balance due for each option at 1, 4, 12, and 52 weeks; that is, 1 week, 1 month, 3 months, and 1 year. Interest is compounded weekly.

In your group, you will be discussing which option is best, assuming these are your only two options.

Option 1: Payday Loan

(1) A payday loan is a small, short-term loan that is intended to cover a borrower's expenses until his or her next payday. Payday loan centers are often located in convenient locations such as shopping centers. Usually, they only require the borrower to have a checking account and a job. It is also assumed that the loan is short term, meaning it will be paid back in a few weeks. In many states, payday loans are somewhat regulated, with a maximum Annual Percentage Rate (APR) of 390%. For this option, assume the APR is 390%. Payday loan centers justify this high interest rate because the loan is short term and high risk.[17]

Although these loans are not exactly exponential (because of loan fees and payment schedules), exponential equations provide a good model to approximate what this loan costs the borrower. There are restrictions (usually around 45 days) for the amount of time you can hold the loan. However, many people just move the balance from center to center, letting the unpaid amount add up for quite a long time. That is, they use a new loan to pay off the old loan, and the balance continues to accumulate.

Step 1: Develop a formula for this scenario. Do this individually before sharing your ideas in your group.

Step 2. Use the formula to complete the table in part (a).

Step 3: Use the information in the table to create a graph.

Step 4: Discuss your findings and record your observations.

(a) Work in your group and use your formula to complete the table below. Round your figures to two decimal places.

Time in Weeks	Amount Owed
1	
4	
12	
52	

(b) Create a graph of the model from (a). **Note**: If completing this problem online, follow the instructions given online to create your graph.

(c) In your group, discuss your thoughts on this option. Record your insights in the space below.

Option 2: Borrow from a Friend

(2) You have a friend who offers to lend you the money. You agree to repay the original amount plus $10 for each week you owe her money.

Step 1: Develop a formula for this scenario.

Step 2. Use the formula to complete the table below.

Step 3: Use the information in the table to create a graph.

Step 4: Discuss your findings and record your observations.

(a) Work in your group using your formula to complete the table below.

Time in Weeks	Amount Owed
1	
4	
12	
52	

(b) Plot the data from the table above on the graph you created in Question 1(b). **Note**: If completing this problem online, follow the instructions given online to create your graph.

(c) What patterns or interesting properties of linear and exponential models have you noticed so far?

FURTHER APPLICATIONS

(3) Finance charges on credit cards are calculated by several different methods depending on the card and the company. None of these methods are exactly exponential, but like payday loans, an exponential model can be used to approximate the charges. According to Wikipedia, finance charges based on a credit card annual percentage rate (APR) of 29.99%, compounded daily, is about equivalent to an effective annual rate of 34.96%.[18]

For this problem, you will explore the finance charges for an average balance of $2,000 of debt on a credit card. Note that in reality, credit card balances go up and down each month. The customer is required to make a minimum payment each month, which lowers the balance, but the customer often makes new charges, which increases the balance.

Calculate the interest for one year on $2,000 with an APR of 29.99% compounded daily. (**Hint**: Remember your answer should be the interest, not the total balance.)

(4) *Investopedia* illustrates credit card interest with the following example.

> Let's say John and Jane both have $2,000 of debt on their credit cards, which require a minimum monthly payment of 3%, or $10, whichever is higher. Both are strapped for cash, but Jane manages to pay an extra $10 on top of her minimum monthly payments. John pays only the minimum.
>
> Each month, John and Jane are charged a 20% annual interest on their cards' outstanding balances. So, when John and Jane make payments, part of the payment goes to paying interest and part goes to the principal.[19]

(a) Assume the interest is compounded monthly. Calculate the interest charged for one month for John and Jane (the interest rate is the same for both John and Jane). Since 20% is the annual interest, the rate for one month is 20% ÷ 12 = 1.67%. Round your answer to the nearest cent.

(b) The minimum payment is $60. John pays the minimum and Jane pays an extra $10 for a payment of $70. Part of this pays the interest you calculated in (a), and the rest pays off the balance. Complete the table below. Round to the nearest cent.

	John	**Jane**
Original balance	$2,000	$2,000
Amount paid on the balance		
New balance after the payment is subtracted		

(c) Calculate the interest that John and Jane each pay in the second month. Remember to use the new balance for each. Round to the nearest cent.

John's interest:

Jane's interest:

(d) You might wonder if the small difference in the interest charges for John and Jane really matters. According to *Investopedia*, if John continues to make the minimum payment and Jane continues to pay an extra $10 each month, it will take John 15 years to pay off the original debt of $2,000, and he will pay a total of $4,240. Jane will pay off her balance in seven years and pay a total of $3,276.

(i) How much interest did John pay?

(ii) What percentage of John's total payment is for interest?

(iii) How much interest did Jane pay?

(iv) What percentage of Jane's total payment is for interest?

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[17] www.paydayloaninfo.org/state-information

[18] http://en.Wikipedia.org/wiki/Credit_card_interest

[19] www.investopedia.com/articles/01/061301.asp#axzz1bo1CMcfb

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4.7.1: Preparation 4.7

(1) Kim is trying to do a better job of managing her money. A book on personal finance recommends that she track how much she spends on different items. Kim knows she spends a lot on “nonessentials” such as buying a cup of coffee or buying small items from street vendors. She finds it hard to track these purchases so she decides to set aside a certain amount of money in cash that she will use for these purchases. Every Sunday, she records how much she has left. She keeps a record for 10 weeks. Her data are approximately linear so she made the following graph in which she extends the model past 10 weeks.

Line graph showing the non-essential expenses. Y-axis represents the amount left to spend in dollars, ranging from 0 to 600. X-axis represents the number of weeks, ranging from 0 to 25. 0 = 500 5 = in between 300 and 400 10 = in between 200 and 300 15 = in between 100 and 200 20 = 0

(a) Complete the table below by matching the following mathematical terms to the appropriate contextual meaning and estimating the values from the graph.

- Slope
- Vertical intercept
- Horizontal intercept

Contextual Meaning	Mathematical Term	Value (including units)
Amount Kim sets aside for non-essential items		
Time it takes Kim to spend the entire amount		
Amount Kim spends per week		

(b) Write an equation for the model. Let A = amount left to spend ($), and w = the number of weeks.

(2) In many computer applications such as spreadsheets, a power is indicated by a *caret* symbol (^). For example, 3^4 would be written as 3^4. As a part of her budget management, Kim has put money into an Individual Retirement Account. She creates a spreadsheet to estimate her earnings over time.
She enters the formula: $= 1500 * \left(1+\frac{0.06}{4}\right) \wedge (4 * A2)$, where A2 = the time in years.

(a) How much did Kim invest?

(b) What annual rate of interest does she expect to earn over time? Write your answer as a percentage.

(c) What is the compounding period?

(i) Annual

(ii) Quarterly

(iii) Monthly

(iv) Daily

After Preparation 4.7 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 4.7, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	Rating from 1 to 5
create and use linear equations, including understanding of vertical and horizontal intercepts and slope in the context of a problem.	

create and use exponential equations, including understanding of vertical intercept and percentage change in the context of an interest-bearing account.	

4.7.2: Exercise 4.7

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Exponential growth starts slowly, but then speeds up. Linear growth is constant.

(ii) Compound interest has a special formula because it is not the same as exponential growth.

(iii) “Compounded weekly” means that the interest will be compounded 52 times in a year.

(iv) Payday loans are not a good way to borrow money because they have very high interest rates.

(2) The graph below is an exponential model of the levels of toxic waste in a lake. The amount of waste is decreasing at a rate of 12% each year.

Line graph showing the levels of toxic waste in Lake Meadows. Y-axis represents the amount of toxic waste in pounds, ranging from 0 to 1,200. X-axis represents the number of years, ranging from 0 to 14. 0 = 1,000 2 = in between 600 and 800 4 = 600 6 = in between 400 and 600 8 = in between 200 and 400 10 = in between 200 and 400 12 = 200

(a) If the trend continues, which of the following statements would be correct?

(i) The decrease in the level of waste from Year 12 to Year 14 will be the same as the decrease from Year 10 to 12.

(ii) The decrease in the level of waste from Year 12 to Year 14 will be less than the decrease from Year 10 to 12.

(iii) The decrease in the level of waste from Year 12 to Year 14 will be more than the decrease from Year 10 to 12.

(b) Would your answer in (a) change if this had been a linear model? Explain.

DEVELOPING SKILLS AND UNDERSTANDING

(3) Match the following descriptions with the graphs (on the next page). Each description may match more than one graph. Some graphs may match more than one situation and some graphs may not match any situation.

(a) This graph represents the monthly cost of a gym membership plan. Its monthly fee is \$20 with an additional charge of \$5 per class, where Y = monthly cost of gym membership and t = number of classes.

(b) This graph represents the monthly cost for a gym membership that has a fixed monthly charge of \$20 with free unlimited classes where Y = monthly cost of gym membership and t = number of classes.

(c) This graph represents the amount in an account that decreases by \$5 each month where Y = balance in the account and t = months.

(d) This graph represents the amount in an account that decreases by 25% each month where Y = balance in the account and t = months.

(e) This is a graph of $Y = -5t + 40$

(f) This is a graph of $Y = 30(1.25)t$

(g) This is a graph of an exponential function.

(h) This is a graph of a linear function.

I. Graph showing an increasing curb, indicating the following: Start: 0 = 30 End: 5 = just above 90	II. Graph showing a decreasing straight-line, indicating the following: Start: 0 = 40 End: 5 = in between 10 and 20
III. Graph showing a straight-line, indicating the following: Start: 0 = 20 End: 5 = 20	IV. Graph showing a decreasing curb, indicating the following: Start: 0 = 50 End: 5 = in between 10 and 20
V. Graph showing an increasing squiggly line, indicating the following: Start: 0 = 30 End: 5 = in between 40 and 50	VI. Graph showing an increasing straight-line, indicating the following: Start: 0 = 20 End: 5 = 45

(4) In a previous collaboration, you saw that a person’s Blood Alcohol Content (BAC) after t hours could be approximated by Widmark’s equation. Paula is part of a research study to verify Widmark’s equation. She weighs 130 pounds. The researchers ask her to drink three beers. After one hour, her BAC is at 0.10. After three hours her BAC is at 0.08.

(a) Find an equation of the line using her BAC measurements where B is her BAC after t hours. **Hint**: The time and BAC can be written as ordered pairs: (1, 0.10) and (3, 0.08). It might help to graph the two points and draw the line between them.

(b) What is the slope of your equation and how would you interpret this value in the context of the situation?

(c) What are the horizontal and vertical intercepts of your equation and how would you interpret these values in this scenario?

(d) Recall Widmark's equation:

$$B = -0.015 \cdot t + \left(\frac{2.84 \cdot N}{W \cdot g} \right)$$

where N is the number of "standard drinks," W is his/her weight in pounds, and g = gender constant (0.68 for men and 0.55 for women). One standard drink is a 12-ounce beer, 5-ounce glass of wine, or 1.5-ounce shot of liquor.

(i) Simplify this equation for Paula's case, leaving t as a variable.

(ii) State the slope and vertical intercept for this simplified equation, and interpret these values.

(e) Looking at the models you created in (a) and (d), compare the slopes of the two equations. Which model has the steepest slope? What does that mean in this context?

(5) Caffeine is another chemical in food and beverages. However, caffeine is processed in the body differently than alcohol. For most people, five hours after eating or drinking something with caffeine only about half of the caffeine is left in the bloodstream. In another five hours (10 hours total) half of that is left, and so on. This situation is exponential because the amount of caffeine decreases by 50% every five hours.[20]

(a) Suppose Fred drinks a tall Café Americano from Starbucks, which has about 150 mg of caffeine.[21] Complete the table below for this situation. Round to the nearest tenth. The first two entries are done for you.

Time in Hours	Amount of Caffeine Left in His Body (mg)
0	150
5	75
10	(i)
15	(ii)
20	(iii)
25	(iv)
30	(v)

(b) Plot the data from (a) on a graph.

(c) Suppose his wife, Wilma, drinks a 12-oz Diet Coke, which has 45 mg of caffeine. However, Wilma is using an oral contraceptive. When someone is on "the pill," it takes about 10 hours (twice as long) for half of the caffeine to leave the bloodstream, and another 10 hours for half of the remaining to leave, etc. Complete the table below for this situation. Round to the nearest tenth. Then plot the amount of caffeine left in Wilma's body on the same graph as in (b).

Time in Hours	Amount of Caffeine Left in Her Body (mg)
0	(i)
10	(ii)
20	(iii)
30	(iv)

(d) Looking at your graph of Fred and Wilma's data, who has:

(i) more caffeine in their body in the first few hours?

(ii) more caffeine in their body in the long term?

(e) Based on your graphical models, is there a time that either Fred or Wilma will have no caffeine left in their body? Explain your reasoning.

MAKING CONNECTIONS ACROSS THE COURSE

(6) In Exercise 4.5, you modeled the world population based on the growth rate in 2023. To plan for future needs, it is necessary to project how many people there will be in a certain place, but it is also necessary to know information about the ages of the people. This type of information is often displayed in a type of graph called a *population pyramid*. The population pyramid shown below is from the CIA World Factbook. Look at it carefully and think about what information is shown before continuing.[22]

United States - 2023

Horizontal bar graph showing the population in millions by age group in the United States in 2023. Y-axis represents the age groups, ranging from 0-4 to over 100. X-axis represents the population in millions, ranging from 0 to 15. Male age groups 0-4, 5-9, 10-14, 15-19, 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59, 60-64, all have a population between 10 and 12.5. Female age groups 0-4, 5-9, 10-14 all have a population around 10. Female age groups 15-19, 20-24, 25-29, 30-34, 35-39, 40-44, 50-54, 55-59, 60-64, 65-69 all have a population between 10 and 12.5. 45-49 = around 10. Male 65-69 = between 7.5 and 10. Female 65-69 = just over 10. Male 70-74 = just under 7.5. Female 70-74 = in between 7.5 and 10. Male 75-79 = just over 5. Female 75-79 = in between 5 and 7.5. Male 80-84 = in between 2.5 and 5. Female 80-84 = close to 5. Male 85-89 = between 0 and 2.5. Female 85-89 = around 2.5. Male 90-94 = about 1. Female 90-94 = about 2.5. Male 95-99 and 100+ = very close to 0. Female 95-99 = about 1. Female 100+ = very close to 0.

This population pyramid is for the U.S. in 2023. The left side of the pyramid represents males; the right represents females. Each bar represents the age group listed in the center column. This pyramid measures the population in millions along the horizontal axis. Sometimes these types of graphs measure the population in percentages instead of numbers. Use this graph to answer the following questions.

(a) Based on the graph, which of the following statements is the most accurate?

(i) In the population under 30 years old, the number of men and the number of women are similar.

(ii) In the population over 79 years old, the number of men and the number of women are equal.

(iii) There are fewer people in the age range of 35–44 than in the range of 45–55.

The graph below illustrates Mexico’s population in 2023.[23]

Mexico - 2023

Horizontal bar graph showing the population in millions by age group in Mexico in 2014. Y-axis represents the age groups, ranging from 0-4 to over 100. X-axis represents the population in millions, ranging from 0 to 8. 0-4: Male = in between 4 and 6; Female = in between 4 and 6. 5-9: Male = in between 4 and 6; Female = in between 4 and 6. 10-14: Male = in between 4 and 6; Female = in between 4 and 6. 15-19: Male = in between 4 and 6; Female = in between 4 and 6. 20-24: Male = in between 4 and 6; Female = in between 4 and 6. 25-29: Male = in between 4 and 6; Female = in between 4 and 6. 30-34: Male = in between 4 and 6; Female = in between 4 and 6. 35-39: Male = in between 4 and 6; Female = in between 4 and 6. 40-44: Male = in between 4 and 6; Female = in between 4 and 6. 45-49: Male = in between 2 and 4; Female = in between 4 and 6. 50-54: Male = in between 2 and 4; Female = around 4. 55-59: Male = in between 2 and 4; Female = in between 2 and 4. 60-64: Male = in between 2 and 4; Female = in between 2 and 4. 65-69: Male = in between 0 and 2; Female = about 2. 70-74: Male = about 1; Female = about 1.5. 75-79: Male = under 1; Female = about 1. 80-84: Male = about 0.5; Female = just over 0.5. 85-89: Male = close to 0; Female = just under 0.5. 90-94: Male = just over 0; Female = just over 0. 95-99: Male = just over 0; Female = just over 0. Over 100: Male = just over 0; Female = just over 0.

(b) Indicate if each statement applies to the United States, Mexico, or both.

(i) Beyond age 65 there are consistently more women than men in each category.

(ii) The number of people 0–9 years old is about the same as the number of people 10–19 years old.

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[20] http://en.Wikipedia.org/wiki/Caffeine

[21] https://www.starbucks.com/menu/product/406/hot/nutrition

[22] https://www.cia.gov/the-world-factbook/countries/united-states/

[23] https://www.cia.gov/the-world-factbook/countries/mexico/

4.8: Green Savings

INTRODUCTION

This collaboration uses a personal finance context (buying a car) to lead you through a complex mathematical modeling exercise. The first half of the exercise is presented in this collaboration, and the second half is presented in Collaboration 4.9. You will be confronted with an open-ended problem, which requires you to identify missing information, make the relevant accommodations, and perform the appropriate calculations to solve the problem.

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- mathematical modeling and quantitative reasoning can be used to help make personal financial decisions.

By the end of this collaboration, you should be able to

- identify missing information.
- demonstrate an ability to determine reasonable values for missing information.
- construct a linear model of a savings scenario.
- describe in words and graphs the effect changes in assumptions have on linear models.

PROBLEM SITUATION: COMPARING THE COSTS OF CARS

You have decided to investigate some pros and cons of buying a hybrid car. A hybrid car gets better gas mileage than a gas-powered car, but it usually costs more. While there are many issues to consider in this choice, at this point you want to investigate the savings at the gas pump for the hybrid vehicle and determine if these savings will offset the higher price of the car. Suppose Internet research yields the following information:

- A 2023 Honda Accord hybrid costs $31,895. This car is supposed to get 46 miles per gallon (mpg) in regular city driving and 41 mpg on the highway.[25]
- A completely gas-powered 2023 Honda Accord costs $27,295. This car is supposed to get 29 mpg in regular city driving and 37 mpg on the highway.

(1) What additional information is necessary to determine how much money will be spent on gas in a year of driving one of these vehicles? Discuss in your group and write down some reasonable values for any missing information.

(2) (a) If you purchase the hybrid Accord, how much money would you save in gasoline purchases the first year? Take a minute to think about this individually before sharing your ideas and calculations in your group. Round figures to two decimal places.

(b) How long would it take for the gasoline savings to make up for the additional cost of the hybrid?

Round to the nearest year.

(3) (a) You have seen in earlier collaborations the importance of being able to model quantities both algebraically (with equations) and graphically. Determine an algebraic model (equation) for the amount of money saved in gasoline. Define all variables used in the model.

(b) Produce a graph of this model. **Note**: If completing this problem online, follow the instructions given online to create your graph.

(c) Explain whether the model is linear, exponential, or neither.

(4) How does the graph change if the price of gas increases?

(5) How does the graph change if the number of miles driven in any given year increases?

FURTHER APPLICATIONS

(6) Below is a graph about Tyrone’s savings account. The coordinates of the first and last points on the graph are provided.

Line graph showing Tyrone’s Savings Account. Y-axis represents the account balance in dollars, ranging from 0 to 400. X-axis represents the number of months, ranging from 0 to 12. 0 = 235 10 = 335

(a) How would you best describe the situation modeled by the graph?

(b) Write an equation for the graph. Be sure to define your variables, and explain your group’s approach.

(c) If the trend continues, how long will it take for Tyrone to have $500? Explain how you found your answer.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[25] https://automobiles.honda.com/accord-sedan

4.8.1: Preparation 4.8

(1) The table below shows data on the prevalence or rate of smoking among adults in the United States over time.[24]

Year	Prevalence of Smoking Among Adults in the United States (as a percentage)
1965	42.40
1970	37.40
1980	33.20
1990	25.50
2000	23.30
2005	20.90
2010	19.30
2013	17.80
2020	12.50

(a) Graph the data on the grid below. Add appropriate labels.

Blank graph. The y-axis ranges from 0 to 45, and the x-axis ranges from 1965 to 2010.

(b) Create a linear equation for the data. Let P = the percentage of adults who smoke and t = the number of years after 1965. Given the two points below, find your model.

Point 1: Year 1970, Percentage 37

Point 2: Year 2013, Percentage 18

Round your vertical intercept to the nearest whole number, and your slope to two decimal places.

Your linear equation:

(c) Describe the situation modeled by your equation. This should include an interpretation of the starting value and the rate of change.

(d) Do you think your model would accurately predict the rate of smoking in 2060? Explain your answer.

(e) Based on these data, can you say that fewer adults in the United States smoked in 2010 than in 2005? Explain your answer.

After Preparation 4.8 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 4.8, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	Rating from 1 to 5
graph data on a coordinate plane.	
graph linear equations.	
solve linear equations.	
create an equation, graph, or table from a verbal situation and use these representations to analyze, predict, and solve a problem.	

[24] www.infoplease.com/ipa/A0762370.html#ixzz1bS71bHKR and https://www.cdc.gov/tobacco/data_statistics/fact_sheets/adult_data/cig_smoking/index.htm

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4.9: Saving Green

INTRODUCTION

In this collaboration, you continue to examine the financial impact of buying a hybrid car versus a non-hybrid car. In Collaboration 4.8, you created a linear model to represent the yearly savings in gas due to driving a fuel-efficient car. In this collaboration, you use what you have learned about compound interest to examine the savings from a different point of view—the person who did not buy the fuel-efficient car. In this scenario, the savings from buying a less expensive vehicle is placed in a bank to earn interest.

Note that the suggested timeline for this collaboration indicates that a lot of personal responsibility is placed on your groups. Unlike previous collaborations, there are not a lot of preparatory questions in the problem situation. You are expected to rely on your group and the work done previously in the course to work through this collaboration. After the initial discussion to make sure the problem situation is understood and reasonable assumptions have been made, you will create, analyze, and use your savings model.

Your instructor will analyze your collaboration work and provide you with detailed feedback.

There are no Preparation or Exercise assignments related to this collaboration.

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- mathematical modeling or quantitative reasoning can be used to help make personal financial decisions.

By the end of this collaboration, you should be able to

- identify missing information.
- demonstrate an ability to determine reasonable values for missing information.
- construct a linear model of a savings scenario.
- describe–in words and graphs–the effect changes in assumptions have on linear models.

PROBLEM SITUATION: COMPARING THE COST OF CARS (A CONTINUATION OF COLLABORATION 4.8)

Suppose you buy the non-hybrid Accord. Since this car is cheaper, you have already saved some money compared with someone who bought the hybrid car. If these savings are invested in a savings account that earns interest, they could continue to grow over time.

(1) Create an algebraic model (equation) for the amount of savings over time. Determine reasonable values for any missing information and define all variables.

(2) Produce a graph of your model and label your axes appropriately. **Note**: If completing this problem online, follow the instructions given online to create your graph.

(3) Now that you have invested time creating and studying two savings models (you created one of these models in Collaboration 4.8), you want to share your analysis with your peers. Suppose you have the opportunity to submit one paragraph on a blog for college students. Write a paragraph that compares the gasoline costs associated with gas-powered and hybrid vehicles. Your paragraph should include a final recommendation on whether someone should buy a gas-powered or hybrid vehicle.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

4.10: Module 4 Review Survey

As with Modules 1, 2, and 3, you should assess your understanding of Module 4 to prepare for the Module 4 test. Your instructor may give you specific assignments for your review in addition to this self-assessment.

Assessing Your Understanding

The table on the following page lists the Module 4 concepts and skills you should understand. After completing it, you will be able to prioritize your review time more effectively.

Assess your understanding:

- Go through the topics list and locate each concept or skill in the Module 4 student workbook or in the online activities.
- If you recently used the skill and feel confident that you did it correctly, rate your understanding a 4 or 5.
- If you remember the topic but could use more practice, rate your understanding a 3.
- If you cannot remember that skill or concept, or still find it difficult, rate your understanding a 1 or 2.

Now that you have done an initial rating of your understanding, it is time to begin reviewing. The goal is to have a confidence rating of 4 or 5 on all the topics in the table when you have finished your review of Module 4.

Things to consider for reviewing:

1. Start at the beginning of the module and review the material in the Preparations, Collaborations, and Exercises, and your notes on the skills and concepts you rated 3 or below.
2. Select a few questions to do.
3. Once you have finished, remember to check the solutions. If you are not sure if you have done the problems correctly, check with your instructor, other classmates, your previous work or work with a tutor in the learning center or online.
4. Rate your confidence on this skill again. If you understand the concept better, rate yourself higher. Begin a list of topics that you want to review more thoroughly.

For topics that you need to review more thoroughly, make a plan for getting additional assistance by studying with classmates, visiting your instructor during office or virtual office hours, working with a tutor in the learning center or online, or looking up additional information on the Internet.

Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Concept or Skill	Rating
Linear Models	
Create, interpret, and use the four representations of a linear model (verbal, table, graph, equation) (4.1, 4.2, 4.3, 4.7, 4.8)	
Translate between the four representations of a linear model (4.1, 4.2. 4.3)	
Identify and interpret vertical intercept, horizontal intercept, and slope from a graph (4.1, 4.2, 4.3, 4.7, 4.8)	
Identify the slope and vertical intercept from an equation (4.1, 4.2)	
Understand that a linear model is defined by a constant rate of change (4.1, 4.2, 4.7, 4.8)	
Identify a model as linear based on any of the four representations (4.1, 4.2, 4.7, 4.8)	
Understand the role of units in a linear equation (4.1)	

Calculate and interpret slope (4.2, 4.3)	
Create a linear equation to model data (4.3)	
Understand the limitations of models based on data (4.3)	
Use equations, tables, and graphs to solve or estimate solutions to problems (4.1, 4.2, 4.3, 4.7, 4.8)	
Write and use linear equations based on a percentage increase or decrease (4.4)	
Exponential Models	
Create, interpret, and use the four representations of an exponential model (verbal, table, graph, equation) (4.5, 4.6, 4.7, 4.9)	
Write and use an exponential model (4.5, 4.6, 4.7, 4.9)	
Understand that an exponential model is defined by a percentage change (4.6, 4.7, 4.9)	
Identify an exponential model as growth or decay (4.6, 4.7, 4.9)	

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CHAPTER OVERVIEW

5: Fundamentals Lessons 1 to 27 - Student Lessons

5.1: Fundamentals 1 - Quantitative Reasoning
5.2: Fundamentals 2 - Order of Operations–Addition and Subtraction
5.3: Fundamentals 3 - Order of Operations–Multiplication and Division
5.4: Fundamentals 4 - Order of Operations–Multistep Calculations
5.5: Fundamentals 5 - Place value
5.6: Fundamentals 6 - Rounding
5.7: Fundamentals 7 - Large Numbers and Powers of Ten
5.8: Fundamentals 8 - Multiples and Factors
5.9: Fundamentals 9 - Prime Factorization, GCF, and LCM
5.10: Fundamentals 10 - Fractions–Basics
5.11: Fundamentals 11A - Fractions–Multiplying and Dividing
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5.13: Fundamentals 12 - Ratios and Proportions
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5.15: Fundamentals 14 - Percents–Just the Basics
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5.17: Fundamentals 16 - Estimation
5.18: Fundamentals 17 - Conversion
5.19: Fundamentals 18 - Real Line
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5.21: Fundamentals 20 - Algebra–Basics
5.22: Fundamentals 21A - Geometry–Perimeter, Area, and Volume
5.23: Fundamentals 21B - Comparison and Inequality Symbols
5.24: Fundamentals 22 - Translating Word Expressions
5.25: Fundamentals 23 - Linear Equations (Part 1)
5.26: Fundamentals 24 - Linear Equations (Part 2)
5.27: Fundamentals 25 - Pattern Recognition
5.28: Fundamentals 26 - Plotting Points
5.29: Fundamentals 27 - Technology and Math

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5.1: Fundamentals 1 - Quantitative Reasoning

LEARNING GOALS

By the end of this lesson, you should understand

- quantitative reasoning is the ability to understand and use quantitative information. It is a powerful tool in making sense of the world.
- relatively simple math can help make sense of complex situations.

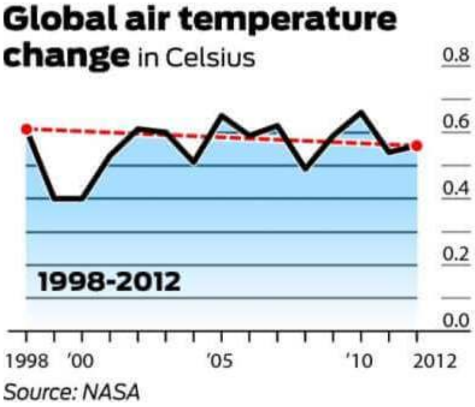
By the end of this lesson, you should be able to

- read each chart to identify quantitative information.
- determine which chart gives a better indication of a trend in climate change.

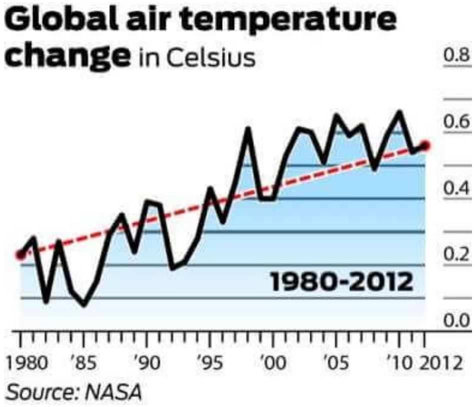
TRY THESE

Global Warming

The graph below[1] is the one most often referenced to disprove global warming. It demonstrates the change in air temperature (Celsius) from 1998 to 2012.



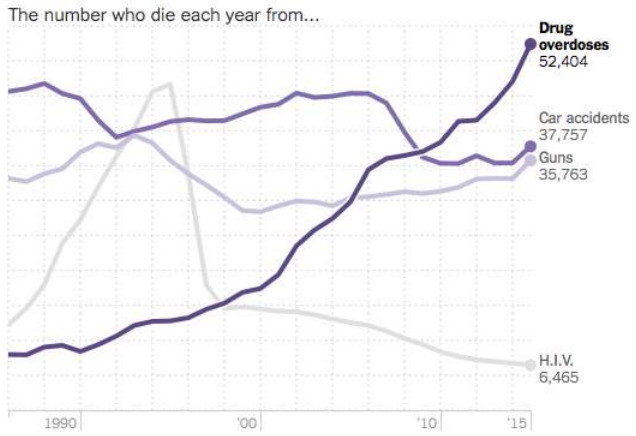
1. What is global warming?
2. Would the graph above more likely be used by a person who believes or does not believe in global warming (also referred to as climate change)?
3. Now take a moment to look at the following graph.[1] What do you and your group members notice about this graph as compared to the first one? What are some major differences?



4. Would this graph more likely be used by a person who believes or does not believe in global warming? Explain.

Questions: Quantitative Reasoning

The following graph[2] presents data on the number who die each year from drug overdoses, car accidents, guns, or H.I.V. for the years 1986 to 2015. Use the graph to answer questions 1-6.



1. In the mid 1990’s (1992 – 1998), the number of people who died each year from ______________________ drastically increased and then decreased.
    (i) Drug Overdoses
    (ii) Car Accidents
    (iii) Guns
    (iv) H.I.V.
2. The number of people who died each year from ______________________ drastically increased from 1986 to 2015.
    (i) Drug Overdoses
    (ii) Car Accidents
    (iii) Guns
    (iv) H.I.V.
3. Which categories had a **decrease** in the number who died if you only compare the data from 1986 and 2015? Select all that apply.
    (i) Drug Overdoses
    (ii) Car Accidents
    (iii) Guns
    (iv) H.I.V.
4. Which categories had an **increase** in the number who died if you only compare the data from 1986 and 2015? Select all that apply.
    (i) Drug Overdoses
    (ii) Car Accidents
    (iii) Guns
    (iv) H.I.V.
5. According to Time Health[3], a total of 63,632 Americans died of a drug overdose in 2016. How large was the increase from 2015 to 2016?

6. According the Center for Disease Control and Prevention and the FBI[4], in 2016, there were a total of 38,658 gun deaths in the US. How large was the increase from 2015 to 2016?

---

[1] https://www.sfgate.com/technology/dotcommentary/article/How-climate-deniers-abuse-statistics-to-mislead-4677033.php

[2] https://www.nytimes.com/interactive/2017/04/14/upshot/drug-overdose-epidemic-you-draw-it.html

[3] http://time.com/5220372/drug-overdose-deaths-cdc/

[4] https://www.thetrace.org/rounds/gun-deaths-increase-2017/

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5.2: Fundamentals 2 - Order of Operations–Addition and Subtraction

LEARNING GOALS

By the end of this lesson, you should understand

- a number line and how to use it to perform addition and subtraction.
- the concept of order of operations is a matter of understanding mathematical notation.
- **order of operations** is an agreement of which operations should be performed first in order to evaluate a given mathematical expression.
- we use order of operations so that we all come up with the same answer.

By the end of this lesson, you should be able to

- use a number line, if needed, in order to add and subtract with whole numbers and integers.
- correctly evaluate a mathematical expression with addition and subtraction.
- set up a mathematical expression and use order of operations to simplify.

FUNDAMENTALS OF THE LESSON

The **number line** is a line where numbers are displayed and marked in order and equally spaced. It can be used as a flexible tool for developing greater ability to perform mental arithmetic and constructing mathematical meaning, number sense, and understandings of number relationships.

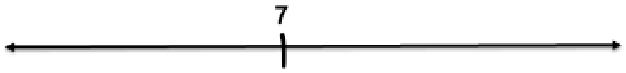
An **open number line** (as opposed to a ruler with fixed numbers and spaces) begins blank and can be used to write numbers and divide the space as needed given the problem situation.

1. You will now use the open number line below to simplify the expression 7 - 8 + 5. In order to simplify this expression, we must first identify the operations contained so that we can determine where we should start or what operations to perform.

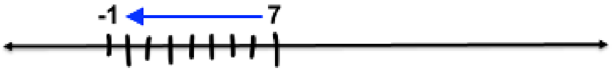
What operations are contained in this expression?

2. Where should we start on the number line?

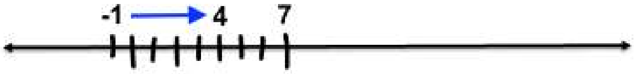
Let's place our starting number, 7, in the middle of the number line:



Since the expression only contains addition and subtraction, which have the same level or order of importance, we can start at the 7 then subtract 8, and we arrive at -1:



And, from the -1 we can add 5:



NEXT STEPS

Managing a Budget

Below is a sample monthly budget. A budget can be used to help you manage money. A budget is an estimate of your income and expenses over a period of time. By creating and using a monthly budget, you can get a better idea of where you spend your money, to whom you owe money, and how much. A budget can help you focus on saving more and can help you reach your financial goals.

The sample budget below has two categories, income and expenses. Income includes money earned, including wages. Expenses include items you spend your money on, like rent and food. The sample budget has four columns: the category, the budget estimate for the income or expense, the actual income or expense amount, and the difference between the estimate and the actual amount.[5]

Category	Estimated Amount	Actual Amount	Difference
Income:			
Wages	\$911	\$892	
Interest	\$232	\$196	
Income Subtotal			\$55
Expenses:			
Taxes	\$386	\$317	
Rent	\$298	\$298	
Utilities	\$99	\$95	
Groceries	\$160	\$171	
Clothing	\$66	\$60	
Household	\$55	\$46	
Entertainment	\$44	\$40	
Misc/Other	\$35	\$30	
Expenses Subtotal			
Net Income = Income - Expenses			

3. Set up an equation that could be used to verify that the difference between the estimated income and the actual income is \$55. Is there more than one way to determine this answer?

4. Compute the following and record in the budget table.

A. Compute the estimated income subtotals.

B. Then compute the actual income subtotals.

C. Compute the estimated expense subtotal.

D. Compute the actual expense subtotal.

E. What do you notice about the **estimated** income subtotal and expenses subtotal?

F. Compare the actual income and expenses subtotals.

G. At the end of the month, does this person have extra money or a deficit?

H. If this person has extra money, how much?

FURTHER APPLICATIONS

5. Yasmine would like to sell homemade pillows on Etsy and is trying to determine what to charge for each pillow to make a profit of \$3.00. The pillow inserts each cost \$4.99, but she has a coupon for \$1.00 off each pillow insert. The fabric costs \$1.75 per pillow, and she wants one decorative button per pillow, which costs \$0.45 each. How much should Yasmine charge for each pillow if shipping will cost her \$2.50?

Questions: Order of Operations–Addition and Subtraction

Simplify each expression:

1. -11 + 14 - 3

2. 15 - 17 - 3 + 8

3. -100 + 85 - 10 - 20

4. You owe your mom $10 and your best friend $5. You get paid $37. Complete the expression below to verify that you have $22 left.

________ - 5 + 37 = 22

Sample student monthly budget:

Sample Student Monthly Budget

Expense Type	Amount
Income	$650 per month
Fixed expenses	
Rent	$0 (live on campus)
Car payment	$125
Cell phone	$75
Total fixed expenses	
Flexible expenses	
Food, in excess of meal plan	$95
Gas	$65
Entertainment	$75
Clothing	$45
Misc	$25
Total flexible expenses	

5. Determine the total fixed expenses. Write in the table above.

6. Determine the total flexible expenses. Write in the table above.

7. Does this student have any extra money to put into savings? If so, how much?

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[5] Modified from: *Money Instructor*, Sample Monthly Household Budget, accessed August 1, 2018, Content.moneyinstructor.com/437/sample-budget.html

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5.3: Fundamentals 3 - Order of Operations–Multiplication and Division

LEARNING GOALS

By the end of this lesson, you should understand

- the concept of order of operations is a matter of understanding mathematical notation.
- **order of operations** is an agreement of which operations should be performed first in order to evaluate a given mathematical expression.
- multiplication and division have the same power, they are performed as they occur left to right.
- we use order of operations so everyone can arrive at the same answer when performing multi-step problems.

By the end of this lesson, you should be able to

- correctly evaluate a mathematical expression with multiplication and division, as well as addition and subtraction.
- set up a mathematical expression and use **order of operations** to simplify.

FUNDAMENTALS OF THE LESSON

When given quantitative information, it is sometimes difficult to figure out what to do first, or if there are different ways to simplify an expression. For the following two situations, read the information carefully on your own. Before starting the problem on your own or in your small group, we will discuss rules of order of operations, when multiplication and division occur.

1. You work as a director at a local television production station. You and your fellow directors are in a meeting with the vice president of the station. She announces that you will each be receiving a bonus from profits the company made the previous year.

A. A total of $12,840 bonus money is to be divided among the three directors. You calculate what your bonus will be. Several days later, the vice president receives an email that there was a mistake, and the actual amount to be given as a bonus is twice as much as she announced. Write an expression that represents the above scenario of your actual bonus. Determine the amount of your actual bonus.

B. What if the vice president had told you all of the information right away? How could you have correctly calculated your bonus in a different way? **Hint**: What would the expression now be to represent the order the information was given?

2. At a financial planning course, 35 people have a total of $245,000 in savings, but they have four times that amount in credit card debt. If each person has about the save amount in savings and debt, about how much credit card debt does each person have?

A. Write an expression that represents the amount each person has in credit card debt given this scenario.

B. Determine the amount each person has in credit card debt.

TRY THESE

When performing arithmetic operations, there can be only one correct answer. We need a set of rules in order to avoid confusion of which answer is correct. Mathematicians have devised a standard order of operations for calculations involving more than one arithmetic operation.

- Rule 1: First perform any calculations inside parentheses or other grouping symbols such as brackets, a fraction bar (which indicates division), and absolute value (which is defined as distance from zero).
- Rule 2: Next perform all multiplications and divisions, working from left to right.
- Rule 3: Lastly, perform all additions and subtractions, working from left to right.

Why do we have these rules? Let’s do a problem to demonstrate.

Our expression from Question 1 was $12840 \div 3 \times 2$, which produces 8,560 when we perform the operations from left to right, according to the rules above.

First, we divided 12,840 by 3, and then we multiplied by 2 to give us 8,560.

Now, let's see what happens if we perform the operations from right to left: we'd start by multiplying 2 by 3, which gives us 6. Then we'd have $12840 \div 6$, which produces 2,140. Wait a minute!!! This is a much smaller bonus! What happened?! Working from right to left does not always lead to the same answer.

For the following practice problems, you will be working to simplify expressions that contain addition, subtraction, multiplication, and division.

3. $10 \div 5 \times 3 \times 3$

4. $10 - 8 \div 2 - 7$

5. $4 \times 8 - 22 \div 11$

6. $35 \div 7 \times 3 - 3 \times 2$

NEXT STEPS

Why is it important to calculate the total daily dosage of opioids?

Opioids are a class of drugs that include the illegal drug heroin, synthetic opioids such as fentanyl, and pain relievers available legally by prescription, such as oxycodone (OxyContin®), hydrocodone (Vicodin®), codeine, morphine, and many others.[6] Patients prescribed higher opioid dosages are at higher risk of overdose death. In a national sample of Veterans Health Administration (VHA) patients with chronic pain receiving opioids from 2004–2009, patients who died of opioid overdose were prescribed an average of 98 MME/day, while other patients were prescribed an average of 48 MME/day. Calculating the total daily dose of opioids helps identify patients who may benefit from closer monitoring, reduction or tapering of opioids, or other measures to reduce risk of overdose. (**Note**: MME = Morphine Milligram Equivalent.[7])

HOW SHOULD THE TOTAL DAILY DOSE OF OPIOIDS BE CALCULATED?

Step 1: DETERMINE the total daily amount of each opioid the patient takes.

Step 2: Using the table below, CONVERT each to MMEs—by multiplying the dose for each opioid (mg per day) by the conversion factor.

Step 3: ADD them together.

Opioid (doses in mg/day)	Conversion Factor
Hydrocodone	1.0
Oxycodone	1.5

7. John suffers from chronic back pain. On days when he is feeling a low amount of pain, he takes 4 tablets of Hydrocodone at 5 mg each. On days that he has a high amount of pain, he takes 8 tablets of the Hydrocodone at 5 mg each and 2 tablets of Oxycodone at 20 mg each.

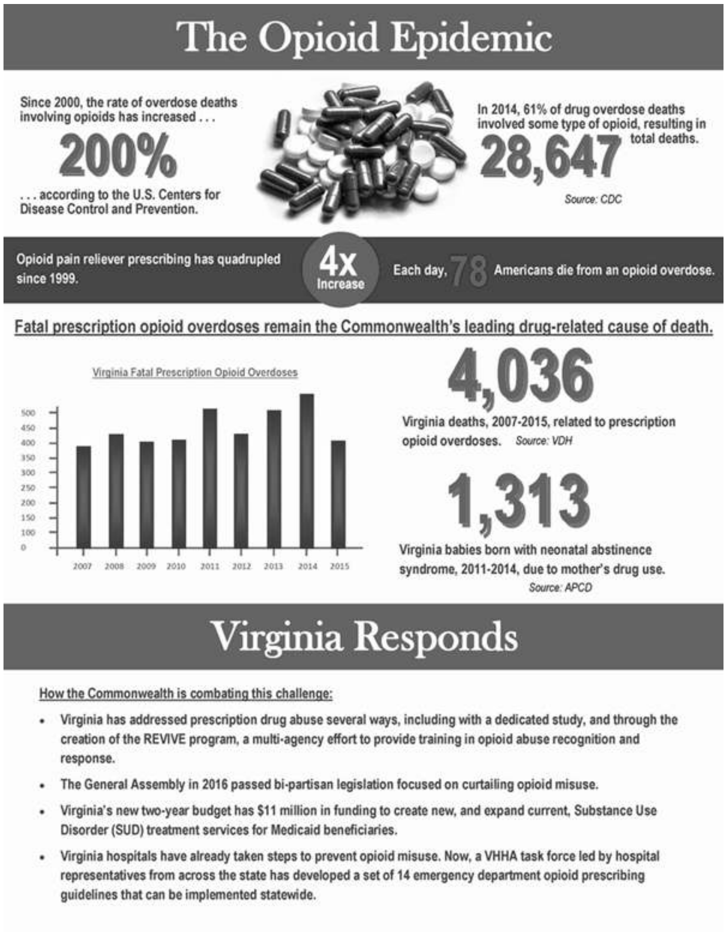
A. Calculate his daily dose of opioids for days with a **low** amount of pain.

B. Calculate his daily dose of opioids for days with a **high** amount of pain.

FURTHER APPLICATIONS

Opioid Epidemic

8 The infographic below[8] states that each day 78 Americans die from an Opioid overdose.



The Opioid Epidemic

Since 2000, the rate of overdose deaths involving opioids has increased . . .

200%

. . . according to the U.S. Centers for Disease Control and Prevention.

In 2014, 61% of drug overdose deaths involved some type of opioid, resulting in **28,647** total deaths.

Source: CDC

Opioid pain reliever prescribing has quadrupled since 1999.

4x Increase

Each day, **78** Americans die from an opioid overdose.

Fatal prescription opioid overdoses remain the Commonwealth's leading drug-related cause of death.

4,036

Virginia deaths, 2007-2015, related to prescription opioid overdoses. Source: VDH

1,313

Virginia babies born with neonatal abstinence syndrome, 2011-2014, due to mother's drug use.

Source: APCD

Virginia Responds

How the Commonwealth is combating this challenge:

- Virginia has addressed prescription drug abuse several ways, including with a dedicated study, and through the creation of the REVIVE program, a multi-agency effort to provide training in opioid abuse recognition and response.
- The General Assembly in 2016 passed bi-partisan legislation focused on curtailing opioid misuse.
- Virginia's new two-year budget has $11 million in funding to create new, and expand current, Substance Use Disorder (SUD) treatment services for Medicaid beneficiaries.
- Virginia hospitals have already taken steps to prevent opioid misuse. Now, a VHHA task force led by hospital representatives from across the state has developed a set of 14 emergency department opioid prescribing guidelines that can be implemented statewide.

A. Assuming there are 365 days in a year, about how many Americans die each year from an Opioid overdose?

B. About how many Americans die each month? Round to the nearest person.

9. From 2007 – 2015 (which is nine years in total), there were 4,036 Virginia deaths related to prescription Opioid overdose. On average, how many deaths were there for each year? Round to the nearest person.

Questions: Order of Operations–Multiplication and Division

Simplify the following:

1. $15 \div 3 \times 6$
2. $18 \times 3 \div 6$
3. $35 - 10 \div 2 \times 3$
4. $75 \div 5 - 3 \times 3 + 8$
5. $11 + 10 \times 3 \div 6 - 4$

6. Maria is planning a party. She figures out that food will cost $20 for each pizza and $5 for each snack that she buys. Maria creates the following expressions for different party scenarios. Simplify them to help Maria determine the potential cost of each party scenario.

$20 \times 3 + 5 \times 3$

$20 \times 10 + 5 \times 10$

$20 \times 2 + 5 \times 4$

7. At each marketing event, Marylou puts 40 brochures from her business on each of the 12 tables in the event room. The brochures come in boxes of 120. Which expression below correctly determines how many boxes of brochures are needed to complete this task?

(i) 120 + 40 + 12

(ii) 120 × 12 × 40

(iii) 40 × 12 ÷ 120

8. During 2016, there were more than 63,600 overdose deaths in the United States. For every 100 drug overdose deaths, 66 involved an opioid.

Determine the number of drug overdose deaths that involved an opioid in 2016.

If there are 365 days in a year, about how many overdose deaths were there each day in 2016?

How many drug overdose deaths that involved an opioid were there each day in 2016?

---

[6] https://www.drugabuse.gov/drugs-abuse/opioids

[7] https://www.nsc.org/home-safety/safety-topics/opioids/treating-chronic-noncancer-pain

[8] https://www.joinbaad.org/facts/2017/2/7/the-opioid-epidemic-in-virginia

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5.4: Fundamentals 4 - Order of Operations–Multistep Calculations

LEARNING GOALS

By the end of this lesson, you should understand

- the concept of order of operations is a matter of mathematical notation.
- **order of operations** is an agreement of which operations should be performed first in order to evaluate a given mathematical expression.
- we use order of operations so that we all come up with the same answer.

By the end of this lesson, you should be able to

- use multiple steps to correctly evaluate a mathematical expression.
- set up a mathematical expression and use order of operations to simplify.

FUNDAMENTALS OF THE LESSON

Often, numerical expressions require multistep calculations in order to be simplified. When performing arithmetic operations, there can be only one correct answer. We need a set of rules in order to avoid confusion of which answer is correct. Mathematicians have devised a standard order of operations for calculations involving more than one arithmetic operation:

Rule 1: First perform any calculations inside parentheses or other grouping symbols such as brackets, a fraction bar (which indicates division), and absolute value (which is defined as distance from zero).

Rule 2: Next perform all multiplications and divisions, working from left to right.

Rule 3: Lastly, perform all additions and subtractions, working from left to right.

1. Let’s try to simplify an expression.

$$1+1+1+1+1+1+1\times 0$$

TRY THESE

With and Without Grouping

Now let’s try to simplify expressions with and without parentheses and other grouping symbols.

2. Below are examples of expressions with grouping symbols other than parentheses. Without these grouping symbols, the expression would simplify to be a different value. Evaluate both to see the difference.

A. Brackets – higher order parentheses: $7-[(5-3)+4]$ vs. $7-(5-3)+4$

B. Fraction bar – which implies division: $\frac{6-4}{2}$ vs. $6-4\div 2$

C. Absolute value – distance from zero: $5-|-3|$ vs. $5-(-3)$

3. Which student(s) correctly simplified the mathematical expression?

Student 1	Student 2	Student 3
5 + 6 × 2	5 + 6 × 2	(5 + 6) × 2
11 · 2	5 + 12	11 · 2
22	17	22
Correct or incorrect?	Correct or incorrect?	Correct or incorrect?

Look at the expression below. Let's read over which operations need to be performed for each step in order to simplify each expression fully.

Expression: $5-6\div 2$

Evaluation	Operation

= 5 - 3	Division: First we divide 6 by 2 to get 3. So, 5 - 3.
= 2	Subtraction: Then we subtract 3 from 5 to give 2.

4. Look at the expressions below. Which operations were performed to result in each resulting expression?

A. **Expression**: $70 \div 7 \times 7$

Evaluation	**Operation**
= 10 × 7	
= 70	

B. **Expression**: 2(11 - 4)

Evaluation	**Operation**
= 2(7)	
= 14	

C. **Expression**: $\frac{|-5|}{10-5}$

Evaluation	**Operation**
$=\frac{5}{5}$	
= 1	

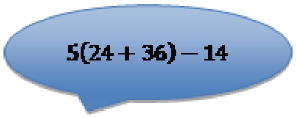
D. **Expression**: $\frac{36-3\times 4}{35-9\times 3}$

Evaluation	**Operation**
$=\frac{36-12}{35-27}$	
$=\frac{24}{8}$	
= 3	

NEXT STEPS

Carlos’ Thoughts

5. Carlos, shown below, is thinking about a problem situation he has. We have no idea what it could be about! Your job is to create a real-world situation that could represent the expression in his head. Be creative. You could use food, money, books, pets, … , in the problem situation Carlos is considering. After you write your real-world situation, make sure you answer the question in context.



TRY THESE

Practice

In case you did not review this in previous lessons, please review the following. You can also view https://www.youtube.com/watch?v=dAgfnK528RA to review more concepts with order of operations. Remember, it's all about agreeing on the final answer!

6. Use order of operations to simplify the following expressions.

A. 18 - [4(6 - 3) - 2]

B. 3 + 6(5 + 4) ÷ 3 - 7

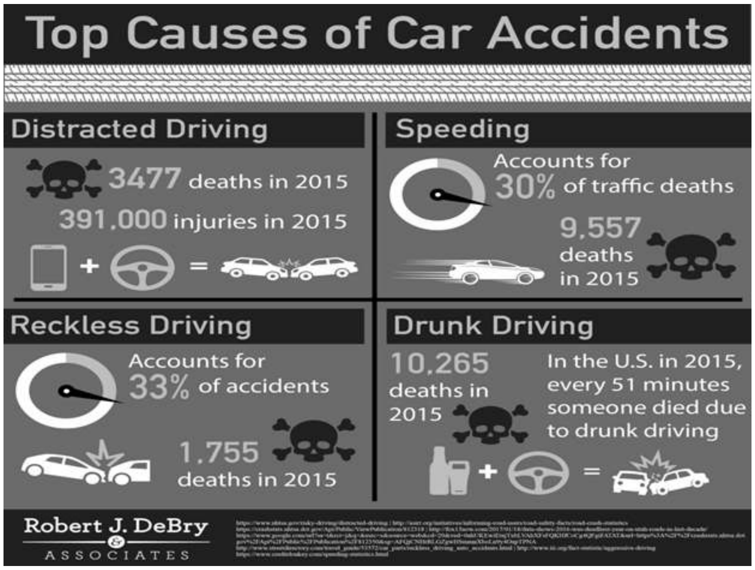
C. 217 ÷ (7 + 3 × 8) - 5

D. 2[9 - 5 ÷ (8 - 3)] + 6

E. $\frac{36-6}{12+3} + |-7|$

FURTHER APPLICATIONS

According to the Center for Disease Control and Prevention, accidents (unintentional injuries) are the fourth leading cause of death in the United States.[9] The top cause of unintentional deaths are falls, poisoning, and automobile accidents.



The infographic above[10] gives the top causes of car accidents for 2015. Use the infographic to answer each of the following questions.

7. If there were a total of 35,092 deaths caused by automobile accidents in 2015, how many were caused by reasons other than distracted driving, speeding, reckless driving, or drunk driving?

8. In 2015, speeding accounted for 30% of traffic deaths, which means if 100 people die from traffic deaths, 30 of them were caused by speeding. If 3,000 people died from traffic accidents, how many would we expect to have died from speeding?

9. A In 2015, if a person died from drunk driving every 51 minutes, about how many died in one day? Round your answer to the nearest person.

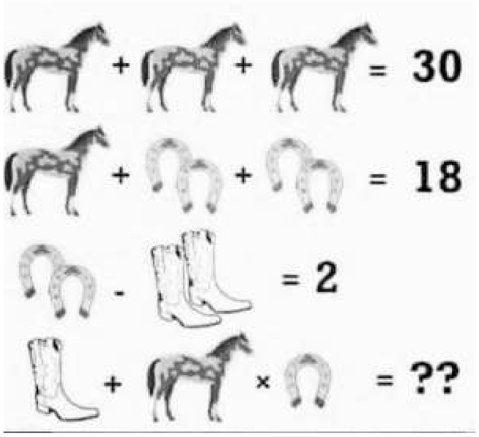
B. Using the answer from part A, about how many died in July? (**Hint**: there are 31 days in July).

10. For a group of 10 people, a caterer charges a flat fee of $250 for food and beverages as well as for set up and clean up. The caterer also charges $30 for each hour of serving food. If x represents the number of hours serving food, write an algebraic expression that will determine her total charge.

11. Using the correct expression from Question 10, what is the total amount charged if the caterer serves food for 2 hours?

Questions: Order of Operations–Multistep Calculations

1. Solve the last equation by carefully using the given equations.



2. Amanda buys 2 shirts which cost $15 each. She also buys a pair of jeans that are normally $42, but have a $3 discount this week. Write a numerical expression and determine the total amount of her purchase. (**Note**: Assume tax is already included).

(i) 2(15) + 42 - 3 = $69

(ii) 2(15) + 42 + 3 = $75

(iii) 2 + 12 + 42 - 3 = $56

3. Jason has $955 dollars in his bank account. It is holiday time, so he is buying each of his four nephews a ten-dollar iTunes gift card. He also plans on spending $50 for a gift for his mom and $50 for a gift for his dad. He knows he will get $75 in cash from his grandparents, which he will deposit into his account. If he does not spend anything more, how much will he have left in his account?

4. If Jason now has $890 in his bank account, how many months will this last if he needs $120 each month for gas and entertainment?

Simplify each expression fully.

5. 7(10 ÷ 2 + 3) - 4

6. 9 ÷ (4 - 1) × (2 + 3)

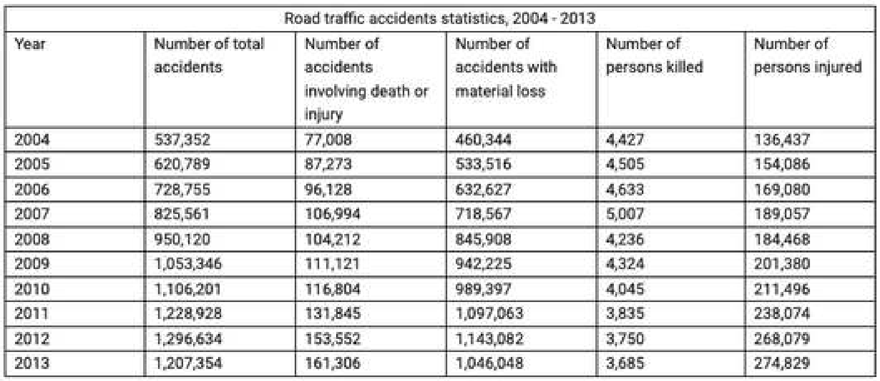
7. 3(4 + 8 ÷ 2 - 6)

8. [(10 - 6 + 5) ÷ 9) + 2

9. $2(\frac{24-6}{3+6})$

10. | - 6| + 8(15 - 2) ÷ 4

The chart below shows road traffic accident statistics from 2004 – 2013 in Turkey.[11]



Road traffic accidents statistics, 2004 - 2013

Year	Number of total accidents	Number of accidents involving death or injury	Number of accidents with material loss	Number of persons killed	Number of persons injured
2004	537,352	77,008	460,344	4,427	136,437
2005	620,789	87,273	533,516	4,505	154,086
2006	728,755	96,128	632,627	4,633	169,080
2007	825,561	106,994	718,567	5,007	189,057
2008	950,120	104,212	845,908	4,236	184,468
2009	1,053,346	111,121	942,225	4,324	201,380
2010	1,106,201	116,804	989,397	4,045	211,496
2011	1,228,928	131,845	1,097,063	3,835	238,074
2012	1,296,634	153,552	1,143,082	3,750	268,079
2013	1,207,354	161,306	1,046,048	3,685	274,829

11. Using the table above, how many road traffic accidents involved death or injury from 2004 to 2013?

12. Using the table above, how many road traffic accidents involved death or injury for all the years from 2004 to 2013 excluding every odd year?

13. Using the table above, determine the **change** in the number of **people** killed **or** injured from 2004 to 2013.

14. Some state penalties for texting while driving are described below.[12] The most expensive penalty is in Alaska, where a penalty costs $10,000! The five other states in the continental part of the US with the greatest penalties for texting while driving are Oregon ($500), Utah ($750), Wisconsin ($400), Indiana ($500), and New Jersey ($400). If you got a ticket for texting while driving in each of these 5 states, what would be the total amount of money you would owe?

15. How much greater is the penalty for texting while driving in Alaska compared to the total penalties for Oregon, Utah, Wisconsin, Indiana, and New Jersey? Express this comparison in two different ways:

1) Alaska's penalty is $__________ greater than the five other states combined.

2) Alaska's penalty is about __________ times greater than the five other states combined. Round to the nearest whole number.

---

[9] https://www.cdc.gov/nchs/fastats/accidental-injury.htm

[10] http://www.iihs.org/iihs/topics/t/general-statistics/fatalityfacts/overview-of-fatality-facts/2015

[11] http://www.turkstat.gov.tr/PreHaberBultenleri.do?id=18510

[12] http://handsfreeinfo.com/.

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5.5: Fundamentals 5 - Place value

LEARNING GOALS

By the end of this lesson, you should understand

- how small or large a number is is determined by its place value.
- each digit in a number has its own place value.

By the end of this lesson, you should be able to

- determine the place value of the digits of whole numbers and decimal numbers.
- order numbers using place value.
- write numbers in word form.

FUNDAMENTALS OF THE LESSON

A digit is a single whole number used alone or in combinations to represent numbers. The single one-digit numerals and their combinations are the numerals of the system to which they belong. The decimal system (base 10) has 10 digits: 0 through 9.

Numbers have values. The number 5 is larger than 4, but 14 is larger than 5. Numbers can be whole, fractional (part of a whole), or a combination or the two. 14 is a whole number because it is not a fraction or decimal. Numbers such as $\frac{1}{2}$, 0.5, and 1.7 are not whole numbers because they represent a part of a whole – a fraction or a decimal.

TRY THESE 1

Comparing numbers

1 Mike and Sabrina are looking at their paychecks. Mike's take home pay is $1,578 and Sabrina's take home pay is $1,598. Which employee had a larger take home paycheck?

To determine how large or small a number is, we need to place each digit of the number into its place value. For large numbers, digits of a number are commonly clustered by commas in groups of three, which helps to identify the place values of the digits. Each group of three numbers is called a period. The period names such as ones, thousands, millions, etc., are listed in the chart below and assist with reading and writing numbers. When reading a number, start from the left of the number, saying a period of digits one group at a time, followed by the group name.

Whole Numbers

Period	**Trillions**			**Billions**			**Millions**			**Thousands**			**Ones**		
Number	Hundred Trillions	Ten Trillions	Trillions	Hundred Billions	Ten Billions	Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones

When we compare two numbers we line up their place values.

2. Let's explore how we can use the chart to help us determine place values.

A. To understand the value or size of the number 78429300175, first place commas to separate out the periods of the number.

B. Second, locate the digit in the hundred thousands place.

3. Write the number 3,422,710 in words. In order to do so, it is helpful to identify the periods which are separated by commas. **Hint**: Placing the digits in the chart may help you correctly read, and then write the number.

Whole numbers can be compared to determine which one is larger, as in problem 1. For larger numbers, to compare we need to line up the place values of each of the numbers. The number that has a digit in the left-most period is largest. If the numbers have the same number of place values, look at the first number, in the highest place value (on the left). Whichever number has the larger value in the highest place value is the largest. If that number is the same, continue looking at the next place values until you see one that is larger.

4. Compare 1,143,345 and 983,839. Explain why one whole number is larger than the other.

5. Compare 62,478 and 62,427. Explain why one whole number is larger than the other.

TRY THESE 2

Comparing decimal numbers

A decimal is a number that contains a decimal point. The decimal point separates the whole number and the fractional part of the number. The whole number is to the left of the decimal point while the fractional part is to the right. The place values of decimals are related to the place values of whole numbers. The whole number part (to the left of the decimal) has the same place value as before. The place values of the digits to the right of the decimal have their own place value.

6. Looking at the place value chart from earlier, what patterns do you see in the place value **names** of the decimals as compared to the **names** of the whole numbers?

To write the name of a decimal:

Step 1: Write the whole number, as earlier in the lesson.

Step 2: Write the word “and” to represent the decimal point.

Step 3: Write the decimal portion of the number as you would if it were a whole number but end with the name of the place value of the **last** digit.

7. Write the numbers 12.27 and 23,754.102 out using words. Use the above chart to help you, if needed.

FURTHER APPLICATIONS

Numbers in our world, big and small

8. According to the US debt clock[13], on July 27, 2018, the population of New York State was 19,742,866 people and the debt of this state was 356,265,275,579 dollars (and growing every second). Write both of these numbers out in words.

9. The largest creature to ever live on earth is the blue whale.[14] A female blue whale weighs about 180,000 kilograms, which is equivalent to about 396,832.072 pounds. Write both of these numbers out in words.

Questions: Place Value

Whole Numbers															
Period	**Trillions**			**Billions**			**Millions**			**Thousands**			**Ones**		
Number	Hundred Trillions	Ten Trillions	Trillions	Hundred Billions	Ten Billions	Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones

Name the place value of the 5 in each of the numbers below. It may be helpful to place commas in between periods for large numbers.

1. 325984110

2. 125971

3. 9473.0153

4. 15.6847

5. 0.05779036

6. How do we express 51400942 in words?

7. How do we express 3.25 in words?

8. How do we express 40257.278 in words?

Compare the following sets of two numbers using the phrases greater than or less than.

9. 9,402 and 9,204

10. 0.337 and 0.307

11. 10.042 and 10.204

12. 845,351,307 and 854,315,370

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[13] http://www.usdebtclock.org/state-debt-clocks/state-of-new-york-debt-clock.html

[14] http://us.whales.org/whales-and-dolphins/facts-about-blue-whales

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5.6: Fundamentals 6 - Rounding

LEARNING GOALS

By the end of this lesson, you should understand

- The purpose of rounding numbers.

By the end of this lesson, you should be able to

- Round whole numbers and decimals to a specified place value.

FUNDAMENTALS OF THE LESSON

Rounding is a key and necessary skill in mathematics, as calculations often result in decimal numbers that have many digits before and after the decimal point. To round effectively, you must consider the context (or purpose) of the calculation. This will help you identify the appropriate number of decimal places to round to.

1. Let’s begin by identifying digits within numbers that are in specific decimal places.

A. 547 Determine the digit in the hundreds place.

B. 476,542 Determine the digit in the thousands place.

C. 26,573,000,000 Determine the digit in the billions place.

D. 782,956,400 Determine the digit in the hundreds thousands place.

E. 372.84 Determine the digit in the tens place.

F. 17.435 Determine the digit in the tenths place.

G. 3.1459 Determine the digit in the hundredths place.

H. 0.03468 Determine the digit in the thousandths place.

2. Given the numbers below, identify the decimal place that each digit of the number is in.

A. The number 2,154,900 is shown in the table below. Write the place values that correspond to each digit. For example, 9 is in the hundreds place, so write Hundreds into the box below the 9 digit.

2	**1**	**5**	**4**	**9**	**0**	**0**	**.**
							Decimal point

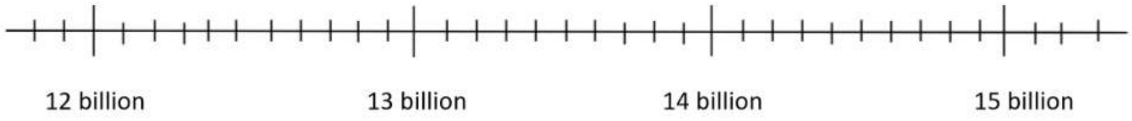
B. The number 76.5438 is shown in the table below. Write the place values that correspond to each digit.

7	**6**	**.**	**5**	**4**	**3**
		Decimal point			

Rounding with the Number Line

Brianna and Rosa live in New York State and are thinking about moving to Ohio. In looking at the two states spending on education, Brianna found that New York was planning to spend $13.1 billion in 2018. Rosa found that Ohio is planning to spend $10,682,738,002. They found it difficult to compare those two numbers so they decided to round them both to the nearest billion.

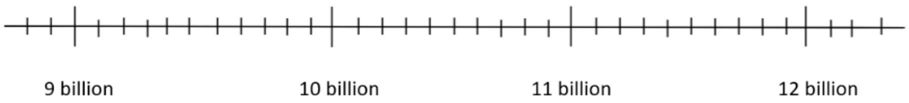
3. A. Place 13.1 billion on the number line below:



B. Which billion marker is 13.1 billion nearest to?

C. We can now say that 13.1 billion rounded to the nearest billion is ____________ billion.

D. Now place $10,682,738,002 on the number line below:


9 billion
10 billion
11 billion
12 billion

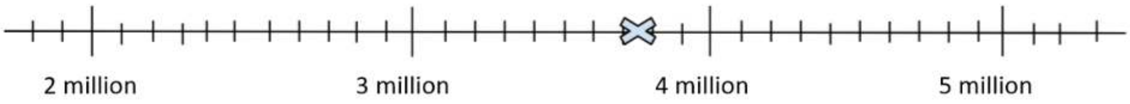
E. Which billion marker is $10,682,738,002 nearest to?

F. We can now say that $10,682,738,002 rounded to the nearest billion is ____________ billion.

G. Now compare the two states’ spending on education, using your prior answers.

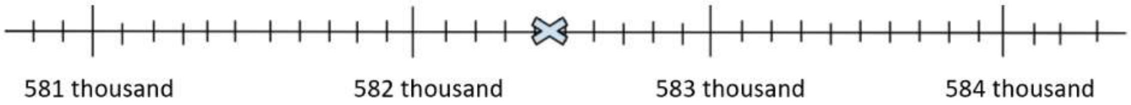
The number line is helpful for rounding numbers. However, it would be useful to have a way to round without having to draw the line. Complete the following examples and then come up with a rule that you can use to round without using the number line.

4. A. Round 3,721,900 to the nearest million. The location of 3,271,900 is displayed on the number line below.


2 million
3 million
4 million
5 million

3,721,900 rounded to the nearest million is ___________________________________

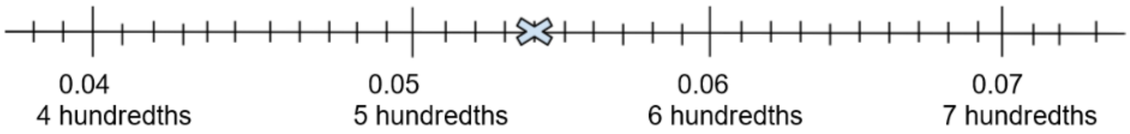
B. Round 582,478 to the nearest thousand. The location of 582,478 is displayed on the number line below.


581 thousand
582 thousand
583 thousand
584 thousand

582,478 rounded to the nearest thousand is ___________________________________

C. What is a rule (strategy) for rounding a number to a specific decimal place that does not involve using the number line? Check that the strategy works for questions 4A and 4B.

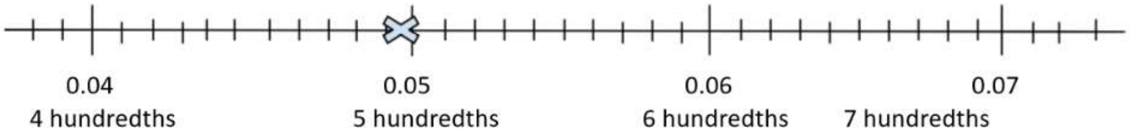
5. Let’s see if the rule you identified works for rounding to places less than one (tenths, hundredths, etc). Jorge is going to Mexico and uses a website to find out that 1 Mexican peso = 0.0548521 US dollars. He wants to round that to the hundredths place (cents).


0.04
4 hundredths
0.05
5 hundredths
0.06
6 hundredths
0.07
7 hundredths

A. Using the number line, which number, rounded to the nearest hundredth, is 0.0548521 nearest to? ________________________

B. 0.0548521 rounded to the nearest hundredths is ___________________________

6. Jorge’s friend suggests an additional trip into Belize. Jorge finds that 1 Belizean dollar = 0.0496315 US dollars. Jorge uses the number line to round 0.0496315 to the hundredths place:


0.04
4 hundredths
0.05
5 hundredths
0.06
6 hundredths
0.07
7 hundredths

A. Using the number line, which number, rounded to the nearest hundredth, is 0.0496315 nearest to? ________________________

B. 0.0496315 rounded to the nearest hundredths is __________________________

C. Does your rule from Question 4 work with decimal places also? Do you need to change the way you worded it? If so, write the new rule below.

7. Use the general rule of rounding to round the numbers below.

A. Round 23,495,000 to the nearest million: _______________________

B. Round 5,890 to the nearest thousand: ___________________________

C. Round 5.728 to the nearest hundredth (this is also called "rounding to two decimal places") ______________________________

D. Round 0.7236 to the nearest tenth (this is also called "rounding to one decimal place") _________________________________

E. Round 5.6666666 to one decimal place: _______________________________

F. Round 7.3449 to two decimal places: __________________________________

FURTHER APPLICATIONS

When performing calculations, you sometimes have to determine on your own the number of decimal places to round to. To do so, you can use the situation to help you make this decision. For example, if you are determining the number of people you want to invite to a party, the number you get should be a whole number. It would not make sense to invite 6.73 people to a party. Sometimes you can round a solution to a problem based on how the numbers that you were given in the problem were rounded.

8. For each problem below, think about the situation, and then round the numbers as you deem appropriate.

A. Maya is selling T-shirts with her own designs. She buys 70 plain T-shirts for $400. How much did each one cost her?

B. Remember that Ohio was planning to spend about $11 billion ($11,000,000,000) on education. Ohio has 1,729,916 children in its public schools. About how much are they planning to spend on each child? Round to the nearest dollar.

C. A school district has 5,942 students. The district's budget for school lunches is $16,950. How much money are they spending per student?

9. Julius is trying to find out which is smaller: $\frac{1}{13}$ or $\frac{2}{27}$. One way to do this is to change both fractions to decimals.Use your calculator to divide the numerator (top number) by the denominator (bottom number). Do this for both fractions. Which fraction, $\frac{1}{13}$ or $\frac{2}{27}$, is smaller?

Questions: Rounding

1. Round 78,381,818 to the nearest million.

2. Round 650,414 to the nearest hundred thousand.

3. Round 46.276 to the nearest tenth.

4. Round 1.333333333 to one decimal place.

5. Round 0.03626 to two decimal places.

6. Round 51.812% to the nearest tenth of a percent.

7. Round 19.59% to the nearest percent.

8. In a recent year, Americans owed approximately $1,510,290,000,000 in student loan debt, spread out among about 44 million borrowers. Round $1,510,290,000,000 to the nearest hundred billion.

9. Many college students graduate college with student loan debt. One student owed $32,056 in student loan debt. Round $32,056 to the nearest thousand.

10. Sarah's student loan debt increased by 5.3106% from 2016 to 2017. Round 5.3106% to the nearest tenth of a percent.

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5.7: Fundamentals 7 - Large Numbers and Powers of Ten

LEARNING GOALS

By the end of this lesson, you should understand

- the meaning of powers of ten.
- the purpose of scientific notation.

By the end of this lesson, you should be able to

- convert powers of ten to numbers and vice versa.
- write numbers in scientific notation and vice versa.

FUNDAMENTALS OF THE LESSON

Many real-world situations involve the use of very large numbers. For example, the distances between planets and stars in our solar system, the amount of data used by people in one day, and the number of credit card transactions made each day are examples of extremely large numbers.

To understand and use very large numbers, we need to understand the following mathematical concepts:

- The place values of digits in numbers
- How to round numbers to specific place values
- How to express numbers as powers of 10

1. Name the following numbers (using words only):

A. 1,000 ____

B. 1,000,000,000 ______________________________________

C. 10,000,000 ____

2. Write the following as numbers. **Use commas to separate the digits** in the hundred, thousands, millions, and billions decimal places. For example, one billion would be written as 1,000,000,000. Numbers written in this form are in the standard notation:

A. One million ____

B. One hundred thousand __________________________________

C. One trillion ____

D. Ten billion ____

3. Write the following as numbers. **Use commas to separate the digits** in the hundred, thousands, millions, and billions decimal places.

A. 3 thousand ____

B. 4 billion ____

C. 50 million ____

D. 700 trillion ____

4. Write the following as a number/word combination (like the numbers in question 3 above). For example, “8,000” should be written as “8 thousand.”

A. 5,000,000 ____

B. 7,000,000,000,000 ____

C. 80,000,000,000 ____

D. 400,000 ____

Power of Ten: Using Powers of 10

When we perform calculations involving very large numbers, to avoid making calculation errors, we can write the numbers as powers of 10. Look at the following examples involving powers of 10.

- 10^2 means 10 times 10 (two tens are multiplied together). So 10^2 = 10 x 10 = 100 (one hundred).
- 10^3 means 10 times 10 times 10 (three tens multiplied together). So 10^3 = 10 x 10 x 10 = 1000 (one thousand).
- 10^5 = 10 x 10 x 10 x 10 x 10 = 100,000 (one hundred thousand).

5. For parts A and B below, write the appropriate power of 10, and write the correct number in words, to create an accurate mathematical statement.

A. 10^4 = 10 x ___________________ = ________ In words: ______________________________

B. 10^7 = 10 x ___________________ = ________ In words: ________________________________

For parts C and D below, write the appropriate number, and write the correct number in words, to create an accurate mathematical statement.

C. 10^6 = ________________________________ In words:_______________________________

D. 10^{12} = ________________________________ In words:_______________________________

6. Now let's practice the reverse process, and write large numbers as powers of ten.

A. 10,000,000 = ____

B. 1,000,000,000,000 = ____

7. Write the following as powers of ten. For example, one thousand = 10^3:

A. One million =

B. One billion =

C. One trillion =

D. What do you notice about the powers of ten for these numbers?

Because we use a decimal number system, multiplying by powers of ten can be easy. The decimal number system is a base-10 system, so every decimal number can be expressed as a multiple of 10 to a power.

8. Write the answers to the following multiplication problems. Use your calculator if necessary. Then write a rule for multiplying by powers of ten. Use commas to separate groups of three digits.

1. 6 x 10^3 = ____
2. 75 x 10^5 = ____
3. 300 x 10^4 = ____

D. Create a rule for multiplying by a power of ten:

9. Let's see if the rule you just created works when the number you are multiplying is not a whole number. Multiply in the expressions below. **Use commas to separate groups of three digits**.

A. 6.7 x 10^3= ____

B. 77.32 x 10^5 = ____

C. 321.8 x 10^4 = ____

D. Based on these example, create a new rule for multiplying a number by a power of ten:

Scientific Notation: Distances in Our Galaxy

Distances between objects in our galaxy are very large. For example, the sun is 94,000,000 miles from the earth. The nearest star to the sun, Alpha Centauri, is 26,000,000,000,000 miles away. We can use **scientific notation** to write these numbers.

In **scientific notation**, numbers are written as a product of a number between 1 and 10 and a power of ten:

$$a \times 10^n$$

a is a number greater than or equal to 1 but less than 10. It can be an integer or a fraction. n is an integer (a positive or negative whole number).

Here are some examples:

7,000,000,000 is written as 7 x 10^9

26,730,000 is written as 2.673 x 10^7

We can convert from standard form to scientific notation by counting the number of decimal places between the decimal point and the number between 1 and 10, not including 10. This is shown below.

7000000000 has 9 decimal places between the decimal point and the number 7.0. 26730000 has 7 decimal places between the decimal point and the number 2.637.

10. Write the following numbers in standard notation. Standard notation is the form we typically use to write numbers. **Use commas to separate groups of three digits**.

A. 7.5 x 10^3 = ____

B. 8.92 x 10^5 = ____

11. Write the numbers below in scientific notation. Remember that the first number must be greater than or equal to 1. Enter in a product of the form: $a \times 10^n$.

A. The sun is 94,000,000 miles from the earth:

94,000,000 = ____

B. Alpha Centauri is 26,000,000,000 miles away:

26,000,000,000 = ____

12. One of the reasons for writing large numbers in scientific notation is that it makes it easier to see which is bigger (or which is smaller). Mars is 141600000 miles from our sun, and Venus is 67200000 miles from our sun. Write these numbers in scientific notation:

A. Distance from Mars to our sun: ____ miles

B. Distance from Venus to our sun: _______________________________________ miles

C. Which is farther away from our sun? How can you tell from the scientific notation?

FURTHER APPLICATIONS

Writing Large Numbers in Different Forms

Jupiter, on average, is 483.6 million miles from our sun. This number could be written in the following ways. All these forms are useful in different situations.

Number	**Form**
483,600,000	Standard Notation
483.6 million	Number/Word Combination
483.6×10^6	Multiple of a Power of 10
4.836×10^8	Scientific Notation

13. The U.S. National Debt as of 8/22/2020 was reaching $26,670,000,000,000. Write this number in scientific notation.

14. Alpha Centauri, on average, is 26 trillion miles from our sun. Write this number in Standard Notation, Multiple of a Power of 10, and Scientific Notation.

15. Think of a situation in which each of the four forms might be best.

Questions: Large Numbers and Powers of Ten

1. Write in standard notation: 10^{10}

2. Write as a power of ten: 1,000,000

3. Write the numbers using words (e.g., 37,000 is “37 thousand”):

A. 36,000,000,000

B. 558,000,000

4. Write in standard notation:

A. 9×10^6

B. $92.31 \text{ x } 10^3$

5. Write in scientific notation: 85,950,000

6. Write in standard notation: 1.59×10^7

7. The average distance from Saturn to the sun is 886,700,000 miles. The average distance from Neptune to the sun is 2,794,000,000 miles. Write each number in scientific notation, and then compare. Which number is bigger, and why?

8. A student was trying to write 33 billion in scientific notation. She wrote $33 \text{ x } 10^9$. Why is this number not in scientific notation? What is the number in scientific notation?

9. In July 2018 , Lebron James signed a 154 million dollar contract with the Los Angeles Lakers. The contract is for 4 years. Assuming Lebron plays 82 regular season games each year, what is Lebron earning per regular season game? Assume all the regular season games are played. Write the large number in standard notation, rounded to the nearest dollar.

10. Mars is extremely far from the Earth. Scientists estimate that it could take astronauts 247 days to travel from Earth to Mars. This is more than half of a year! Suppose astronauts travel to Mars when the distance between Earth and Mars is 108 million miles. If the trip takes 247 days, on average, how many miles would be traveled each day? Write the large number in standard notation, rounded to the nearest mile.

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5.8: Fundamentals 8 - Multiples and Factors

LEARNING GOALS

By the end of this lesson, you should understand that

- if one number is a multiple of a second number, then the second number is a factor of the first,
- two numbers always have a least common multiple (LCM),
- there are relatively easy to use divisibility criteria for some divisors like 2, 5, 10, 100, 3, and 9 but others.

By the end of this lesson, you should be able to

- find the least common multiple (LCM) of two numbers by listing their multiples,
- recognize multiples of 2, 5, 10, 100, 3 and 9.

TRY THESE 1

Adding The Same Number Multiple Times

1. Jerry set his alarm for 7 AM with a 5-minute snooze button. He kept pressing the snooze button every time the alarm rang. He finally got up as late as he could **before** 8 AM.

A. How many minutes after 7 AM did the last alarm ring?

B. How many times did the alarm ring?

2. Mariana manages a hotel. She is in charge of ordering sets of tables with chairs for the dining hall. Each set contains 4 chairs.

A. How many chairs will the hall have if she buys 16 sets?

B. How many if she buys 25 sets?

C. How many sets should Mariana buy if she needs to accommodate 250 guests?

3. Ruchi is baking several wedding cakes. She needs a total of 100 eggs. She buys eggs that are sold by the dozen.

A. Will 5 dozen be enough?

B. Does she need to buy 10 dozen?

C. How many dozen does she need to buy?

A number is called a **multiple** of another number, if the first number can be written as a repeated addition of the second number. In this case, the first number can be divided by the second number without remainder. For example, the number 6 can be written as 2 + 2 + 2, a repeated addition of 2, and as 3 + 3, a repeated addition of 3. Therefore, 6 is a multiple of 2 and a multiple of 3. We also write 6 = 3 x 2 = 2 x 3. Because the number 6 can be divided by 2 without remainder, we say that 2 is a **factor** or **divisor of** 6. We also say that 6 is **divisible by** 2. The number 3 is also a factor or divisor of 6. In the first example, the number of minutes past 7 AM is a multiple of 5. And 5 is a factor of the number of minutes past 7 AM.

4. Jamal serves in his community's Soup Kitchen center. The center organizes dinners every Saturday. Jamal is also a member of a group of runners who get together to run every three days at 7 PM. In 2017, January 1st was a Sunday and the runners group had trained the day before, December 31, 2016. On what date in 2017 did Jamal have his first conflict between his running group and his Soup Kitchen commitment? In other words, what was the first date in 2017 that his running group and his Soup Kitchen dinner happened at the same time?

The day of the year that you found as the answer to Question 4 is the smallest number divisible by both 3 and 7. This number is called the **least common multiple** (LCM) of 3 and 7.

5. Find the least common multiple of 6 and 18.
6. Find the least common multiple of 20 and 24.

TRY THESE 2

Multiples of 2, 5, and 10

7. Write two numbers that are multiples of 2 but end in different digits.
8. Write two numbers that are multiples of 5 but end in different digits.

9. Complete the following sentences:

A. A number is divisible by 2 if and only if:

B. A number is divisible by 5 if and only if:

C. A number is divisible by 10 if and only if:

D. A number is divisible by 100 if and only if:

TRY THESE 3

Multiples of 3 and 9

Rita was experimenting with the number 85. She wanted to know if 85 could be divided by 9 without carrying out the division. She did the following:

$$85 = 8 \times 10 + 5 = 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 5 = 8 \times 9 + 8 + 5$$

Rita noticed that 85 would be a multiple of 9 if 8 + 5 could be written as 9 times a number - and only if this were true. She also noticed that 8 + 5 is the sum of the digits of the number 85. It turns out that:

- A number is divisible by 9 if and only if the sum of its digits is a number that is divisible by 9.

In the above example, 8 + 5 = 13 and 13 is not divisible by 9. You can even use the rule to show this since 1 + 3 = 4 and 4 is not divisible by 9. A number such as 27, however, is divisible by 9 since 2 + 7 = 9 and 9 is divisible by 9.

Rita then wrote the following:

$$85 = 8 \times 9 + 8 + 5 = 8 \times 3 \times 3 + 8 + 5$$

This calculation lead her to:

- A number is divisible by 3 if and only if the sum of its digits is a number that is divisible by 3.

Again, in the above example, 8 + 5 = 13 and 13 is not divisible by 3. You can even use the rule to show this since 1 + 3 = 4 and 4 is not divisible by 3 either. A number such as 27, however, is divisible by 3 since 2 + 7 = 9 and 9 is divisible by 3.

10. Are the following numbers divisible by 3? Are they divisible by 9? Use the previously stated criterion to check.

A. 675183

B. 44445555

C. 99999999999913

D. 87655678

Questions: Multiples and Factors

Find the least common multiple of the following pairs of numbers:

1. 5 and 3

2. 4 and 7

3. 10 and 15

4. 6 and 14

5. 20 and 4

6. 7 and 42

7. Is the following statement True or False? "The number 98888835 is not divisible by 2 but is divisible by 5."

8. Is the following statement True or False? "The number 135490 is odd and not divisible by 5.”

9. Is the following statement True or False? "The number 5433372 is divisible by 9."

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5.9: Fundamentals 9 - Prime Factorization, GCF, and LCM

LEARNING GOALS

By the end of this lesson, you should understand that

- it is possible to list all of the factors of a number.
- a number can always be written as a product of primes (prime factorization).
- two numbers always have a least common multiple (LCM) and a greatest common factor (GCF).

By the end of this lesson, you should be able to

- find all the factors of a number.
- write the prime factorization of a number.
- find the greatest common factor (GCF) and least common multiple (LCM) of two numbers using the prime factorization.

FUNDAMENTALS OF THE LESSON

A number is called a **multiple** of another number, if the first number can be written as a repeated addition of the second number. In this case, the first number can be divided by the second number without remainder. For example, the number 6 can be written as 2 + 2 + 2, a repeated addition of 2, and as 3 + 3, a repeated addition of 3. Therefore, 6 is a multiple of 2 and a multiple of 3. We also write 6 = 3 x 2 = 2 x 3. Because the number 6 can be divided by 2 without remainder, we say that 2 is a **factor** or **divisor of** 6. We also say that 6 is **divisible by** 2. The number 3 is also a factor or divisor of 6.

1. List all of the factors of 12.
2. List all of the factors of 15.
3. List all of the factors of 29.
4. Which is **not** a factor of 100?

   (i) 1

   (ii) 10

   (iii) 50

   (iv) 25

   (v) 20

   (vi) 30

TRY THESE 1

School groups (factors, listing all of the factors of a number)

5. There are 60 first graders in Forest Hills elementary school. The principal wants to divide them into different groups of the same size. If no students are left out of a group, what is a possible size of the groups in this situation? For example, one answer could be "1," where all students are in a group of their own. What could be another group size?
6. Mr. Jones and Mrs. Sandoval teach elementary school. There are 20 students in Mr Jones’ class and 24 in Mrs. Sandoval’s class. Each teacher wants to create groups of students in their classes. They would like to have the same size groups in both classes and they would like the groups to be as large as possible. How many students should the teachers put into each group?
7. Mrs. Singh has 23 students in her class. She wants to divide her students into groups of the same size. What size groups can she make?

The answer you obtained in question 5 is the list of all divisors (factors) of the number 60. The answer you got in question 6 is the greatest number that divides both 20 and 24 without remainder. It is called the **greatest common factor (GCF)** of 20 and 24. A **prime number** is a number that is only divisible (without remainder) by 1 and itself. As you found out in question 7, the number 23 is a prime number.

8. List all the prime numbers between 1 and 50.

TRY THESE 2

Writing as a product (multiplication)

Mel was given the task of writing the number 6 as a product (multiplication). She wrote 6 = 2 x 3. **Factoring a number** means writing the number as a product. Mel factored the number 6. She was then asked to factor the number 24 as much as possible. She wrote:

$$24 = 4 \times 6 = 2 \times 2 \times 6 = 2 \times 2 \times 2 \times 3.$$

Mel realized that the numbers or **factors** involved in the final multiplication were all prime. She couldn't factor them anymore without writing a 1. Mel's final product is called the **prime factorization** of 24.

9. Write the prime factorization of the following numbers (Divide the work with the members of your group):

A. 27

B. 60

C. 98

TRY THESE 3

Using Prime Factorization to Find the GCF of Two Numbers

10. Reread question 6 and answer the questions below.

A. Write the prime factorization of 20 and 24.

B. Write the prime factorization of your answer to Question 6.

C. What do you notice in the answers to 10A and 10B?

D. Now write the prime factorization of the numbers 105 and 140 and find their greatest common factor.

TRY THESE 4

Using the Prime Factorization to Find the LCM of Two Numbers

11. If a number is divisible by 6, should this number be divisible by 3? Explain.
12. Should the least common multiple of two numbers be divisible by all the factors of both numbers?

FURTHER APPLICATIONS

An **exponent** indicates how many times a number is a multiple. For example, when you see 2^4, 4 is the exponent. This can be expanded to read 2 x 2 x 2 x 2.

13. A. Calculate $2^4 \times 3 \times 7$

B. Calculate $2^2 \times 3^3 \times 5$

C. Use the prime factorizations from 13A and B to find the greatest common factor of these two numbers.

14. Use the conclusion you made in question 13 to find the least common multiple of the numbers $2^4 \times 3 \times 7$ and $2^2 \times 3^3 \times 5$.

Questions: Prime Factorization, GCF, and LCM

Find the prime factorizations of the following numbers:

1. 28
2. 625
3. 100
4. 13

Find the GCF and the LCM of the following pairs of numbers (you can leave your answers in factored form):

5. 2 x 5 x 11 and 5 x 7
6. $2^3 \times 3$ and 2×3^2
7. 3 and 4
8. 4 and 6

9. 5 and 7
10. 9 and 45
11. 12 and 18
12. 50 and 30

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5.10: Fundamentals 10 - Fractions–Basics

LEARNING GOALS

By the end of this lesson, you should understand

- Fraction terminology: numerator, denominator, fraction bar
- The meaning of a fraction

By the end of this lesson, you should be able to

- Convert from division calculation to a fraction, and vice versa
- Estimate with unit fractions
- Determine if two fractions are equivalent
- Simplify a fraction

FUNDAMENTALS OF THE LESSON

A division occurs when one number is divided by another. We can represent division using a fraction. For example, ten divided by five can be written as $\frac{10}{5}$. Ten is called the numerator and five is called the denominator. The horizontal line is called the fraction bar.

1. Identify the numerator or denominator in the following fractions:

A. $\frac{3}{4}$ Denominator________________

B. $\frac{5}{6}$ Numerator __________________

C. $\frac{4}{12}$ Denominator_________________ Numerator _____________

2. Write the following division calculations as a fraction:

A. 12 divided by 7

B. Six divided by two

C. 27 divided by 15

3. Write the following fractions out in words:

A. $\frac{9}{3}$ ____

B. $\frac{4}{8}$ ____

C. $\frac{4}{5}$ ____

Fractions are of two types: proper and improper. A fraction is called **proper** if its numerator is less than its denominator. Proper fractions have values less than 1. A fraction is **improper** if its numerator is greater than or equal to its denominator. These fractions are often written as **mixed numbers**. A mixed number contains an integer and a proper fraction. For example, 3/2 can be written as $1\,\frac{1}{2}$, which means $1 + \frac{1}{2}$.

Improper fractions can be converted to mixed numbers by dividing the numerator by the denominator.

Example: Convert 11/5 to a mixed number.

Solution: Divide 11 by 5 using long division. The fraction 11/5 represents the division: $11 \div 5$.

Divide 11 by 5 using long division. 5 is the divisor. 11 is the dividend. 2 is the quotient. and 1 is the remainder.

The resulting mixed number is $2\,\frac{1}{5}$. In the mixed number, the quotient (2) is the whole number, the remainder (1) is the numerator in the fraction, and the divisor (5) is the denominator in the fraction.

4 Write the following improper fractions as a mixed number:

A. 4/3 _____________________

B. 12/7 ______________________

C. 19/10 ____________________

Division Difficulty

Rosa works in an elementary school. During her work day, Rosa had to do several calculations. She had to divide six homework problems into two sets, 24 cupcakes between 12 children, and assign 12 children into groups of four. She had no problem with any of those calculations. The next day she had four pizzas to divide amongst 12 children. She found that division to be more challenging.

5. Can you explain why the pizza problem seems more difficult?

6. Write Rosa's problems as division problems (using the fraction bar) and solve them. The first division is shown below:

Six problems divided by two: $\frac{6}{2}$, so 3 problems in each set.

A. 24 cupcakes divided between 12 children:

B. 12 children assigned to groups of four:

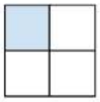
C. Four pizzas divided by 12:

NEXT STEPS

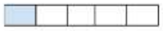
Unit Fractions

Joe read that one-fifth of all Americans live in rural areas. He wanted to estimate how many people in his home state of South Carolina live in rural areas. One fifth is an example of a unit fraction. Examples of unit fractions are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, where the numerator is one.

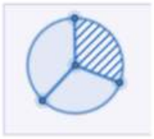
7. Here are some pictures to illustrate unit fractions. Write the fraction underneath. The first one is done for you.



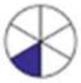
$\frac{1}{4}$



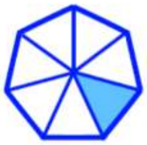
A____________



B____________



C________



D________________

Unit fractions can be used to determine a portion of an amount. To find a unit fraction of a number, we divide. For example, ½ of an amount is the amount divided by 2. Half of $800, is $800/2, which is $400. To find the unit fraction of a number, we divide the number by the denominator of the unit fraction.

8. For each expression below, determine the corresponding division:

A. $\frac{1}{3}$ of a number = the number divided by ______

B. $\frac{1}{4}$ of a number = the number divided by ______

C. $\frac{1}{5}$ of a number = the number divided by ______

9. Find unit fractions of the numbers below:

A. $\frac{1}{3}$ of 15 = _______

B. $\frac{1}{4}$ of 28 = _______

C. $\frac{1}{5}$ of 45 = _______

In the previous examples, the fractions simplified to whole numbers. $\frac{1}{3}$ of 15 = $\frac{15}{3}$ = 5. Since 15 is a multiple of 5, when we divide 15 by 5, we get 3, a whole number. What about when the numbers do not divide exactly. For example, what is $\frac{1}{3}$ of 20? This is equal to 20 divided by 3. We can estimate this to be about 7, since $\frac{20}{3}$ is close to $\frac{21}{3}$, and $\frac{21}{3}$ equals 7.

10. For each expression below, determine the corresponding fraction, and estimate the value.

A. $\frac{1}{4}$ of 9: It's about _________________. Exact fraction:______

B. $\frac{1}{6}$ of 17: It's about _________________. Exact fraction:______

C. $\frac{1}{10}$ of 98: It's about _________________. Exact fraction:______

11. Let's return to the problem Joe was investigating. There are about 5 million people in South Carolina. $\frac{1}{5}$ of people in South Carolina live in a rural area.

A. About how many people in South Carolina live in rural areas?

B. Joe also read that in New York state, there are 19,800,000 people. What is one fifth of the people in New York State? First estimate, then calculate.

TRY THESE

Non-Unit Fractions

There are several ways to think of a fraction that does not have one as its numerator. Let's look at two thirds (2/3):

We can think of this fraction and represent it in two ways. As shown in image (1) below, we could divide a quantity into three equal parts, and take two of the three parts. Or, as shown in image (2) below, we could take two quantities and find one-third of each of them.

Rectangle with three equal parts. Select 2 of 3 parts. With two rectangles each made up of three equal parts, select 1 of 3 parts from 2 wholes.

Since the whole quantities are the same in both images, these images represent the same amount.

12. Draw similar diagrams to represent each fraction below. There may be more than one correct answer.

A. $\frac{2}{4}$

B. $\frac{3}{5}$

Finding a Fraction of an Amount

When we find a non-unit fraction of a number, we must multiply and divide. We can do so in either order. For example, to find of $\frac{2}{3}$ an amount, we can find $\frac{1}{3}$ of the amount, and then multiply by two. Or, we can multiply the amount by 2, and then find $\frac{1}{3}$ of the result.

Example: Find $\frac{2}{3}$ of 9 .

Solution: We can find this amount in two ways. Both methods are shown below.

Method 1: Find $\frac{1}{3}$ of 9 (divide 9 by 3), and then multiply by 2. This is shown below.

$$\frac{2}{3} \text{ of } 9 = \left(\frac{1}{3} \text{ of } 9\right) \text{ x } 2 = \frac{9}{3} \text{ x } 2 = 3 \text{ x } 2 = 6$$

Method 2: Multiply 9 by 2, and then find ⅓ of the result (divide by 3). This is shown below.

$$\frac{2}{3} \text{ of } 9 = (9 \text{ x } 2) \text{ x } \frac{1}{3} = 18 \text{ x } \frac{1}{3} = \frac{18}{3} = 6$$

13. Find the fractions of the following amounts. Use both methods introduced in the prior example:

A. $\frac{3}{4}$ of 16

B. $\frac{2}{5}$ of 25

C. $\frac{3}{10}$ of 100

NEXT STEPS

Equivalent Fractions

Jenna has 25 chairs in her classroom. Five of them need fixing. One of her students says that this means $\frac{5}{25}$ of the chairs need mending. Another student says $\frac{1}{5}$ of the chairs need mending. Who is right?

Diagram 1 (showing 5 of 25 chairs broken)	Diagram 2 (showing 5 groups of chairs with 1 of 5 broken)
5 by 5 grid. Top row of 5 squares is shaded.	Five columns of five squares. Top square in each column is shaded.

*The gray boxes represent broken chairs, and the white boxes represent unbroken chairs.

You can see that both students are right. $\frac{5}{25}$ and $\frac{1}{5}$ are **equivalent fractions**. Equivalent fractions have the same value even though they have different numerators and denominators.

Two fractions are equivalent if you can get from one to the other by multiplying or dividing the top and bottom of the fraction by the same number. For example, the fractions $\frac{1}{3}$ and $\frac{3}{9}$ are equivalent since you can multiply the top and bottom of $\frac{1}{3}$ by 3 to get $\frac{3}{9}$ or, you can divide the top and bottom of $\frac{3}{9}$ by 3 to get $\frac{1}{3}$.

Multiplying numerator and denominator of 1 over 3 by 3 gives 3 over 9.

14. Decide whether the following fractions are equivalent or not. If the fractions are equivalent, determine the number that you can multiply one of the fractions by to get the other fraction, or determine the number you can divide one of the fractions by to get the other fraction.

A. $\frac{3}{6}, \frac{1}{2}$

B. $\frac{3}{9}, \frac{2}{3}$

C. $\frac{16}{12}, \frac{4}{3}$

D. $\frac{12}{30}, \frac{2}{5}$

E. $\frac{24}{45}, \frac{3}{5}$

Simplifying Fractions

We can simplify a fraction by reducing it to lowest terms. This means that we find an equivalent fraction that cannot be reduced any further. To simplify a fraction, we divide the top and bottom by the same number.

FURTHER APPLICATIONS

15. Joe found out that there are only about one million people living in rural areas in New York State. He wanted to know what fraction that is of the New York State population, which is about 20 million. Complete the following sentences:

A. _____________ people in New York State live in rural areas out of _____________________ total people in New York State.

B. If we simplify the fraction, we find that __________ person out of every __________ people in New York lives in a rural area.

In the above problem, twenty million is called the **reference value**. It is the total number (of people in this case) in which you are interested. One million is called the **comparison value**. It is the number that you are comparing to your reference value.

If one person out of every twenty people in New York State lives in a rural area, then $\frac{1}{20}$ is the fraction of people in New York State who live in rural areas.

16. A. There are 25 students in Rosa's class. Ten of them are in the after school program. What fraction of Rosa's class is in the after school program? Simplify the fraction.

B. Eighteen teachers in the school are over thirty-five years of age. There are 30 teachers in the school. What fraction of the teachers in the school are over thirty-five years of age?

C. There are about 5,000 students in the school district. About 500 of them are below grade level in reading. What fraction of the students in the school district are below grade level in reading?

Questions: Fractions–Basics

1. Determine which pairs of fractions below are equivalent fractions.

A. $\frac{1}{4}, \frac{5}{28}$

B. $\frac{2}{6}, \frac{3}{9}$

C. $\frac{4}{12}, \frac{5}{21}$

D. $\frac{3}{4}, \frac{12}{16}$

2. Keisha has made 9 pizzas. She is having a party for 27 people.

A. Are there enough pizzas for at least one-half pizza for each person? Explain.

B. How many pizzas are there for each person?

3. Raul runs a pet shop. He has 18 gallons of salt water for some of his fish. If he divides it equally among his 45 tanks for these fish, how much water will each tank get?

4 Find the following:

A. $\frac{1}{12}$ of 108

B. $\frac{1}{10}$ of 30

C. $\frac{1}{7}$ of 1190

5. Estimate the following. For each expression, determine the whole number that is closest to the actual result:

A. $\frac{1}{6}$ of 31

B. $\frac{1}{8}$ of 23

C. $\frac{1}{50}$ of 346

6. Find the following:

A. $\frac{2}{6}$ of 42

B. $\frac{5}{9}$ of 54

C. $\frac{2}{100}$ of 800

7. There are 30 days in April. Last year it rained on $\frac{1}{2}$ of them. How many days did it rain?

8. There are 30 days in June. Last year it was sunny on $\frac{3}{5}$ of the days in June. How many days was it sunny?

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5.11: Fundamentals 11A - Fractions–Multiplying and Dividing

LEARNING GOALS

By the end of this lesson, you should understand

- multiplying and dividing fractions does not require a common denominator
- when you multiply a number by a fraction that is less than 1, the result is smaller than the original value

By the end of this lesson, you should be able to

- multiply fractions
- cancel common factors to simplify fractions before multiplying.
- divide fractions by multiplying by the reciprocal.

FUNDAMENTALS OF THE LESSON

A **fraction** is a quantity that is made up of two parts called the **numerator** and **denominator**. The **denominator** is the bottom number of the fraction and it represents how many equal parts your item was split into. The **numerator** is the top number of the fraction and it represents how many of those equal parts you are concerned with (for example, how many of the pieces someone ate, or how many of the pieces are colored in).

Sharing a Sweet Treat

Imagine that you are holding a chocolate bar in your hand. A friend walks up to you, so you decide to share the chocolate bar by splitting it into two equal parts. She walks away with her portion of the chocolate bar. Then, before you have eaten any of your portion, a second friend walks up to you. You split what you have left into two equal parts and give him one.

1. Draw a picture to show how the chocolate bar has been split among you and your two friends.

2. How can you describe how much of the original chocolate bar each of you ends up with? Can you use fractions in your descriptions? **Hint**: Use fractions to describe the amount of the bar that you have left and the amounts given to your friends.

3. When we multiply by a fraction (or a decimal or a percent), we can think of the word "of" as where the multiplication sign goes. For example, we can say that your second friend received $\frac{1}{2}$ of $\frac{1}{2}$ of the chocolate bar, since you gave him one half of your half. Now let's replace the word "of" with a multiplication sign. We will use the dot symbol for multiplication. Write $\frac{1}{2}$ of $\frac{1}{2}$ as a mathematical expression.

The first friend received $\frac{1}{2}$ of the chocolate bar. The second friend received $\frac{1}{4}$ of the chocolate bar, which is $\frac{1}{2}$ of $\frac{1}{2}$. To multiply $\frac{1}{2}$ by $\frac{1}{2}$, we simply multiply across the numerators, and multiply across the denominators. This is shown below.

1 over 2 times 1 over 2 = 1 times 1 over 2 times 2 = 1 over 4. We multiply the numerators. We multiply the denominators.

4. Let's consider a different scenario. You have a chocolate bar, and you give half of the bar to a friend. A second friend comes along. Before eating any of the remaining bar, you break it into three equal parts, and you give your second friend one of the pieces. What fraction of the original chocolate bar did you give to your second friend?

NEXT STEPS

Passing An Exam

The Pre-Algebra class at Whitmore Community College had an exam at the end of last week. $\frac{2}{3}$ of the students in the class decided to get extra tutoring help before the exam. $\frac{5}{6}$ of those students who got extra help passed the exam. Let's figure out what fraction of the entire class passed the class after receiving the extra tutoring.

5. The fraction of all students in the class who received extra tutoring is a fraction of a fraction. Fill in the blanks below.

______ of the class received extra tutoring. ______ of those students pass the class. So, ______ of

______ of the class are in this group.

6. Write the multiplication that represents this situation, and find the results. Explain the meaning of the results.

You might notice that the multiplied answer is not in its simplest form. We can find an **equivalent fraction** by multiplying or dividing the numerator and denominator by the same number. If we can divide the numerator and denominator by a common factor other than 1, then that will help us put the fraction in simplest form. A fraction is in simplest form, or lowest terms, when the only factor that the numerator and denominator have in common is 1.

7. A. What is the greatest factor that the numerator and denominator in the fraction $\frac{10}{18}$ have in common?

B. Divide the top and bottom of the fraction $\frac{10}{18}$ by the greatest common factor. What fraction do you get?

We also could have simplified our fractions **before** multiplying. Let's look back at the multiplication problem we were working with:

$$\frac{5}{6}\cdot\frac{2}{3}$$

Any time we are multiplying two fractions and we can find a common factor between any numerator and any denominator, we can cancel that factor from those two numbers before multiplying.

C. In this example, do you see a numerator and denominator that have a common factor? What is that common factor?

We can *cross cancel* by crossing out the 2 and 6, as shown below. When we cross cancel, we divide a number in the numerator and a number in the denominator by the same number. Below, both 2 and 6 are divided by 2, resulting in 1 and 3 respectively. Then we multiply.

5 over 6 times 2 over 3 = 5 over 9. The 2 and the 6 cross cancel by dividing each by 2 to give 5 over 3 times 1 over 3.

When we simplify before multiplying, we obtain the same product as when we multiply first and then simplify.

8 Multiply the following fractions. First cross cancel any common factors, then multiply.

A. $\frac{3}{10}\times\frac{5}{4}=$

B. $\frac{4}{9}\times\frac{5}{12}=$

C. $\frac{12}{13}\times\frac{1}{18}=$

FURTHER APPLICATIONS

Dividing fractions is very closely related to multiplying fractions! In fact, whenever we have a division problem, we will rewrite it in a certain way to turn it into a related multiplication problem.

9. Dan is scooping ice cream for the campers in his group at summer camp. He is not sure there will be enough for everyone. He knows that he has 8 cups of ice cream in the container, and each camper is receiving $\frac{1}{2}$ of a cup in his or her bowl. How many campers can he feed?

Remember that when we want to split something into equal parts, that is dividing. In this case we want to take the 8 cups and divide it into portions of $\frac{1}{2}$ of a cup and see how many there are.

A. Try to write this out as a division problem. (**Hint**: How can you write a whole number such as 8 as a fraction?)

We need to solve 8 divided by ½. To figure out how many times ½ goes into 8, we can first think about how many times ½ goes into other numbers.

- How many times does ½ go into 1? The result is 2. $1\div\frac{1}{2}=2$
- How many times does ½ go into 2? The result is 4. $2\div\frac{1}{2}=4$
- How many times does ½ go into 3? The result is 6. $3\div\frac{1}{2}=6$

Usually when we divide, we obtain a smaller number. However, when we divide by a proper fraction, we obtain a larger number.

B. What is $8\div\frac{1}{2}$?

We can divide by a fraction by changing the division problem into an equivalent multiplication problem. In order to do that, we must keep three steps in mind:

1. **Keep** the first number (divided) as it is.

2. **Change** the division sign to multiplication.

3. **Flip or invert** the fraction we are dividing by (divisor), so that it is upside-down (This is called the **reciprocal** of the original fraction).

In the example that we just solved, we can convert the division to multiplication as follows. Notice that the number 8 has been written as the fraction 8/1.

8 over 1 divided by 1 over 2 = 8 over 1 times 2 over 1 = 16 over 1 = 16. 8 over 1 is the dividend. 1 over 2 is the divisor.

C. Suppose Dan decides to scoop the ice cream into larger servings. He still has 8 cups of ice cream in the container, and now each camper will receive $\frac{2}{3}$ of a cup in his or her bowl. How many campers can he feed?

Questions: Fractions

Multiplying and Dividing

1. Write two expressions that are equivalent to the division: $7 \div \frac{1}{8}$

2. Write two expressions that are equivalent to the division: $\frac{35}{5} \div \frac{8}{6}$

3. Tanya makes $\frac{2}{4}$ of the foul shots she attempts during basketball practice. If she took 36 shots, how many did she make?

4. A pizza store offers 30 different topping choices. If $\frac{2}{5}$ of these choices are non-meat options, how many meat options does the store offer?

5. Janita brings $\frac{3}{5}$ of her birthday cake to school with her. She and her classmates eat $\frac{2}{3}$ of what she brought. What fraction of the original cake did they eat? Write the fraction in simplest form.

6. What fraction of the original cake will Janita be bringing home from school? **Hint**: Start with the amount that they did not eat, and then determine the total amount she will be bringing home.

7. According to https://www.catster.com/cat-food/how-much-should-i-feed-my-cat, a guideline for feeding is that an 8-lb cat should be given about $\frac{4}{5}$ cups of dry cat food per day. If you have a cat of this size and have a bag of cat food that contains 52 cups of food in it, how many days will that last your cat at this feeding rate?

8. Alex is making muffins and has $\frac{14}{6}$ sticks of butter. Each batch of muffins will take $\frac{2}{6}$ sticks of butter. How many batches of muffins can he make?

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5.12: Fundamentals 11B - Fractions–Adding and Subtracting

LEARNING GOALS

By the end of this lesson, you should understand

- fractions with the same denominator can be added (or subtracted) by adding (or subtracting) the numerators and keeping the common denominator
- in order to add (or subtract) fractions with different denominators, we must find a common denominator and convert the fractions to equivalent fractions with the common denominator

By the end of this lesson, you should be able to

- add and subtract any two fractions.

FUNDAMENTALS OF THE LESSON

Two fractions are said to be **equivalent** if they represent the same quantity. Equivalent fractions can be obtained by multiplying the numerator and denominator of a fraction by the same number. (**Note**: By doing so, we are essentially multiplying by "1.")

Example: $\frac{7}{4}$ and $\frac{35}{20}$ are equivalent fractions. They both equal the decimal number 1.75. Also, when we multiply both the numerator and denominator of $\frac{7}{4}$ by 5, we get the fraction . This is shown below.

$$\frac{7}{4}=\frac{7\times 5}{4\times 5}=\frac{35}{20}$$

1. Every fraction has an infinite number of equivalent fractions. We can find additional fractions that are equivalent to $\frac{7}{4}$ by multiplying the numerator and denominator by other numbers.

A. What is the equivalent fraction if you multiply the numerator and denominator of $\frac{7}{4}$ by 2?

B. What is the equivalent fraction if you multiply the numerator and denominator of $\frac{7}{4}$ by 4?

C. What is the equivalent fraction if you multiply the numerator and denominator of $\frac{7}{4}$ by 10?

We will use the concept of equivalent fractions to add and subtract fractions when they have different denominators. But, before we do this, let's explore some situations involving adding and subtracting fractions with common denominators.

2. Fill in the blanks in order to obtain the correct answer on the right:

A. 1 apple + 2 apples = _______ apples

B. 1 fourth + 2 fourths = _______ fourths

C. $\frac{1}{4}+\frac{2}{4}=\frac{}{4}$

D. $\frac{2}{5}+\frac{3}{5}=\frac{}{5}=$

E. $\frac{6}{7}-\frac{4}{7}=\frac{}{7}$

Leftover Pizza

Sarah has some leftover pizza from a party. She has 1/3 of one pizza and 1/2 of another pizza. The pizzas, before being eaten, were the same size. She wants to feed 5 children who came over to eat the leftovers.

3. How can Sarah divide the leftover slices if she wants each child to get the same amount of pizza while at the same time giving away all of the pizza?

Hint: Try to figure your problem out with the help of pictures. What leftover amount is larger? Which one is smaller? How should you divide each of them into slices for the children?

wi pie charts. Left chart is split into two equal parts with one shaded. Right chart is split into three equal parts with one shaded.

4. Given how the pizza was divided above, how do you know that each of the five children will receive the same amount? How can you show mathematically that the amount of pizza that each of the five children receive are the same?

5. Write fractions that represent the following amounts of pizza

- The smallest leftover amount was ________ of the pizza.
- The largest leftover amount was ________ of the pizza.
- The total leftover amount before the children ate it all was _______ of the two pizzas.
- The amount each child received was _______ of the pizzas.

6. What do you notice about the denominators of your answer to the previous question? How are they related?

The least common multiple (LCM) of the denominators of two fractions is called the **least common denominator (LCD)** of the two fractions. 6 is the LCD of 1/3 and 1/2. To add ⅓ and ½, we must convert these fractions to equivalent fractions with 6 in the denominator.

1 over 2 = something over 6 1 over 3 = something over 6.

To find the fraction equivalent to $\frac{1}{2}$ with the **6 in the denominator**, we must multiply the denominator (2) by 3. So, we multiply the numerator and denominator by 3.	To find the fraction equivalent to $\frac{1}{3}$ with the **6 in the denominator**, we must multiply the denominator (3) by 3. So, we multiply the numerator and denominator by 3.

We multiply the numerator and denominator of 1 over 2 by 3 to get 3 over 6. We multiply the numerator and denominator of 1 over 3 by 2 to get 2 over 6.

7. The sum of $\frac{1}{2}+\frac{1}{3}$ is equivalent to the sum $\frac{3}{6}+\frac{2}{6}$. What is the sum of these fractions? This sum represents the total amount of pizza left over.

8. Let's look at a different example. Find the LCD of the fractions 5/4 and 7/6.

9. Add 5/4 and 7/6.

A. What numbers should we multiply the top and bottom of each fraction by so the products have the least common denominator of 12?

$$\frac{5}{4}+\frac{7}{6}=\frac{5\times}{4\times}+\frac{7\times}{6\times}=\frac{}{12}+\frac{}{12}$$

B. After multiplying, what are the equivalent fractions?

C. What is the sum?

NEXT STEPS

More Leftover Pizza

Paul ordered one pepperoni pizza and one cheese pizza, both of the same size. He had $\frac{2}{3}$ of the pepperoni pizza left over, and had $\frac{3}{4}$ of the cheese pizza left over. Then Luis and Akisha came to visit. He gave the pepperoni pizza leftovers to Luis and gave the cheese pizza leftovers to Akisha. He wants to determine how much more pizza Akisha received than Luis.

10. What mathematical operation does Paul need to perform to solve this problem?

11. To subtract $\frac{2}{3}$ from $\frac{3}{4}$, we must convert the fractions to equivalent fractions with a common denominator.

A. What is the least common denominator (LCD) of the fractions $\frac{2}{3}$ and $\frac{3}{4}$?

B. $\frac{3}{4}-\frac{2}{3}=\frac{3\times}{4\times}-\frac{2\times}{3\times}$

C. After multiplying, what are the equivalent fractions?

D. What is the difference?

TRY THESE

Similar to negative integer numbers, there are also negative fractions. We can also use the LCD of negative fractions to carry out an addition or a subtraction problem, and the negative sign is assigned to the numerator of the fraction. Below is an example:

$$-\frac{8}{9}+\frac{5}{6}=-\frac{8\times 2}{9\times 2}+\frac{5\times 3}{6\times 3}=\frac{-16+15}{18}=\frac{-1}{18}=-\frac{1}{18}$$

We can write negative fractions in several ways, because there are several expressions involving a division that yield the same result. For example, the fraction $-\frac{1}{18}$ can be written in the following three ways.

The negative sign can be placed before the fraction, in the numerator, or in the denominator.

12. Hannah bought 3 pizzas for a family gathering. Her relative ate $\frac{11}{6}$ of the 3 pizzas. How much pizza does Hannah have left?

13. Perform the following calculations:

A. $-\frac{14}{3}+\frac{5}{3}=$

B. $-\frac{5}{7}+(-\frac{4}{3})=$

C. $2-\frac{13}{5}=$

D. $-\frac{3}{2}-(-\frac{11}{4})=$

E. $-\frac{7}{6}-\frac{3}{8}=$

Questions: Fractions–Adding and Subtracting

1. For each pair of fractions, find the least common denominator (LCD). **Hint**: What are the multiples of the denominators? What is the least common multiple?

A. Find the LCD of $\frac{2}{7}$ and $\frac{1}{6}$.

B. Find the LCD of $\frac{1}{9}$ and $\frac{2}{6}$.

C. Find the LCD of $\frac{1}{6}$ and $\frac{1}{3}$.

D. Find the LCD of $\frac{6}{25}$ and $\frac{5}{10}$.

2. Liz needs to use $\frac{3}{4}$ of a cup of flour to make a sponge cake. She wants to cover it with a cream recipe that calls for $\frac{1}{6}$ of a cup of flour.

A. How much flour will she need in total?

B. Liz has 4 cups of flour. If she uses $\frac{11}{12}$ cups of flour for the cake recipe, how much flour will she have left over? Write your answer as a fraction.

3. Lou bought $\frac{3}{4}$ of a meter of ribbon to decorate a vase, then he found out he only needed $\frac{1}{6}$ of a meter. How much ribbon did he have left after he finished his decoration?

4. Jamal decided to make a long trip on foot. On the first day he walked $\frac{2}{9}$ of the way, but he got really tired. On the second day he decided to hitchhike and traveled $\frac{1}{4}$ of the way.

A. What total fraction of the trip did Jamal cover on the first two days?

B. What fraction of the trip does he still need to travel in order to finish his journey?

5. Carla decided to share 14 yards of lace she inherited from her mother with her sisters. She decided to give them the amount they needed for their projects: $\frac{11}{4}$ of a yard to Maria, $\frac{8}{3}$ of a yard to Cecilia and $\frac{6}{5}$ of a yard to Ana.

A. How much lace did Carla give away? Write as a fraction.

B. How much lace does she have left? Write as a fraction.

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5.13: Fundamentals 12 - Ratios and Proportions

LEARNING GOALS

By the end of this lesson, you should understand

- understand the difference between a **ratio** and a **proportion**
- a ratio compares two quantities.
- a ratio can be written in different ways and still be equal to the same value.
- a ratio with different units represents a rate.
- a proportion shows that two ratios are equal.
- there are different ways to set up a proportion.
- you can use different operations to compare fractions or determine the value of a missing quantity.

By the end of this lesson, you should be able to

- recognize when two ratios are equivalent to each other.
- create two equivalent ratios.
- use various techniques, including cross multiplication, to find a missing quantity.
- include units in your ratio or final answer.

FUNDAMENTALS OF THE LESSON

Imagine in a bag of M&M’s there are 2 red M&M's and 5 blue M&M's. We say that the **ratio** of red M&M's to blue M&M's is **2 to 5**. In other words, in this bag *for every 2 red M&M’s, we have 5 blue M&M’s*. We can write this ratio as **2 : 5**.

Five blue M&M's and two red M&M's.

Now, imagine you have a second bag with 8 red M&M’s and 20 blue M&M’s. In this bag, the ratio of red M&M’s to blue M&M’s is **8 : 20**.

Eight red M&M's and 20 blue M&M's.

You can divide the red M&M’s into 4 groups of 2 and the blue M&M’s into 4 groups of 5. This shows that in this second bag you also have 2 red M&M’s for every 5 blue M&M’s.

Five blue M&M's and two red M&M's.	Five blue M&M's and two red M&M's.	Five blue M&M's and two red M&M's.	Five blue M&M's and two red M&M's.

The **ratio** of red M&M’s to blue M&M’s for the second bag is the same as the first bag. We say that **the ratio of 2 : 5 is equivalent to 8 : 20**. You can obtain the second ratio by multiplying both numbers of the first ratio by 4. You can obtain the first ratio from the second ratio by dividing both numbers by 4. These are the operations we use to get equivalent fractions. That is why ratios are also written as fractions.

For example, we can write the ratio of red M&M’s to blue M&M’s in the second bag as 8 : 20. Or, we can write it as an equivalent fraction. Since $\frac{8}{20}=\frac{8\div 4}{20\div 4}=\frac{2}{5}$, we can express the ratio in lowest terms as $\frac{2}{5}$.

1. Set up a ratio to compare 12 feet to 48 feet. Express this ratio in reduced form.
2. Express the ratio of 220 miles to 4 hours in reduced form.
3. At the local grocery store, eggs cost $2.40 per dozen. What is the cost of 84 eggs?
4. An orange drink calls for 2 cans of orange concentrate for every 3 cans of soda. We could say that the ratio of cans to soda is 2 : 3. Complete the table below. Compare the cans of orange concentrate to the cans of soda. What do you notice?

Cans of Orange Concentrate	2	4	6	8	10	12
Cans of Soda	3	6	9			

NEXT STEPS

If the World Were 100 People

Imagine the world only had 100 people… Watch the video entitled “If the World Were 100 People” https://www.youtube.com/watch?v=QFrqTFRy-LU

From the video “If the World were 100 People”, there were 12 people who spoke Mandarin, 6 spoke Spanish, 5 spoke English, 3 spoke Hindu, 3 spoke Arabic, and the rest spoke 6,500 other languages. Use this information for questions 5 and 6.

5. How many people speak the 6,500 other languages?
6. Express a ratio for:

A. Spanish speaking to the number of people in the world.

B. English speaking to those that speak languages other than English.

C. From the video “If the World Were 100 People”, 21 people were overweight, 63 were at a healthy weight, 15 were malnourished, and 1 person was starving. Find the ratio of malnourished people to those at a healthy weight.

D. What if the world population doubled? If everything else doubled as well, what would the new ratio be for malnourished people to those at a healthy weight? How does this ratio compare to your answer from part (a)?

Which Is Better?

7. You are trying to determine the better deal for buying soda. A local supermarket offers a pack of 12 sodas for $3.48. The local discount warehouse offers a pack of 36 sodas for $11.52.

A. Express each deal as a ratio.

B. Which is the better deal?

8. A car travels about 26 miles on 1 gallon of gas while a truck travels about 250 miles on 14 gallons of gas. Which gets the better gas mileage?

Making Sure You Have Enough

Sometimes, recipes need to be adjusted for different serving sizes or batches. A bread recipe calls for 5 tablespoons of sugar for every 8 cups of flour.

9. Express this comparison of sugar to flour as a ratio.

Solving for an Unknown in a Proportion

Many real-world situations involve the use of proportions. A proportion is an equation showing the equality of two ratios. When we are solving for an unknown in a proportion, we can use a process called cross multiplication. Consider the example below.

Example: A bread recipe calls for 5 tablespoons of sugar for every 8 cups of flour. How many tablespoons of sugar are required for 20 cups of flour?

Solution: Let’s use the variable x to represent the unknown number of tablespoons of sugar in the new ratio. The proportion is shown below on the left. The proportion shown below on the right does not include the units.

5 tablespoons over 8 cups = x tablespoons over 20 cups. 5 over 8 = x over 20

We can solve for x by cross multiplying. We multiply diagonally to create an equivalent equation. This is shown by the green arrows below.

5 over 8 = x over 20 cross multiples as 5 times 20 = 8 times x. This produces 100 = 8x, so x = 12.5

The result is that we need 12.5 tablespoons of sugar for 20 cups of flour.

10. Let’s find out how many tablespoons of sugar are required for 4 cups of flour.

A. Create a proportion involving the unknown quantity to solve this problem. Use the variable x to represent the unknown quantity. Do not place the units in the proportion.

B. Solve the proportion for x using cross multiplication. Write the answer below.

11. Returning to our bread recipe, 5 tablespoons of sugar are needed for every 8 cups of flour. Let’s now find out how many tablespoons of sugar are needed for 32 cups of flour.

A. Create a proportion involving the unknown quantity to solve this problem. Use the variable x to represent the unknown quantity. Do not place the units in the proportion.

B. Solve the proportion for x using cross multiplication. Write in the answer below.

FURTHER APPLICATIONS

12. Kiki, Tre, and Julie made $2,520 for working at a tree nursery, but they did not work the same amount of hours. Kiki worked 30 hours, Tre worked 50 hours, and Julie worked 60 hours. They divided the money in proportion to the number of hours worked. If they all earn the same rate of pay, how much money did each person earn?
13. A fire truck can hold 3000 gallons of water. A firefighter can deliver 160 gallons of water every 2 minutes from the truck.

A. How much water will be delivered in 10 minutes?

B. How long will it take for the firefighter to empty the tank?

Questions: Ratios and Proportions

1. Write two ratios that are equal to $\frac{1}{4}$.

2. Write each as a ratio in lowest terms:

A. 15 miles to 20 miles

B. 136 hours to 20 days

3. Fill in the missing value to make an equivalent statement.

A. $\frac{3}{2}=\frac{?}{18}$

B. $\frac{4}{5}=\frac{12}{?}$

4. Determine if each of the following statements is a proportion:

A. $\frac{3}{5}=\frac{6}{9}$

B. $\frac{10}{9}=\frac{29}{27}$

C. $\frac{5}{7}=\frac{15}{21}$

D. $\frac{8}{9}=\frac{16}{27}$

5. Solve each proportion:

A. $\frac{a}{8}=\frac{5}{2}$

B. $\frac{20}{15}=\frac{x}{14}$ (Round to 2 decimal places.)

6. A pancake recipe that feeds three people uses 2 cups of flour and 1 cup of milk.

A. Express this as a ratio with units (flour to milk).

B. Create an equivalent ratio that would feed 12 people.

7. There are 20 pieces of fruit in a bowl. Three pieces are bananas. Use this information to express the ratio of:

A. Bananas to total fruit.

B. Total fruit to bananas.

C. Bananas to fruit other than bananas.

8. Michael takes 1 hour to type 5 pages of an essay. Determine the number of hours he would need to type 27 pages. Create and use a proportion to solve this problem to the nearest tenth.

9. A 16-oz grande Shaken Sweet Tea at Starbucks has 100 calories.[15] How many calories are there in a 24-oz venti Shaken Sweet Tea? Create and use a proportion to solve this problem.
10. If a 170-pound person weighs approximately 65 pounds on Mars, how much does a 9000-pound satellite weigh on Mars? Create and use a proportion to solve this problem. Round to the nearest pound.

---

[15] Starbucks, Shaken Sweet Tea, accessed August 1, 2018, https://www.starbucks.com/menu/drink...?foodZone=9999

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5.14: Fundamentals 13 - Rates

LEARNING GOALS

By the end of this lesson, you should understand

- a **rate** is a special ratio that compares two quantities of different units
- a **unit rate** is a ratio where the denominator is a unit of one.
- units can add meaning to specific situations.

By the end of this lesson, you should be able to

- write a rate as a fraction.
- simplify a rate by a unit factor.
- use unit rates to compare values.

FUNDAMENTALS OF THE LESSON

A **ratio** is a comparison of two quantities. If a bowl has 12 pieces of fruit in it and 5 of those are apples, the ratio of apples to the fruit in the bowl is **5 to 12**. This ratio could also be written as 5:12 or as a fraction, 5/12.

A **rate** is a special ratio that compares two quantities of different units. If you buy 10 apples for $3.00, we say the cost is $3.00 per 10 apples. Rates are often used in our everyday lives. How much money you make per hour, how many miles per hour you are driving, and how much you pay for food per pound are examples of rates. Rates can be used to determine costs, like gas prices (dollars per gallon), or be used to express certain measurements in healthcare, like pulse rates (beats per minute).

A **unit rate** means that we have a rate of one of some quantity. For example, what is the price of one apple, if you paid $3.00 for 10 apples? You are paying 30 cents for 1 apple. Both $3.00 for 10 apples and $0.30 for 1 apple are rates, but $0.30 for 1 apple is a *unit rate*.

What is the rate?

Rob has a mowing service and charges $280 to mow eight different spaces in a local petting zoo. It usually takes Rob 7 hours to mow the lawn spaces before he is finished working for the day. The petting zoo has 62 different species and over 300 animals in it.

1. What is Rob's price for mowing each lawn space? **Hint**: Assume the rate he charges for each lawn space at the petting zoo is the same.

2. What is Rob's hourly wage?

3. A. How much should Rob charge if he mows 5 lawn spaces?

B. How much will Rob make an hour if it takes Rob 2.5 hours to mow these 5 spaces?

NEXT STEPS

Do you have enough?

You travel to the local Farmers Market for fresh produce. You have $20 left in your grocery budget for the month and want to buy the items in the left column in the table below. The right column shows the price for each item.

4. Do you have enough money to purchase the following list of items at the Farmers Market? **Hint**: You can use the quantity of an item and the cost of an item ($ / item) to find the price of the item.

Produce List	Farmers Market Pricing

6 sweet potatoes 8 ears of corn 8 apples 2.5 lbs. bananas 0.5 lbs. broccoli 1 lb. mushrooms 3 zucchini 1.5 lbs. carrot 2 lbs. onions	Apples: 5 for $1.00 Bananas: $0.68/lb. Broccoli: $1.30/lb. Carrots: $2.50/lb. Corn: 5 for $2.00 Mushrooms: $0.45 per ¼ lb. Onions: $3.00/5 lbs. Sweet Potatoes: $0.70 each Zucchini: 12 for $3.00

A. Cost of sweet potatoes =

B. Cost of corn =

C. Cost of apples =

D. Cost of bananas =

E. Cost of broccoli =

F. Cost of mushrooms =

G. Cost of Zucchini =

H. Cost of carrots =

I. Cost of onions =

J. Total =

FURTHER APPLICATIONS

5. The 2019 VW Jetta has a highway rating of 41 miles per gallon. You plan to make a summer road trip from Morrisville, New York to Myrtle Beach, South Carolina. The distance is about 788 miles one way.

A. If the primary route to Myrtle Beach is I-95 south (highway driving), how many gallons of fuel will you need for the whole trip, there and back? Round to one decimal place.

B. If fuel costs an average of $2.35 a gallon, use your rounded answer from part A to determine the fuel cost for the whole trip.

6. Catrina needs to ride her bike to her friend's house 96 miles away. She is riding at an average rate of 15 miles per hour. She has 6 hours to get there. Will she make it? Explain.

Questions: Rates

1. A Honda Civic car dealer has 6 black cars, 8 white cars, and 14 silver cars. What is the ratio of black cars to silver cars? Simplify the ratio.

2. Four pounds of meat cost $5. Choose the best choice that represents the rate given.

   i. $\frac{1\ lb}{4\ lb}\ and\ \frac{5\ lb}{\$4}$
   ii. $\frac{4\ lb}{\$5}\ and\ \frac{\$5}{4\ lb}$
   iii. $\frac{\$4}{5\ lb}\ and\ \frac{5\ lb}{\$4}$

3. 3 kiwis cost $1.80.

A. What is the cost per kiwi?

B. At this rate, what is the cost of 13 kiwis?

4. If David earns $100 in 8 hours, how much will he earn in 3 hours at the same rate? Round to the nearest cent, when appropriate.

5. If a car is traveling at a rate of 50 miles per hour, how far will it travel in 4 hours?

6. A plane flies 750 miles in 5 hours. How fast did the plane fly in one hour?

7. A box of crackers contains 7 servings and has a total of 420 calories.

A. How many calories are in 1 serving?

B. How many calories are in 4 servings?

8. A store sells bundles of shirts in three ways: 6 shirts for $25.50, 4 shirts for $18.00 or 5 shirts for $21.00.

A. Which is the better buy?

B. What is the unit rate of the best buy?

9. Sarah can make 38 dozen cupcakes in 8 days. How many dozen cupcakes can Sarah make in 5 days? Round to the nearest dozen cupcakes.

10. A pizza place sells pepperoni pizzas to cheese pizzas in a ratio of 9:4. If 14 cheese pizzas are sold in one day, how many pepperoni pizzas were sold on that day? Round to the nearest whole pizza.

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5.15: Fundamentals 14 - Percents–Just the Basics

LEARNING GOALS

By the end of this lesson, you should understand

- The meaning of percent.

By the end of this lesson, you should be able to

- Write a percent from context.
- Compare percents.

FUNDAMENTALS OF THE LESSON

Percent means ‘per 100’ or, ‘out of 100’. It is a portion of the whole amount. For example, 3 percent, written as 3%, means ‘3 per 100’. If the whole is 100, then 3% of the whole is 3. In other words, 3% of 100 = 3.

But, what if the whole does not equal 100. For example, what if the whole is 500? What is 3% of 500? Well, by the definition, 3% of 500, means 3 per each hundred contained in 500. How many hundreds are in 500? There are 5 hundreds in 500. 3 percent is 3 per one hundred, so since the whole of 500 has 5 hundreds, 3% of 500 is 3 x 5 = 15. Therefore, 3% of 500 = 15.

1. Write down the operations used in the example above without doing any calculations.

2. How would you describe 100% of an amount?

3. Let’s use the method described above to find the percentages below.

A. What is 5% of 200? Use the same approach as described above. Write down the operations and find the answer.

B. What is 12% of 1000? Use the same approach as described above. Write down the operations and find the answer.

C. What is 8% of 50? Use the same approach as described above. Write down the operations and find the answer.

4. If 2 out of 5 dentists like Limburger cheese, how many dentists in 10 are expected to like Limburger cheese? (**Hint**: the diagram below could help). Explain your steps.

For question 4, you could have considered that since 10 is twice 5, there should be twice as many dentists out of 10 as out of 5, or 2 x 2 = 4 dentists liking Limburger cheese. You could even create a visual, such as the one below, to help.

Two sets of five cartoon teeth. In each set, two teeth are circled.

5. If 4 out of 10 dentists like cheddar cheese, how many dentists in 100 are expected to like cheddar cheese?

NEXT STEPS

An advertisement states: “Four out of five dentists recommend *Flossy* brand dental floss.” This implies that on average, if 5 dentists are found together, 4 of them are expected to recommend Flossy.

6. In a group of 10 dentists, how many are expected to recommend Flossy? Explain your reasoning.

7. In a group of 20 dentists, how many are expected to recommend Flossy? Explain your reasoning.

8. A. In a group of 100 dentists, how many are expected to recommend Flossy? Explain.

B. Based on the advertisement, what percentage of dentists recommend Flossy? Explain.

9. A. An advertisement states, “3 out of 15 dentists use Risco brand toothpaste.” If this is true, how many dentists out of 60 are expected to use Risco? Explain.

B. How many dentists out of 90 are expected to use Risco? Explain.

C. What percent of dentists are expected to use Risco?

10. A. If 12 out of 16 dentists prefer tap dancing to the Lindy, how many dentists out of 100 are expected to prefer tap dancing?

B. What percentage of dentists are expected to prefer tap dancing?

TRY THESE

Comparing Ratios

Imagine that you find out that 4 out of 32 mathematicians prefer apples to oranges, while 3 out of 8 mathematicians have a fear of the dentist.

11. Which percentage is higher, the percentage of mathematicians who prefer apples, or the percentage of mathematicians who fear the dentist? Explain how you arrived at your answer.

12. Make the calculation 4 ÷ 32 followed by the calculation 3 ÷ 8. What do you notice?

13. If 200 mathematicians are gathered together, how many are expected to prefer apples to oranges? How many are expected to fear the dentist? Explain.

Questions: Percents

Just the Basics

1. What is 31% of 100?

2. What is 90% of 300?

3. About 8% of children in the United States have asthma. Asthma is a medical condition which restricts people’s airways in their lungs making breathing difficult. If 400 U.S. children are surveyed, how many of them would you expect to have asthma?

4. What mathematical expression equals 35% of 200?

(i) $35 \times \frac{200}{100}$

(ii) $35 \times \frac{1}{100}$

(iii) $200 \times \frac{1}{200}$

(iv) $1 \times \frac{35}{100}$

5. What mathematical expression equals 13% of 20?

(i) $1 \times \frac{13}{100}$

(ii) $20 \times \frac{1}{20}$

(iii) $13 \times \frac{1}{100}$

(iv) $13 \times \frac{20}{100}$

6. In 2016, 7 out of 10 students attended college following graduating from high school. If 170 of these students are chosen, how many of them do we expect to attend college following their graduation from high school?

7. If 5 students out of 15 are expected to get an “A” on a math test, how many students are expected to get an “A” if the class has 28 people? Round your answer to the nearest number.

8. If 5 students out of 15 are expected to get an “A” on a math test, how many students are expected to get an “A” if the class has 70 people? Round your answer to the nearest number.

9. If 23 students out of a class of 100 get an “A” on a math test, what percentage got an "A"? Round to the nearest percent.

10. If a major league baseball team hopes to win 96 out of their 160 games (normally there are 162 games, but imagine two have been canceled), how many games out of 10 do they need to win on average? Round to the nearest whole number.

11. If a team wants to win the amount of games found in Question 10, what percentage of their games do they need to win?

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5.16: Fundamentals 15 - Percents–Operations

LEARNING GOALS

By the end of this lesson, you should understand

- the relationship between decimals and percents.

By the end of this lesson, you should be able to

- convert between decimals and percents.
- perform arithmetic operations with percents.

FUNDAMENTALS OF THE LESSON

"Percent" comes from the latin *Per Centum*. The latin word *Centum* means 100. For example, a century is 100 years. A penny (a cent) is part of a dollar since there are 100 cents in one dollar. It is a portion of the *whole* amount.

40 percent, or 40%, means '40 out of 100'. 40% of the whole means 40 out of each hundred contained in the whole. Since 40% can be understood as 40 out of one hundred, it can also be represented as a fraction as well as a decimal. For example:

$$40\% = \frac{40}{100} = 0.40$$

If 40 is divided by 100, the decimal equivalent of 0.40 is obtained. Likewise,

$$15\% = \frac{15}{100} = 0.15 \; or \; 2\% = \frac{2}{100} = 0.02$$

$$100\% = \frac{100}{100} = 1.00$$

Notice this is equal to 1 as 100% indicates the whole.

Percentages can be greater than 100%. For example:

$$110\% = \frac{110}{100} = 1.10$$

110% is more than 1. 100% is the whole, so 110% is actually more than the whole. A percent can be interpreted as a ratio of a portion and a whole. Whenever the portion is greater than the whole, the percentage will be greater than 100%.

This activity will cover three basic operations involving percents: **addition, subtraction, and multiplication**.

Addition or subtraction happens when **two portions of the whole are added or subtracted together**. For example, if you are holding two quarters (each quarter is 25 cents, or 25% of a dollar), what percentage of a dollar do you have? To calculate the percentage of a dollar that you are holding, you can add 25% and 25% to get 50% of a dollar, which is 50 cents.

Problems may involve multiplying percentages. This occurs when you are **taking a percent of a percent**. For example, you know that 50% of the time you lose in the game "rock, paper, scissors". You also know that 30% of the games that you lose, you are holding out "rock". You can use this information to determine the percentage of all games that you lose AND are holding out "rock". This can be calculated by multiplying 50% x 30%. This is 0.50 x 0.30 = 0.15 = 15%. So, 15% of the time that you play the game you lose and are holding out "rock".

1. Let's begin by practicing converting percentages to decimals.

A. Write 20% as a decimal.

B. Write 5% as a decimal.

C. Write 250% as a decimal.

D. Will 95% be greater than 1 or less than 1, and why?

NEXT STEPS

Working with percents

2. Saranne, Antonio, and Kera are work-study students in the admissions office of their college. One of their main tasks is filing new student information.

A. On a certain workday, Saranne files 50% of the information while Antonio files 50%. What percentage of the student information has been filed?

B. Antonio files 35% of the documents and his two associates each file 10%. What percent has NOT been filed yet?

C. While filing student information, Kera notices a note that states that students are charged a 2% late fee for tuition if a payment arrives late but within 10 days of the due date. That is, 2% of the amount due is added to the account if the payment is late but arrives within 10 days of the due date. If a payment is more than 10 days late, the late fee percentage is tripled. Suppose a student sends in their tuition payment 15 days late. What percentage of the tuition payment will be charged as a late fee?

D. Kyle submitted his tuition payment 18 days late. His tuition payment was $978. What late fee will Kyle be charged?

3. Harper wants to raise money for breast cancer awareness. Her goal is to raise $10,000 from her community. She works so hard she ends up raising $11,000. Her friends tell her that she raised 110% of her goal. Is her friend right? If so, what does that mean? **Hint**: What is her fundraising goal? What is 110% of her goal?

4. 70% of students who apply to Upstate New York Community College are originally from New York State. Of these New York State applicants, 30% are from Madison County. What percent of students who apply to Upstate New York Community College are from Madison County? Write the answer as a percentage. Hint: Do we add percents, or find a percent of a percent to solve this problem?

5. Approximately 2% of the applicants who apply to Upstate New York Community College are international students. Of the international applicants, about 75% of them are from Japan or China. What percentage of all of the students are from Japan or China?

6. Antonio makes quarterly payments to his insurance company and is charged a 2% late fee if his payment is received after the due date. If his payment is $558, how much will he be charged if his payment is late?

FURTHER APPLICATIONS

7. Kera is a college-student who lives at home with her parents. Her parents are generous and pay for most of her expenses so that she can save her earnings from her work-study job. The bank closest to both her home and college is Simplicity Bank. Banks calculate the interest earned on a savings account in a fairly complicated way. Interest is the amount of money the bank pays the customer for investing their money with the bank. The annualized percentage yield (APY) describes how much money a savings deposit earns in interest in one year. If the APY applied to Kera's account at Simplicity Bank is 1.05%, then the amount of interest Kera will earn in a year is 1.05% of the amount of money she deposited. Kera's account balance after one year can be found by adding the interest she earned to the initial amount she deposited.

A. If Kera only invests $100 in Simplicity Bank over an entire year, how much money should she expect to have in her account after 1 year?

B. Kera would like to earn more interest and decides that she needs to save and invest more money. What would her account balance be after one year if she invested $1000?

C. How much would she have if she invested $5000 for one year?

Questions: Percents–Operations

1. There are several types of colleges and universities in the United States. Colleges can be public or private, and can be for-profit or non-profit. The table below displays the percentages of recent high school graduates from a large city who were enrolled in college in 2012 and 2016, and the type of college the graduates were enrolled in.

	2012	2016
Private 4-year non-profit	16.9	17.8
Public 4-year	38	35.9
Public 2-year	35.5	40.1

A. In 2012, what type of college were most recent high school graduates enrolled in?

B. Notice the percentages for 2012 do not add to 100%. What percentage of recent graduates in 2012 are not represented in this data?

C. There were 15,492 recent graduates in the city’s 2012 class. How many of them enrolled in a private 4-year non-profit college? Round to the nearest student.

D. Which type of college had the greatest increase in percent of enrollment from 2012 to 2016?

E. What is the increase in the percentage of enrollment in public 2-year colleges from 2012 to 2016? Write the increase in terms of percentage points.

2. The table below contains nutrition facts for Almond Milk and 1% Lowfat Milk. The percent daily values of a nutrient indicates the percentage of a daily 2000-calorie diet which a 1-cup serving of each product covers. For example, 1 cup of Almond Milk covers 10% of the amount of Vitamin A a person should consume in one day.

See the source image

A. Per 1 cup serving, which product provides a greater percentage of calcium? How much more?

B.If you had 5 servings of Almond Milk in one day, would you be meeting or exceeding your percent daily value of calcium for a 2,000 calorie diet?

C. What percentage of your total calories would you have met for a 2,000 calorie diet if you had 5 servings of Almond Milk?

D. If you had 5 servings of 1% Lowfat Milk in one day, would you be meeting or exceeding your percent daily value of calcium for a 2,000 calorie diet?

E. What percentage of your total calories would you have met for a 2,000 calorie diet if you had 5 servings of 1% Lowfat Milk?

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5.17: Fundamentals 16 - Estimation

LEARNING GOALS

By the end of this lesson, you should understand

- The meaning and purpose of estimation.
- What is meant by “reasonable” for an answer.

By the end of this lesson, you should be able to

- Estimate simple addition, subtraction, multiplication and division problems mentally.
- Determine if a calculated answer is reasonable.

FUNDAMENTALS OF THE LESSON

1. A useful strategy when estimating is to remember that finding a unit fraction of something is the same as dividing by the denominator. For example, Finding $\frac{1}{4}$ of a quantity is the same as dividing by 4. Try these:

Finding $\frac{1}{5}$ of a quantity is the same as dividing by __________

Finding $\frac{1}{10}$ of a quantity is the same as ___________________

2. Try to quickly estimate the following:

A. $\frac{1}{2}$ of 20

B. $\frac{1}{3}$ of 300

C. $\frac{1}{6}$ of 12

D. $\frac{1}{10}$ of 500

3. Some percentages are the same as (or close to) unit fractions. Convert the following percentages to unit fractions. For example, 50% = $\frac{1}{2}$. We can show this equivalence by converting 50% to a fraction, and then simplifying the fraction. 50% = $\frac{50}{100} = \frac{1}{2}$.

A. 25% =

B. 33% =

C. 20% =

D. 10% =

NEXT STEPS

Why Estimate?

Estimates are useful for many reasons. One is to be able to do calculations quickly in your head on the spot. That might happen when you are shopping and items are on sale.

4. Martha has decided she can afford to spend $50 on new jeans. Her favorite store is having a sale. Decide if she can afford (by estimating):

A. Jeans regularly priced at $69.99 that are 50% off

B. Jeans regularly priced at $75.99 that are $20 off

5. Juan works for a company that makes phones. When a certain phone is discontinued, employees may buy the phone for one-third of the regular price. How much would he pay for a phone that regularly costs:

A. $29.99

B. $54.99

6. Malik is shopping for a new coat. The store is having a sale. Estimate the cost of the following coats.

A. Regular price: $129.99, now 50% off

B. Regular price: $85.99, now $\frac{1}{3}$ off

C. Regular price: $85.99, now 25% off

D. Regular price: $62.00, now 10% off

Checking your calculations

Another reason for estimating is so that you know if your calculated answer is reasonable. If you enter the numbers incorrectly into your calculator you will be able to see that your answer cannot be right.

7. A. Estimate an answer to the following problem (without writing down any calculations). There are 19,795,791 people living in New York state. If 50% of them live within 20 miles of New York city, estimate how many people that is.

B. If Marisol uses her calculator and gets an answer of 989,790 can she be right? Explain to her (using just your estimate from part A) why or why not.

8. Estimate the answer to the following questions and then circle whether the calculated answer could be correct or could not be correct:

A. About one third of adults in the United States have a bachelor's degree. If there are 236 million adults in the US, estimate how many have a bachelor's degree. (**Note**: Do NOT calculate the answer.)

Estimate: ________________________________

B. For the calculation in part A above, is the calculated answer of 78,667,000 reasonable or unreasonable?

C. In a recent year, there were 4,933,100 high school students in the U.S. If 81% of high school students graduate from high school, estimate how many of these students will graduate from high school.

Estimate: ________________________________

D. For the calculation in part C above, is the calculated answer of 2,544,200 reasonable or unreasonable?

E. Lakeland School District in New York serves 5,800 students. If the district's budget is $61,174,165, how much is that per student?

Estimate: ________________________________

F. For the calculation in part E above, is the calculated answer of $105,473 reasonable or unreasonable?

FURTHER APPLICATIONS

Calculations often involve multiple numbers. One strategy for estimating a sum (total) of numbers is to group the numbers before adding them together. This may allow you to get a sense of what the sum (total) should be.

9. Arianna is buying small gifts for party favors. She wants to estimate the total. The six prices are listed below:

$2.45 $6.99 $5.05 $2.90 $4.99 $2.50

She decides to group them like this:

$6.99 and $2.90 $5.05 and $4.99 $2.45 and $2.50

A. Using these groups of numbers, what is an estimate for the total cost?

B. Estimate the total of the following holiday gifts that Gustavo plans to give his family. Group the numbers first to make the mental calculation as easy as possible. Explain your steps.

$49.99 $20.05 $61.50 $39.00 $50.25 $79.99

10. Sometimes it is important to know if your estimate is bigger or smaller than the calculated answer. This can be useful in many situations.

A. Cereal is on sale for 3.79 per box, including tax. Santiago estimates how much 8 boxes will cost by multiplying 4 by 8 to get $32. Is the calculated answer going to be more or less than $32? So if Santiago only has $32 exactly, will he have enough? How do you know?

B. There are 12,254 students at Yera’s college. Each student pays $32 for a lab fee each semester. Yera estimates the total cost for lab fees as 12,000 times $30, or $360,000. Is her estimate more or less than the calculated amount? How do you know?

Questions: Estimation

1. New York State spent $6.02 billion dollars on its roads in 2015. There were 7,166,325 households in the state. Katrin calculated how much money was spent on roads for each household. She found that the State spends $145 on its roads for each household. Is Katrin’s calculation reasonable? Use estimation to decide if her calculated value is reasonable.

2. Sofia does not want to spend more than $20 on a T-shirt. She finds some that are 40% off the regular price of $44.99. Are they in her price range?

3. Sam is looking for a new TV. He finds one whose regular price is $909. It is marked as 25% off. Estimate how much it will cost.

4. Jaime and her two friends have agreed to split the cost of gas for a trip to the beach. If the gas costs $17.16, estimate how much each of them will pay.

5. Julio is going to cook a turkey for Thanksgiving. His recipe tells him to cook it for 20 minutes for each pound. Julio’s turkey weighs 16.01 pounds. Estimate the cooking time.

6. Daniel buys a coffee every day that costs $2.99. Estimate how much he spends on coffee in a month (assume there are 30 days in a month).

7. If lunch at a nearby cafe costs $8.65, estimate how much Sunita will save if she brings lunch from home instead of eating in the cafe every work day for a month. Assume making lunch at home only costs her $2 per day and that she works 20 days in a month.

8. Briana is having some friends over. She wants to estimate how much it will cost. Pizzas cost $8.99 each and she will need 6 of them. Sodas cost $7.20 for a 12-pack and she needs 5 12-packs. Estimate how much this will cost.

9. Students at Matias’s college pay $98.50 in fees each semester. There are about 15,000 students at the college. Matias calculated that the total fees collected by the college is $1,477,500. Use estimation to determine if her calculation is correct.

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5.18: Fundamentals 17 - Conversion

LEARNING GOALS

By the end of this lesson, you should understand

- a percent is out of a hundred.
- 100% represents the entire quantity.
- percentages are an important part of our everyday lives.
- percentages are handy ways of writing fractions.
- percentages can be compared more easily than fractions.

By the end of this lesson, you should be able to

- recognize and convey the meaning of a percent.
- convert a percent to a fraction.
- convert a percent to a decimal.
- compare percentages in different ways.

FUNDAMENTALS OF THE LESSON

The word percent means "per hundred, or, "out of a hundred", and is represented by the % symbol. Since percent means out of 100, we can think of percents as special kinds of fractions that have a denominator of 100. So, 1% means 1 out of 100, or one hundredth of a whole. One hundred percent (100%) represents the entire quantity, 100 out of 100, or 1 whole.

Let's take a look at 25%. This means 25 out of 100 and can be represented as the fractions: begin fraction 25 over 100 end fraction or begin fraction 1 over 4 end fraction. Both begin fraction 25 over 100 end fraction and begin fraction 1 over 4 end fraction can be represented as the decimal number 0.25. Percents, fractions, and decimals are all related. Often, we have to convert from one form to another to correctly understand a situation and solve a problem.

Fraction → Percent

One way to convert a fraction to a percentage is to find the equivalent fraction out of 100. Then, the numerator gives the amount of the percent. For example:

begin fraction 1 over 4 end fraction equals begin fraction 1 times25 over 4 times25 end fraction equals begin fraction 25 over 100 end fraction equals 25 percent sign

Note that begin fraction 1 over 4 end fraction and begin fraction 25 over 100 end fraction are equivalent fractions. They equal the same number.

Fraction → Decimal → Percent

Another way to convert a fraction to a percent is to convert the fraction to a decimal, and then convert the decimal to a percent.

When a fraction has a denominator that is a power of 10 (for example: 10, 100, 1000), you can convert the fraction to a decimal by using place value. You can then convert the decimal to a percent by multiplying the decimal by 100 (or move the decimal point two places to the right). Then, attach a % symbol to the result. For example:

begin fraction 7 over 10 end fraction open bracket r e a d s a s 7 t e n t h s close bracket equals 0 .70 equals 70 percent sign

begin fraction 27 over 1000 end fraction open bracket r e a d s a s 27 t h o u s a n d t h s close bracket equals 0 .027 equals 2 .7 percent sign

When a fraction has a denominator that is not a power of 10, convert the fraction to a decimal by dividing the numerator by the denominator. You can then convert the decimal to a percent by multiplying the decimal by 100 (or move the decimal point two places to the right). Then, attach a % symbol to the result. For example:

begin fraction 1 over 4 end fractionor (1 divided by 4) = 0.25 = 25%

begin fraction 5 over 8 end fraction or (5 divided by 8) = 0.625 = 6.25%

TRY THESE

1. Complete the chart below. In your own words, write how to convert decimals, fractions, and percentages in different ways.

A. How do you change a percent into a decimal?	**B. How do you change a decimal into a percent?**

C. How do you change a decimal into a fraction?	**D. How do you change a fraction into a decimal?**
E. How do you change a percent into a fraction?	**F. How do you change a fraction into a percent?**

2. Fill in the chart below by converting to percents, decimals, and fractions.

Percent	Decimal	Fraction
12%	0.12	A
35.5%	0.355	B
C	1.43	$1\frac{43}{100}$
0.2%	0.002	D
12.5%	E	$\frac{1}{8}$
345%	3.45	F

Organizing Information

The number of mobile phone users in the world is growing rapidly and by 2019, it was estimated that there would be over 5 billion users. A small survey was done to gather information about what type of mobile phones are currently being used by mobile phone users. The table in the next question shows the responses of 40 mobile phone users. The responses will help to provide information to cell phone carrier companies about certain brands of phones being used by the consumers.

Types of smartphone owned									
I	A	I	A	W	I	A	A	I	W
W	A	I	A	I	F	W	I	A	F
I	A	F	I	I	W	I	A	I	A
A	I	W	F	W	F	I	W	A	I
I = iPhone		A = Android		W = Windows phone			F = Amazon Fire		

3. Use the data in the table above to answer the following questions.

A. What percentage of phone users owned an iPhone?

B. What percentage of phone users owned an Android?

C. What percentage of phone users owned an iPhone or an Amazon Fire phone?

D. What percentage of phone users owned a Windows phone?

NEXT STEPS

Understanding and Using Percents, Fractions, and Decimals

Almost everyday we encounter a percent. For example, when we shop at a store and see a 30% off sale. Or, when listening to a basketball game and hearing about a team making 85% of their free-throw shots. Percents are used all the time in money calculations, like when credit card companies advertise that their credit card pays 2% cash back on purchases, or when people in a state are told that 7.65% of their paycheck goes towards federal and state taxes.

Read the following situations carefully. Use the given information to solve the problems.

4. Taylor put one dollar in the vending machine for a $0.85 candy bar and did not get change back. What percent of Taylor’s dollar did she lose?

5. Sam bought a new car for $17,500. A 5.5% down payment was required. How much did Sam have to pay for the down payment? **Hint**: How can we use 5.5% to answer this question? Would it help to write the percentage in a different form?

6. Steph made 27 out of 45 three point shots in a basketball game. What percentage of 3-point shots did Steph make?

7. Thirty shirts have to be sold for a T-shirt company to make its daily profit projection. Six shirts have been sold so far. What percentage of shirts are left to sell?

FURTHER APPLICATIONS

8. A policy at a restaurant states that servers must “tip-out” a certain dollar amount of the tips they earn to the host and the kitchen staff. From the tips they receive, the server must give 5% of the food bill to the kitchen staff and 1% of the tip to the host. If the total **food bill** was $30, and a $6 **tip** was left, what was the “tip-out” amount given to the kitchen staff and to the host? Explain your work.

9. Pam scored 92% on her last math test. If she answered 23 questions correctly, how many problems were on the test? (**Note**: assume each problem was worth the same amount, 1 point). Explain your work.

Questions: Conversions—Percents, Decimals, and Fractions

1. Express the decimal 0.25 as a percentage.
2. Express the decimal 0.3333 as a percentage.
3. Express the decimal 3.53 as a percentage.
4. Express the decimal 0.0035 as a percentage.
5. Express the decimal 5 as a percentage.
6. Express the percent 70% as a decimal.
7. Express the percent 51% as a decimal.
8. Express the percent 781% as a decimal.
9. Express the percent 0.55% as a decimal.
10. Express the percent 60% as a fraction in lowest terms.
11. Express the percent 125% as a fraction in lowest terms.
12. Express the fraction $\frac{1}{3}$ as a percentage. Round to the nearest tenth of a percent, if needed.
13. Express the fraction $\frac{12}{8}$ as a percentage. Round to the nearest tenth of a percent, if needed.
14. Fill in the blank to find another expression for 2%. ______ for every 100
15. Fill in the blank to find another expression for 3%. ______ for every 50
16. Fill in the blank to find another expression for 8%. 1 for every _____
17. Fill in the blank to find another expression for 4%. 0.5 for every _____
18. Which represents the greater percent: begin fraction 325 over 500 end fraction o r begin fraction 600 over 1000 end fraction
19. Mary has 60 candy bars to sell. She has sold 75% of the candy bars so far. How many candy bars has Mary sold? Round to the nearest bar.
20. Barry sold 180 newspapers out of 200. Paul sold 85% of his 260 newspapers.

A. Who sold the most newspapers?

B. Who sold the greatest percentage of newspapers?

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5.19: Fundamentals 18 - Real Line

LEARNING GOALS

By the end of this lesson, you should understand

- that the real line is used to represent numbers graphically,
- the distance from 0 to a number on the line is called the absolute value of this number.

By the end of this lesson, you should be able to

- plot fractions, mixed numbers and decimals on the number line,
- calculate the absolute value of a number.

FUNDAMENTALS OF THE LESSON

You may have seen in a previous lesson that a fraction is called **improper** if its numerator is greater or equal to its denominator. You may also have seen **mixed numbers**, which are numbers that contain integers and proper fractions.

1. The following fractions are improper fractions. Write them as whole or mixed numbers.

A. $\frac{30}{10}$

B. $\frac{5}{4}$

C. $\frac{17}{3}$

D. $\frac{24}{3}$

E. $\frac{7}{3}$

F. $\frac{11}{4}$

2. Write the following decimals as mixed numbers.

A. 1.5

B. 2.75

C. 4.6

D. 10.125

TRY THESE

Walking on A Straight Path

Peggy decided to walk from point O to point P on a straight path. The distance between O and P is one mile. Bill and Alicia walked along the same path starting at O, but Bill covered only half the distance Peggy walked. Alicia walked half of the distance Bill covered. Cesar also walked the same path starting at point O in the direction of P, and stopped right in between Bill and Peggy's points of arrival.

3. A. The path on which Peggy walked is represented below. Label the points at which Alicia (A), Bill (B), Cesar (C) and Peggy (P) arrived.

Number line with O labeled at the left and P labeled at the right.

B. What distances did Alicia, Bill, Cesar, and Peggy travel? In the segment, plot their points of arrival and label them with the corresponding distances. Make sure you write the distances in as many ways as you can (decimals, fractions). Which label should we use for the point O? And for point P?

(i) Alicia (decimal) =

(ii) Bill (fraction) =

(iii) Cesar (fraction) =

C. Jamal traveled $1\frac{1}{2}$ miles on the same path starting at O. Label the point where Jamal stopped.

Number line with O labeled on the left, P in the middle and arrow point right at the far right.

4. A. Luis also decided to walk a straight path starting at O but in the opposite direction from Peggy's. He walked begin fraction 3 over 4 end fraction of a mile. Mark and Label his point of arrival.

umber line with arrows at each end. 0 is labeled in them middle and 1 is labeled on the right.

B. Write the distance Luis traveled as a decimal. In order to show that Luis is traveling in the opposite direction to the others, we write a **negative sign** in front of the distance.

In Questions 3 and 4, you started constructing what we call the **real line**. The real line is a straight line, where each point to the right of 0 is labeled with the distance from 0 to the point, and each point to the left of 0 is labeled with the opposite of the distance from 0 to the point.

An **open real line** is a real line where starting units have not been given.

5 Let's say we want to mark out numbers that range from -3 to 3 on an open real line. We know that -3 will go to the left and 3 will go to the right. Mark -3 and 3 on the open line below. Then, mark the points (A) 0 and (B) 1, and then label (C) 2, (D) -1, and (E) -1.5.

Number line with arrows on the left and the right. 13 line units are marked on the line.

6. The segment of the real line from 0 to 1 is called a **unit segment**.

A. In the unit segment below, label the point $\frac{1}{3}$.

B. In the unit segment below, label the point $\frac{2}{3}$.

Number line with arrows on the left and right. 0 is labeled on the left and 1 is labeled on the right.

C. How would you label the points A, B, and C in the unit segment below?

Arrowed number line. 0 is labeled on the left and 1 is labeled on the right. Between 0 and 1, there are five line unit marked. A is labeled to the right of 0. B is in the middle. And C is labeled to the left of 1.

7. A. Label the point on the line below where we'd find $\frac{2}{6}$.

B. Label the point on the line below where we'd find $\frac{4}{6}$.

Number line with arrows on the left and right. 0 is labeled on the left and 1 is labeled on the right.

The unit segment below shows points and labels for begin fraction 1 over 2 end fraction, begin fraction 2 over 4 end fraction, begin fraction 3 over 4 end fraction, begin fraction 5 over 8 end fraction, begin fraction 6 over 8 end fraction. Note that begin fraction 1 over 2 end fraction equals begin fraction 2 over 4 end fractionand begin fraction 3 over 4 end fraction equals begin fraction 6 over 8 end fraction.

Unit segment with 1/2, 2/4, 3/4, 5/8, and 6/8 marked.

8. On the number line below, label:

A. $2\frac{1}{3}$

B. $-1\frac{1}{2}$

C. $-2\frac{3}{4}$

Number line from -3 to 3.

9 On the line below, plot the following improper fractions:

A. -6/6

B. 13/5

C. -14/3

D. 43/10

E. -9/4.

Number line from -5 on the left to 5 on the right.

On the real line, the **absolute value** of a number is defined as the distance from 0 to the point labeled with this number. The absolute value of a number a is denoted as |a|.

10. First, plot the following point:

A. 3.5

Number line with 0 labeled in the middle.

B. Now use the distance from 0 to find the absolute value of 3.50.

|3.5| =

C. Now plot on the line above where we would find the point -4.

D. Use the distance from 0 to find the absolute value of -4.

|-4| =

11. Can you think of any properties of the absolute value of a number? Can it be negative? Can it be zero? Can it be the same for two different numbers? Explain.

Questions: Real Line

1 Are each of the following pairs of fractions equivalent? Write yes or no.

A. $\frac{1}{2}, \frac{3}{6}$

B. $\frac{-8}{10}, \frac{-9}{15}$

C. $\frac{4}{6}, \frac{10}{15}$

D. $\frac{-21}{7}, \frac{-9}{3}$

E. $\frac{14}{7}, \frac{-10}{5}$

On the real line plot the following numbers. Remember that you can convert improper fractions to mixed numbers before trying to plot them.

2. $\frac{5}{6}$

3. $\frac{-1}{4}$

4. $1\frac{2}{5}$

5. 1.4

6. $-2\frac{1}{3}$

7. $\frac{13}{4}$

8. $\frac{-16}{5}$

Number line with 0 labeled in the middle.

9 Calculate the values of the following expressions:

|17 - 8|

|9-13|

-|11.5|

-|-26|

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5.20: Fundamentals 19 - Units and Measurement

LEARNING GOALS

By the end of this lesson, you should understand

- importance of using the correct units
- the size difference between different units of measure
- simple unit rates

By the end of this lesson, you should be able to

- measure using a ruler with both centimeters and inches
- use units of distance, weight, and capacity
- perform simple unit conversions using dimensional analysis

FUNDAMENTALS OF THE LESSON

When we measure something we need to give the units in which we are measuring. There are different units for different kinds of measurement. Complete the following table. There could be more than one answer and you should put as many as you can.

1. Complete A-H in the table below. Note that the first row has been completed as an example.

Unit:	Are used to measure:
Feet	Distance, length, height
Ounces	(A)
Degrees Fahrenheit	(B)
(C)	Cost, amount of money
(D)	Time
Centimeters	(E)
Gallons	(F)
(G)	Area
(H)	Volume

TRY THESE 1

The importance of knowing what units are being used

In 1999 NASA lost a Mars spacecraft because the engineering team used English units of measurement while NASA used the metric system. The spacecraft came too close to the surface of Mars and burned up in its atmosphere instead of going into orbit around Mars as planned.[16]

The spacecraft was supposed to stay at least 85,000 meters above the planet. Meters are bigger than yards.

2. If the team was measuring the distance in yards instead of meters, would the spacecraft be closer to Mars or further away?

Spacecraft and Mars with 85,000 distance between them.

3. Knowing what units are being used is very important. Often we can guess from the context of the situation what units are being used. What units do you think are being used in each of the following situations?

A. Jack went to the corner shop near his home in New York City and spent 5 ____________

B. Sheela, who lives in Atlanta, just spent 25 ________ on a brand new car.

C. My two year old daughter is 3 _________ tall.

4. Sometimes it is not clear what units are being used unless they are clearly stated. It can be especially difficult to know what unit to use if you are not familiar with the context. Select two options that could work in each situation.

A. When Keisha was on holiday in Europe she spent 50 __________ on a new purse.

Unit 1_______________________ Unit 2_______________________

B. The okapi, an animal that lives in Eastern Africa, is about 4 _____________ long.

Unit 1_______________________ Unit 2_______________________

C. I am going to my grandmother's house tomorrow. She lives 3 _________ away from me.

Unit 1_______________________ Unit 2_______________________

D. People eat more seafood these days. On average, they eat 2.3 ___________ every month.

Unit 1_______________________ Unit 2_______________________

TRY THESE 2

Measuring Length

Find a ruler or measuring tape. Consider the units found on each side of the measuring device. As you saw earlier, it is important to pay attention to which units you are using when measuring.

Find three objects nearby that you can measure. Most of the time things that we measure will not have a length of exactly a whole number of centimeters or inches, but will be in-between two numbers. How could you label the small lines between the centimeter and inch markings? Hint: Think about a number line!

5. Measure the three objects and record your measurements in the table below:

Object name	Length in centimeters	Length in inches

TRY THESE 3

Units for Area and Volume

Area (the measurement of the flat surface that an object covers) is measured in square units such as square feet or square inches. (**Note**: These concepts are covered in more detail in the Fundamentals Lesson *Geometry: Perimeter, Area and Volume*).

6. A. Write in the given space some times in your everyday life when you have heard these units used.

Let's think about how big each of these units is. A square inch is just what it sounds like – it is the area of a square that is 1 inch long on each side. A square foot is the area of a square that is 1 foot long on each side.

B. Think of 3 objects that would be small enough to be covered by 1 square inch.

C. Now think of 3 objects that are bigger than 1 square inch, but are smaller than 1 square foot.

Capacity (how much something can hold) and volume (how big something is in three dimensions) can be measured in cubic units (like cubic inches, cubic meters) or in units like liters and gallons.

Some measures of volume that you are probably familiar with are gallons and quarts (milk comes in those sizes), liters (soda sometimes comes in liter or 2-liter bottles), and cups, teaspoons, and tablespoons for cooking.

7. Circle the unit that you think would be best to measure:

A. Flour for baking cookies: gallons / cups / teaspoons

B. Baby oil for a baby's legs: gallons / cups / teaspoons

C. Lemonade for a family party of twenty people: gallons / cups / teaspoons

Read about the following units for volume or capacity and then answer the questions below.

A cubic inch (in^3) is a the volume of a cube measuring an inch on each side:

Cube with dimensions 1 in x 1 in x 1 in.

A cubic meter (m^3) is the volume of a cube measuring one meter on each side (shown below). A meter is a bit longer than 3 feet, so 1 m^3 is MUCH bigger than 1 in^3, about 61,000 times bigger).

Cube with dimensions 1m x 1m x 1m.

Cubic centimeters (cm^3), cubic feet (ft^3), etc., work the same way.

A liter is about 60 cubic inches, or about a quart (you may know how big a quart is from buying milk which is often sold in quarts).

A cup is about 14 cubic inches (about how much will fill a regular drinking cup).

8. Circle the unit that you think would be best to measure:

A. Water in a water tank for a five-story apartment building: cubic inches / liters / cubic meters

B. The volume of a box of matches: cubic inches / liters / cubic meters

C. The amount of coffee in an cafe’s coffee urn: cubic inches / liters / cubic meters

FURTHER APPLICATIONS

Some things that we want to measure need more than one unit. For example, if we want to measure the speed of an object we need to say how far the object goes and how much time it takes. So we use units like “miles per hour” (or mi/hr). A speed of 30 mi/hr means the object would go 30 miles in one hour. Since the second quantity is always one, these are called *unit rates*.

Unit rates are like fractions:

$$\frac{miles}{hour}$$

9. A. If a car goes 100 miles in two hours, what is its average speed?

____________________ _________________

Number units

B. Juan spends $12,000 on his rent for a year. How much does he spend each month?

____________________ _________________

Number units

C. Leila brought 36 cookies for her class of twelve students. How many cookies for each student?

____________________ _________________

Number units

Unit conversions

Sometimes we would like to switch between one set of units and another set of units. We call this *converting* between units. If we know how the two units are related to each other, we can convert from one to another.

10. At Six Flags America, park guests must be 48 inches tall to ride alone on THE PENGUIN’S Blizzard River raft ride.[17] Five-year-old Andre wants to know how tall that is in feet, since he knows he is 3.5 feet tall.

A. How are feet and inches related? In other words, how many inches are in 1 foot?

_____________ inches = 1 foot

Now we can take our original number of 48 inches and multiply it by a fraction made up of our relationship between inches and feet. In order for the units we don’t want to keep (inches) to cancel out, they have to be on opposite parts of the fraction.

$$\frac{48\ inches}{1} \cdot \frac{1\ foot}{12\ inches} =$$

Now cross off the inches, multiply straight across the top and bottom, then simplify your answer.

B. 48 inches is the same as __________ feet.

C. Can Andre get on this ride alone?

11. Try a similar process to solve this problem: Two hundred ten minutes is how many hours?

Questions: Units and Measurement

1. What units could you use to measure the amount spent on a pair of jeans?

2. What units could you use to measure the length of the guttering on the edge of the roof of your building?

3. What units could you use to measure the area of a school playground?

4. Latoya used inches to measure the height of her son. The doctor used centimeters. Which number will be bigger: Latoya's number of inches or the doctor's number of centimeters?

5. Alfonso is trying to drink more water to be healthy. When asked how much water he had drunk that day, he said, "About 6." What is wrong with his answer? Give a better answer.

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[16] http://edition.cnn.com/TECH/space/9909/30/mars.metric.02/

[17] Six Flags of America, Attractions by Height, accessed August 1, 2018, https://www.sixflags.com/america/att...t-restrictions.

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5.21: Fundamentals 20 - Algebra–Basics

LEARNING GOALS

By the end of this lesson, you should understand

- The meaning of vocabulary words: constant, variable, term, coefficient
- That evaluating an expression means to use a specific value of the variable

By the end of this lesson, you should be able to

- Use the given vocabulary while talking about expressions
- Evaluate an expression containing one or more variables at given values of the variable(s)

FUNDAMENTALS OF THE LESSON

A **variable** is a letter or symbol that we use to hold the place of a value that can change from one situation to the next. For example, if I were counting the number of birds on my birdfeeder each evening, this value would change from night to night. I could say that I was going to use the letter b to represent the number of birds on my birdfeeder at 7:00 pm each evening, and one day b might be 5 (if there are 5 birds) but another day b could be 12.

A **constant** is a value that does not change. Numbers are constants. The number 7 always has a value of 7 and does not change! The number of bird feeders I have is 3. The number of feeders I have does not change; it is constant.

A **term** is something in an algebraic expression that is separated by a plus or minus sign. Constants are terms and variables by themselves are a term. We can also get a term when we multiply variables and constants together. For example, 3b. The **coefficient** is the constant part of the term (in this example, 3). If there is a variable in the term (in this example, b), the coefficient is being **multiplied** by the variable (in this example, 3b would represent multiplying 3 by the number of birds).

Given the expression, 3b + 1, 3 is the **coefficient**, b is the **variable**, 1 is the **constant**. There are 2 terms: 3b and 1.

TRY THESE

Getting to know algebra vocabulary

Let’s say that I want to give pencils to each of the students in my classes. I want to give each student 3 pencils, and I want to bring the correct amount of pencils to each class. Because I have a different number of students in each of my classes, I will call the number of students in each class x. x is a *variable* representing a value that changes. I know that I need 3 pencils for each student times the number of students in that class. I can write this as the term 3x. I can put more than one term together to form an **expression**. If I want to bring each student 3 pencils and also leave 5 extra pencils in the classroom, then I could use the expression 3x + 5 to represent the total number of pencils that I will bring to each class. The 5 is a *constant* that does not change even if the number of students in the class changes.

1. Given the expression x + 3y + 5z + 2, answer the following questions.

A. How many terms does this expression have?

B. Which term only has a constant and no variable in it?

C. What is the coefficient of the 2nd term?

D. What is the variable in the 3rd term?

2. Write an expression that has 3 terms, has 2 different variables, has a constant term of 6 **and** has a term with a coefficient of 12.

NEXT STEPS

Finding the Total Price

You are running the concession stand at a local music festival. In the middle of the show, your cash register suddenly breaks! Oh, no! You can keep selling, but you will just need to figure out the customers’ totals in your head.

Here is the price list:

Hot Dog $2 Water $1

Hamburger $3 Hot Cocoa $2

Note: These prices include tax, so you do not need to add tax to them.

Let's pretend for a minute that you **only** sold hot dogs. In that case, each person's total would be $2 times however many hot dogs they bought. We could use a *variable* to represent the number of hot dogs bought, since that will change (or *vary*) for each person. We could call the total cost for each person 2d, where d represents the number of hot dogs bought. (Remember that when a number and a variable are right next to each other, that means that they will be multiplied.)

3. How much would you charge someone who bought 6 hot dogs? Explain how you arrived at your answer.

When we substitute a specific value in place of a variable, that is called **evaluating** the expression for that value of the variable. For example, in the last problem, you substituted the 6 in place of d, then multiplied it by 2.

We don't really want to just sell hot dogs, though, so let's see how we could figure out each person's total with the four items that you do sell.

We could represent the entire cost with the following **expression** made up of 4 **terms**:

$$2d + 3b + w + 2c$$

4. What do you think each of the variables in this expression represent?

d =

b =

w =

c =

5. The following four customers come to the concession stand. Figure out each of their totals using the above expression.

A. Bob has 3 waters, 2 cocoas, 4 hot dogs

B. Liliana is buying 2 hot dogs, 3 hamburgers, and 3 waters

C. Filipe is buying 1 hamburger and 2 hot cocoas.

D. Stefanie is buying 2 hamburgers, 3 hot dogs, 2 waters, and 3 hot cocoas.

6. Evaluate each of the following expressions for the given value of the variable:

A. $3x + 4$ for $x = 2$

B. $7(x + 3) + 5$ for $x = 18$

C. $2(x + 15) + y^2 + z$ for $x = 3$, $y = 2$, $z = 10$

FURTHER APPLICATIONS

Notice that in Next Steps, the expression was written for you. Now try to write your own expression for the following situation.

7. Entrance tickets for an amusement park cost $30 for adults and $22 for children. We want to figure out an expression that we can use to find the total amount of money the park made from ticket sales each day.

A. What are your variables, and what does each one stand for? (In other words, what are the values that will change from one day to the next?)

B. Write your expression for the total amount of money the park made from ticket sales each day.

C. This expression can also be used to find the total cost for each family who buys tickets to the park. Use your expression to find the total cost for a family of 4 adults and 7 children. Show your work.

Questions: Algebra Basics

1. Answer the following questions for the expression $5x + 3y + 4$.

How many terms does this expression have?

What is the coefficient of the first term?

What is the variable in the second term?

2. Answer the following questions for the expression $145g^2 + 37jk + m$.

A. How many terms does this expression have?

B. Does this expression have a constant term? Explain your answer.

3. The term x^2 does not look like it has a coefficient, but it actually does. Can you explain what the coefficient is? (**Hint**: think about buying water in the expression used in the lesson.)

4. Evaluate 16a + 15b for a = 3 and b = 8. Show your work.

5. Evaluate 5(a + 2b) + 4bc for a = 5, b = 6, and c = 3. Show your work.

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5.22: Fundamentals 21A - Geometry–Perimeter, Area, and Volume

LEARNING GOALS

By the end of this lesson, you should understand

- perimeter is the length of the outline of a polygon.
- perimeter is measured in linear units.
- area is measured in square units.
- the difference between perimeter, area, and volume.

By the end of this lesson, you should be able to

- find the perimeter or area of a rectangle given its length and width.
- find the volume of a rectangular prism given its length, width, and height.
- find the length or width of a rectangle given its perimeter.
- find the length or width of a rectangle given its area.

FUNDAMENTALS OF THE LESSON

The perimeter of a shape is the distance around the shape. This distance is the length of the line that outlines the shape. For a rectangular figure, the perimeter is found by using the formula:

Perimeter equals 2 times the length plus 2 times the width

OR

P equals 2 times length plus2 times width

One way to visualize the concept of perimeter is to imagine a fence bordering all sides of a field. The perimeter of the field is the length of the fence.

The area of a shape is the amount of space inside the shape. For a rectangular figure, the area is found by using the formula:

Area equals length times width

One way to visualize the concept of area is to imagine the amount of space that the field takes up inside the fence. A square is a special rectangle where all sides are equal. The area of a square can be found using the formula:

Area equals side times side

We can find the area of other shaped figures too, such as a triangle or trapezoid.

The area of a triangular figure is found by using the formula:

Area equals begin fraction 1 over 2 end fraction times length times width

The area of a trapezoidal figure can be found by using the formula:

Area equals begin fraction 1 over 2 end fraction times open bracket base plus base close bracket times height

Image showing formula to calculate area of trapezoid.

Volume is three dimensional space. For a rectangular figure, usually referred to as a rectangular prism, the volume is found by using the formula:

Volume equals length times width times height

To visualize the volume of a three-dimensional region, you can imagine the three-dimensional space of the field if you consider the height of the fence.

1. Find the perimeter of each polygon below.

A	B
Polygon with square bas 10 x 10 and two triangles on top with height 12 and hypotenuse 13.	Polygon with 12ft by 28ft base. Triangle on top at the left has height 12ft with hypotenuse 13ft. Length is 26ft across the top.

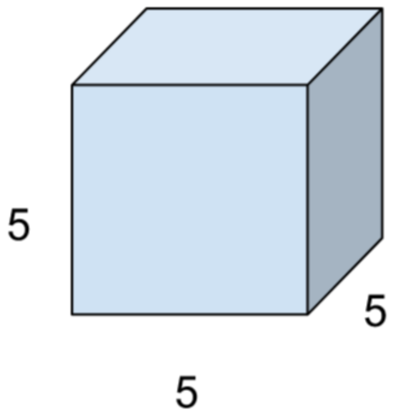
2. Find the total area of each figure above.

A.

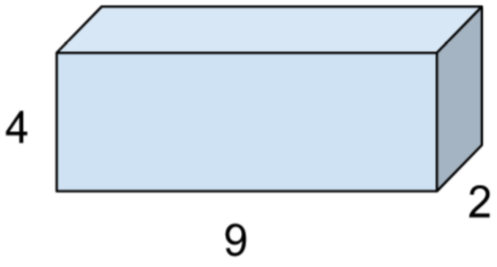
B.

3. Find the volume of each figure below.

A



B



TRY THESE

Room Makeover

Walter and Emily have just finished painting their guest bedroom. Now, they are ready to install new wood trim to the tops of all four walls and install a new beige carpet. The dimensions of the room are 16 feet long by 12 feet wide.

4. How many linear feet of trim will be needed for the room?

5. The carpet installation will cost $6.00 per square foot. What is the total cost of installation of carpet for the guest bedroom?

Walter is building a small closet in the basement. He wants the floor to have an area of 40 square feet with a perimeter of 26 feet.

6. If the dimensions of the closet are whole numbers, what are the length and width of the closet? Explain your thinking. (**Hint**: A drawing might be helpful).

7. What will the volume of the closet be if it is going to be 6 feet high? Include units.

FURTHER APPLICATIONS

8. Rob is making a rectangular concrete countertop to fit on top of a cabinet that is 36 inches wide by 24 inches deep. He plans to use this as a small working island in the kitchen. The countertop must have a 3 inch overhang on all four sides to properly fit the cabinet. (Hint: the answers below will require converting inches to feet).

A frame needs to be made for the concrete countertop out of lumber and an extra foot is necessary for framing overlap.

A. What is the total amount of lumber (in linear feet) needed to build this rectangular frame?

B. How much countertop space will be available to work on when the countertop is finished?

C. The countertop should be 3 inches thick. How many cubic feet of concrete will Rob need to make the countertop? Round to one decimal place.

Questions: Geometry—Perimeter, Area, and Volume

1. A rectangular park measures 10 meters by 15 meters. What is the perimeter of the park?
2. What is the area of the park from Question 1?
3. A rectangle has a length of 8 inches and a perimeter of 22 inches. What is the width of the rectangle?

4. To warm up at soccer practice, Austin and his team jog around a rectangular soccer field that is 77 yards by 120 yards. How many yards do Austin and his team jog if they go around the field four times?
5. David wants to install crown molding around the ceiling of his living room. The length of the room is 17 feet and the width is 12 feet. How many linear feet of crown molding should David order?

6. Joshua made a 3-dimensional cube such that each face was a square measuring 13 inches on each side. What is the volume of his cube?

7. Lindsay has a square with sides that are double the length of the sides of the square faces on Joshua's cube (see the previous question). What is the area of her square?

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5.23: Fundamentals 21B - Comparison and Inequality Symbols

LEARNING GOALS

By the end of this lesson, you should understand

- that a number can either be less than, greater than, or equal to another number.
- that visual representations using length or symbols can be helpful when trying to compare two numbers.
- that often mathematical symbols are used in place of words to express the comparison between two numbers.

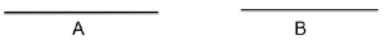
By the end of this lesson, you should be able to

- identify and use mathematical symbols of less than, greater than, less than or equal to, and greater than or equal to in order to compare two numbers.
- read information and determine how values compare to each other.

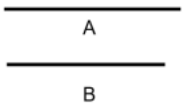
FUNDAMENTALS OF THE LESSON

Comparing line segments: visually, with words, and with symbols

If we inspect line segments A and B as they lie side by side, it is difficult to say which one is longer.



If line segments A and B are placed one below another, it is easier to compare them to each other.



1. What statements of comparisons can be made about line segments A and B? Symbolically, how can these statements of comparison be translated?
2. If line segment A has a length of 52 mm and line segment B’s length is 48 mm, how can you mathematically prove that one line segment is greater than the other?

TRY THESE

How do our numbers compare?

Bobby works as a police officer in Shotokan Township. She believes that she is not being paid fairly as compared to her coworkers. At her yearly performance review, she brings this up to her captain, but is still unsure if her salary is fair given that she has been on the job about the same number of years as her two fellow officers, Jed and Todd. She is aware that public employees' salaries are available to the general public. When she looks up her salary to compare to Jed and Todd’s, she discovers that she does not make less than Jed, but she does make less than Todd.

3. Using B for Bobby’s salary, J for Jed’s, and T for Todd’s, set up statements of inequality to compare:

A. Bobby’s salary to Jed’s.

B. Bobby’s salary to Todd’s.

C. Can we compare Todd and Jed’s salaries?

4. If Jed makes \$55,100 a year, and Todd makes \$61,200 a year, what could Bobby possibly earn? Circle all that apply. (**Note**: More than one answer may be acceptable.)

    (i) Bobby could earn \$55,200.

    (ii) Bobby could earn \$61,200.

    (iii) Bobby could earn \$61,201.

    (iv) Bobby could earn \$55,000

(v) Bobby could earn $55,100.

5. Besides your answer(s) from Question 4, what other salary could Bobby earn?
6. Of all three police officers, who makes the most? Who makes the least? If they all have the same rank and similar time of service, do you believe that any of the police officers are being paid unfairly (either too little or too much, as compared to the others)? Do we have enough information?

NEXT STEPS

Changes in generation between the 115th and 116th House of Representatives

The 115th House of Representatives was one of the oldest in history, but the 2018 midterms also ushered in a wave of younger Gen X'ers and Millennials elected to Congress. The average age of an incoming member of Congress is 47, a full decade lower than the average age of the 115th Congress. Below is a table which shows the makeup of the 115th and 116th House, by generation; Silent (1928 – 1945), Baby Boomers (1946 – 1964), Generation X (1965 – 1980), and Millennials (1981 – 1996).

Generation	**Before election (115th House)**	**After Election (116th House)**	**Percent Change**
Silent (1928 – 1945)	42	37	-13.5%
Baby Boomer (1946 – 1964)	270	234	-13.3%
Gen X (1965 – 1980)	118	138	17%
Millennial (1981 – 1996)	5	26	420%

7. Use the table to answer the following questions.

A. Which generation had the greatest **positive** change in representation?

B. What was the change?

C. Which generation had the greatest **negative** change in representation?

D. What was the change?

E. Which generation had the greatest **positive percent** change in representation?

F. Which generation had the greatest **negative percent** change in representation?

8. Use words to express the comparison of the number of representatives from the Silent generation to the number of Millennials for the 115th House and then the 116th House. Now, using S for Silent and M for Millennials, write this comparison symbolically.
9. Use words to express the comparison of the percent change in representation for Silent and Millennials.

Questions: Comparison and Inequality Symbols

1. Write a mathematical statement of comparison for 17 and 27.
2. Write a mathematical statement of comparison for 120,000,000 and 12,000,000.
3. Let $x = 9$. How does x compare to 7 and 11? Write this comparison as a single mathematical statement using x.

In 2019, there were changes for both the Democratic and Republican parties between the 115th and 116th House of Representatives.

Party	**Before election (115th House)**	**After Election (116th House)**	**Percent Change**
Democrat	193	235	21.8%
Republican	235	200	-14.9%
Vacant	7	1	-85.7%

4. Which party had the greatest change in number of representatives? Fill in the blanks in the statement below.

The Democratic Party had ___________ a change than the Republican Party with an increase of

____________ representatives compared to a decrease of ___________ representatives.

5. Which party had the greatest percent change in number of representatives? Fill in the blank in the statement below.

The percent change for the Democratic Party was _____________ the percent change for the Republican Party.

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5.24: Fundamentals 22 - Translating Word Expressions

LEARNING GOALS

By the end of this lesson, you should understand

- that expressions from real life can be expressed using algebraic symbols.

By the end of this lesson, you should be able to

- translate word expressions into algebraic notation.
- write statements and relations using algebraic notation.

FUNDAMENTALS OF THE LESSON

1. Write the symbols for addition, subtraction, multiplication, and division.
2. Write the symbols denoting equality, less than, greater than, less than or equal to, and greater than and equal to.
3. Evaluate $\frac{x+13}{4}$ for x = 31.
4. Determine whether the following inequalities are true of false:

A. $6.5 > 7$

B. $-10 \leq -3$

C. $8 \geq 8$

D. $-7 < -6.5$

5. Evaluate:

A. 6^2 = _______

B. $(-5)^2$= _______

TRY THESE

Expressions from School

Mathematics is so useful because it helps us solve real-life problems. **Algebra** is the area of mathematics where **symbols** for quantities, unknowns, operations, and relations are introduced. Algebra also encompasses the study of the rules for manipulating those mathematical symbols. Before we can solve a real life problem, we need to learn how to translate real-life expressions into the language of algebra. Let’s try translating some common expressions you may have been exposed to in the past.

6. A. How would you translate “The area of a rectangle equals the length times the width.” into algebra?

B. The letters you wrote are called **variables** because they represent different numbers involved in the formula for the area. Why did you choose those letters for your answer in part A? Indicate what each letter (variable) means.

7. Translate “The sum of the interior angles of a triangle equals 180 degrees.”

8. Another famous statement you may recall from school is the Pythagorean Theorem: “The sum of the squares of the legs of a right triangle equals the square of the hypotenuse.” Can you translate this theorem into algebraic language? Draw a picture and label. Explain what your variables mean.

9. The perimeter of a rectangle equals twice the sum of its length and its width. How would you write this statement in algebraic notation?

10. Translate the following expressions into algebraic notation.

A. A number plus 30. Use x to represent the number.

B. A number decreased by 55. Use x to represent the number.

C. The product of a and b.

D. Half of the sum of x and y.

E. Three times the difference between s and t.

F. A third of a number is less than 5. Use x to represent the number.

G. A number divided by the sum of x and 2 is greater than or equal to 4. Use y to represent the number.

H. 45% of the average of x, y and z.

FURTHER APPLICATIONS

11. **Consecutive numbers** are integers which follow each other in increasing order without gaps. For example: 12, 13, 14, or -5, -4 and -3. Using algebraic notation, and only **one** variable, write the statement “The sum of three consecutive numbers is equal to 51.”

12. One important factoring formula from algebra can be explained as “the sum of two terms times the difference of the same two terms equals the difference of the squares of the terms”. Can you translate the statement between quotation marks into algebraic notation?

13. Different countries use different units to measure temperature. In the US, we use degrees Fahrenheit (ºF), while in Europe degrees Celsius (ºC) are preferred. Translate the following statement into algebraic notation: “The temperature in degrees Celsius (C) equals five ninths of the temperature in degrees Fahrenheit after it is decreased by 32.” Do not simplify.

Questions: Translating word expressions into Algebraic Notation

Translate the following expressions/statements into algebraic notation:

1. The sum of a number and half of another number. Use a and b as the variables.
2. 15% of the difference between the adjusted gross income and $9,075. Use i as the variable. Do not simplify.
3. The square of the sum of a and b equals the square of a plus twice the product of a and b plus the square of b.
4. The area of a triangle (A) equals half of the product of the base (b) times the height (h).
5. The area of a circle (A) equals the product of and the square of the radius (r).
6. The area of a trapezoid (A) equals half the sum of its bases (a and b) times its height (h).

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5.25: Fundamentals 23 - Linear Equations (Part 1)

LEARNING GOALS

By the end of this lesson, you should understand

- the difference between an expression and an equation.
- that addition undoes a subtraction and subtraction undoes an addition.
- how to work a simple addition or subtraction problem backwards.

By the end of this lesson, you should be able to

- solve one-step linear equations involving a sum or a difference by working the problem backwards,
- solve one-step linear equations by adding to or subtracting from both sides of the equation.

FUNDAMENTALS OF THE LESSON

Reminder: In algebra, we use

- numbers and letters to represent quantities known and unknown,
- expressions to represent operations involving numbers.

TRY THESE

How Much Money Was In The Purse?

Mary enjoys long walks, but when she goes out she never knows how much money she has in her purse.

1. Last Monday, Mary went out for a walk. She stopped at Dunkin Donuts and spent $2 on a coffee. When she got back home, she counted how much was left in her purse. She had a total of $6.

A. How much money did she have before she left?

B. What operation did you use to get that result? With what numbers?

2. Let x represent the amount of money Mary had in her purse right before she went out for her walk.

A. Using x, how would you write an expression for the amount she had when she came back? Remember that she had x dollars and spent $2.

B. How would you write that the amount you wrote in part A is equal to 6?

C. Without carrying out any calculations, using x - 2 = 6 how would you write what x is equal to? Remember the operation you did. Do not do the calculation, just indicate it.

D. From your answer to part C, determine what x is equal to.

What you obtained in question 2A is a mathematical expression. A **mathematical expression** is a group of symbols including numbers, variables and operations, used to show the value of something.

An **equation** is a statement saying that two mathematical expressions are equal to each other. Examples of equations are your answers to 2B, 2C, and 2D.

3. Now you have the equation x - 5 = 9.

A. Can you find x by performing operations that are similar to the ones in question 2?

B. What operation was originally being done with 5 on the left side of the equation? How did the 5 “move” to the right?

C. Now substitute the value you obtained for x into the equation x - 5 = 9. What do you observe?

In Question 3 you found the value of x that turned the equation x - 5 = 9 into an equality which is a true statement. That value is 14. 14 is **the solution of the equation** x - 5 = 9 because when you replace x with 14, the equation becomes an equality. **Solving an equation** means finding its solution.

4. Practice what you just learned and solve the following equations. Make sure you master this method.

A. y - 12 = 35

B. n - 5.2 = 7.9

C. 3 = x - 12

5. Sometimes students are taught to “add to both sides” to solve a problem like x - 5 = 9. In your opinion, what are the advantages and disadvantages of this method compared to what we practiced above?

6. Mary went for a walk again. Once more, she didn’t know how much money she had in her purse. During her walk, Mary met her friend Pablo who gave her $3 which he owed her. She went back home and noticed that she now had a total of $10 in her purse.

A. How much money did Mary have before she met Pablo?

B. What operation did you do in order to find that amount?

C. If you were to denote by x the amount of money Mary *initially* had in her purse, how would you write the fact that Pablo gave her $3, and when she got home she had a total of $10?

D. **Without doing any calculations**, how would you write what x is equal to? Remember the operation you did. Do not finish the calculation.

E. From your answer to part D, determine what x is equal to.

7. Now you have the equation x + 8 = 12.

A. Can you find x by performing operations that are similar to the ones in question 6?

B. What operation was originally being done with 8 on the left side of the equation? How did the 8 “move” to the right?

8. Solve the following equations:

A. y + 11 = 47

B. n + 9 = -52

C. 32 = 5 + x

FURTHER APPLICATIONS

Solve the following equations that include fractions. Try using the method you learned today:

9. $x-\frac{2}{3}=\frac{5}{7}$

10. $y-\frac{3}{2}=\frac{1}{4}$

Questions: Linear Equations (Part 1)

Solve the following equations using the primary method from this lesson.

1. x - 9 = -16
2. -5 = x - 14
3. y - 10.8 = 5.6
4. $-\frac{1}{2}+y=\frac{3}{2}$
5. n + 9.7 = 4.6
6. 23 + n = -81
7. 14 = y + 25

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5.26: Fundamentals 24 - Linear Equations (Part 2)

LEARNING GOALS

By the end of this lesson, you should understand

- that multiplication undoes a division and division undoes a multiplication;
- how to work a simple multiplication or division problem backwards.

By the end of this lesson, you should be able to

- solve one-step linear equations involving a product or a quotient by working the problem backwards,
- solve one-step linear equations by multiplying or dividing both sides of the equation by the same number.

FUNDAMENTALS OF THE LESSON

Reminder: In algebra, we use

- numbers and letters to represent quantities known and unknown,
- expressions to represent operations involving numbers.

TRY THESE

How Much Money Were They Making Before The Changes?

Eddy has been working at the same small company for a long time. His hourly wage has varied.

1. In the year 2000, Eddy received a promotion. His hourly rate tripled, coming to be \$24 an hour.

A. How much was Eddy making right before his hourly rate tripled?

B. What operation did you use to get that result? With what numbers?

2. Let x represent Eddy's hourly rate before his promotion.

A. Using x, how would you write an expression for Eddy's hourly rate after his promotion? Remember that he was making x, and his rate tripled.

B. How would you write that the amount you wrote in part A is equal to 24?

C. **Without carrying out any calculations**, how would you write what x is equal to? Remember the operation you did. Do not do the calculation, just indicate it.

D. From your answer to part C, determine what x is equal to.

What you obtained in 2A is a mathematical expression. A **mathematical expression** is a group of symbols including numbers, variables and operations, used to show the value of something.

An **equation** is a statement saying that two mathematical expressions are equal to each other. Examples of equations are your answers to 2B, 2C, and 2D.

3. Now you have the equation $2x = 14$.

A. Can you find x by performing operations that are similar to the ones in question 2?

B. What operation was originally being done with 2 on the left side of the equation? How did the 2 "move" to the right?

C. Now substitute the value you obtained for x into the equation $2x = 14$. What do you observe?

In Question 3 you found the value of x that turned the equation $2x = 14$ into an equality which is a true statement. That value is 7. 7 is **the solution of the equation** $2x = 14$ because when you replace x with 7, the equation becomes an equality. **Solving an equation** means finding its solution.

4. Practice what you just learned and solve the following equations. Make sure you master this method.

A. $5y = 55$

B. $-4n = 25$

C. $-5.1 = -6x$

5. Sometimes students are taught to “divide both sides” to solve a problem like 2x = 14. In your opinion, what are the advantages and disadvantages of this method compared to what we practiced above?

6. In the year 2009, a recession hit. Eddy’s company had to slash wages and his hourly rate became half of what it was right before the recession. After the cut, Eddy started making $25 per hour.

A. How much was he making right before the cut?

B. What operation did you do in order to find that amount?

C. If you were to denote Eddy’s hourly wage before the cut by x, how would you write the fact that his new hourly rate is $25?

D. **Without doing any calculations**, how would you write what x is equal to? Remember the operation you did. Do not finish the calculation.

E. From your answer to part C, determine what x is equal to.

7. Now you have the equation x ÷ 4 = 5.

A. Can you find x by performing operations that are similar to the ones in Question 6?

B. What operation was originally being done with 4 on the left side of the equation?

C. How did the 4 “move” to the right?

8. Practice what you just learned and solve the following equations.

A. y ÷ 3.2 = 8.5

B. $\frac{2n}{3} = -10$

C. $-\frac{x}{6} = -7$

D. $187 = \frac{11x}{5}$

FURTHER APPLICATIONS

Solve the following equations that include fractions. Try using the first method you learned today:

9. $\frac{2}{3}x = \frac{5}{7}$

10. $-\frac{3}{4}y = \frac{3}{10}$

11. $-\frac{10}{3}z = -\frac{15}{9}$

Questions: Linear Equations (Part 2)

Solve the following equations using the primary method from this lesson.

1. 8x = -33
2. -51 = -3x
3. 0.25y = 10
4. $\frac{y}{8} = \frac{3}{2}$
5. $\frac{n}{0.1} = 6.4$
6. $\frac{3n}{10} = -1.2$
7. $\frac{5}{3}y = 7$

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5.27: Fundamentals 25 - Pattern Recognition

LEARNING GOALS

By the end of this lesson, you should understand

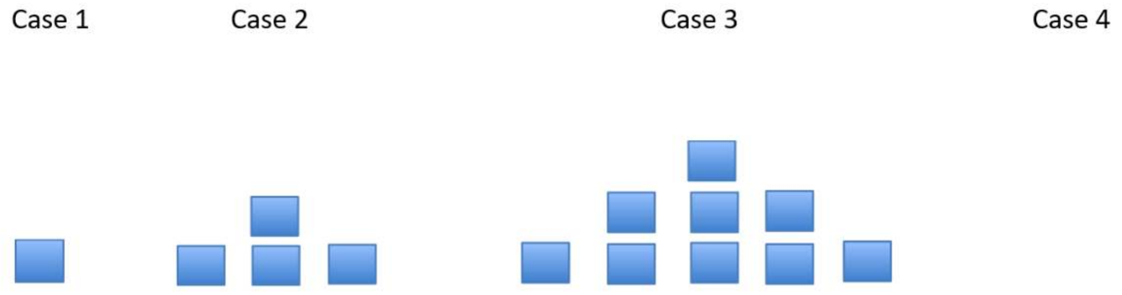
- that different methods can be used to recognize patterns.
- that patterns often occur naturally.
- that visual methods of pattern recognition can be connected with numeric patterns.

By the end of this lesson, you should be able to

- identify the visual or numeric pattern in order to answer specific questions, such as “What will come next?” , “How many (items) are in the 5th case, the 100th case, the nth case?”
- recognize that individuals (or small groups) will be able to develop their own way of seeing pattern development.
- listen to and respect each other’s thinking when attempting to identify or understand patterns.
- represent patterns in multiple ways (visual, symbolic, or numeric).

FUNDAMENTALS OF THE LESSON

Take a few moments to view the shapes below[18] for the 1st, 2nd, and 3rd case.



1. A. How many cubes are in the 1st case?

B. How many cubes are in the 2nd case?

C. How many cubes are in the 3rd case?

D. What would the 4th case look like? How many cubes would there be in the 4th case? Draw it.

2. What are different ways that you could represent the pattern growth? That is, how do you see the shapes growing?
3. Let’s explore further cases...

A. How many cubes would be in the 5th case?

B. How many cubes would be in the 10th?

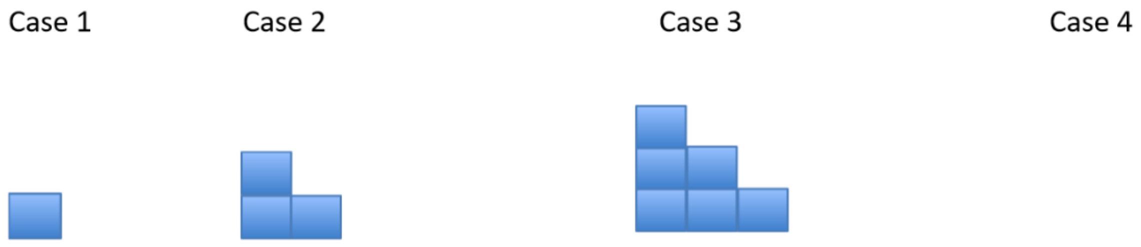
C. How many cubes would be in the 100th?

D. How many cubes would be in the nth?

TRY THESE 1

Staircase

Take a few moments to view the shapes below[19] for the 1st, 2nd, and 3rd case.



4. What pattern do you notice?
5. Let’s explore...

A. How many cubes are in the 1st case?

B. How many cubes are in the 2nd case?

C. How many cubes are in the 3rd case?

D. How many cubes would there be in the 4th case? What would the 4th case look like?

E. How many cubes would there be in the 5th case?

F. How many cubes would there be in the 10th case?

G. How many cubes would there be in the 100th case?

H. How many cubes would there be in the nth case?

TRY THESE 2

Fibonacci

Patterns can be found in many areas of our everyday lives such as in nature and architecture. One such naturally occurring pattern is known as the Fibonacci sequence which is found throughout nature in such objects as the structure of pinecones, number of branches in a tree, the pattern of seeds in a sunflower, and the population growth of animals.

6. The numeric Fibonacci sequence begins with:

1, 1, 2, 3, 5, 8, 13, 21 …

A. Take a few moments to try to determine the pattern in this sequence. Explain the pattern you see.

B. What are the next three numbers in the sequence?

7 Now take a look at the image below which is known as the nautilus spiral. How does this visual representation relate to the numeric sequence?

Image result for fibonacci spirals

FURTHER APPLICATIONS

What Comes Next?[20]

Related image

8. Before we answer the question of “What number comes next?”, take a few moments to write what each number below the images mean, and how the numbers relate to the visual representation.
9. What number comes next?

10. A. For an image that continues this pattern with a total of 64 squares, how many squares would be white?

B. What about for an image with a total of 81 squares?

C. What about for an image with a total of 100 squares?

D. What about for an image with a total of n^2 squares?

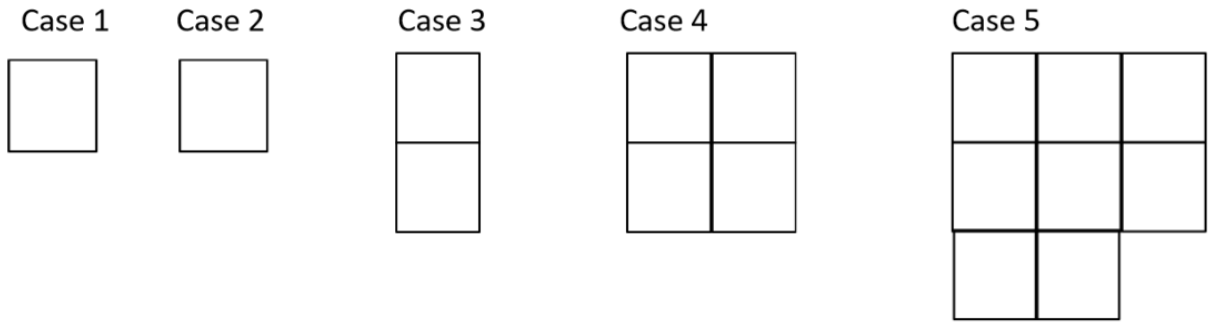
Questions: Pattern Recognition

What number comes next?

1. 5, 9, 13, 17, 21, 25, ?
2. 6, 8, 12, 20, 36, ?
3. (**Hint**: Think about the sum of the numbers in each row).

4	3	7	1
6	2	3	4
8	1	3	5
9	1	4	?

4. Find the number of cubes we would find in Case 6 based on the previous cases drawn below.



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[18] Adapted from an activity by Parker, R, *Mathematics Education Collaborative*.

[19] Jo Boaler, Mathematical Mindsets, 2016

[20] https://brilliant.org/practice/pattern-recognition-intro/

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5.28: Fundamentals 26 - Plotting Points

LEARNING GOALS

By the end of this lesson, you should understand

- the Cartesian coordinate system is used for graphing ordered pairs and locating points.
- the horizontal x-axis intersects the vertical y-axis at the origin.
- the system is separated into 4 sections called quadrants.
- the first number in an ordered pair is known as the x-coordinate; the second number is known as the y-coordinate.
- the positive side of the x-axis is to the right of the origin and the negative side is to the left.
- the positive side of the y-axis is above the origin and the negative is below.

By the end of this lesson, you should be able to

- understand key vocabulary associated with plotting points such as ordered pair, x-coordinate y-coordinate, origin, x-axis, y-axis and quadrant.
- correctly plot points.
- name ordered pairs for given points, and correctly identify the corresponding quadrant in which the point lies.
- use number sense to determine an exact location of a point.

FUNDAMENTALS OF THE LESSON

The Cartesian coordinate system is a grid that was developed to graph ordered pairs. An **ordered pair** is a pair of numbers used to locate a point and is written in the form (x, y), where x is the x-coordinate and y is the y-coordinate. The coordinate system is made up of two perpendicular number lines that meet at the point (0, 0). This special point is called the origin. Each perpendicular line is an axis. The horizontal line is called the x-axis and the vertical line is called the y-axis.

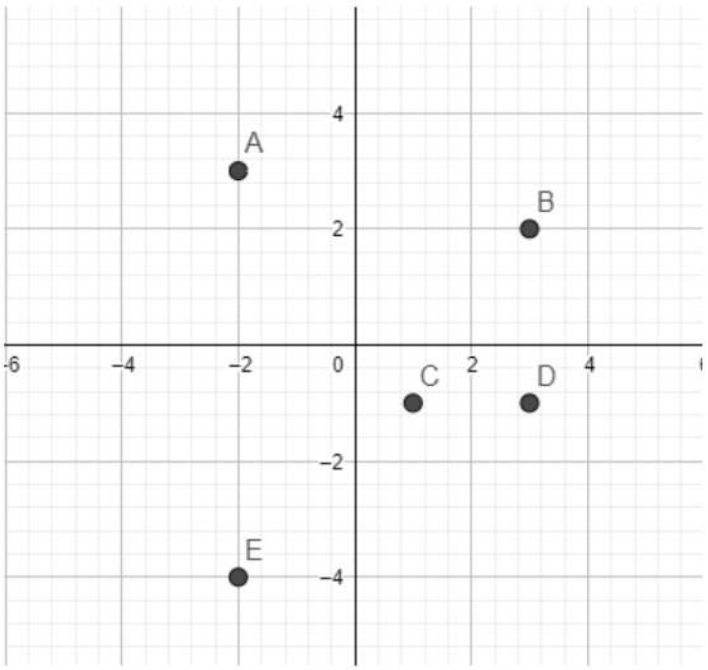
The coordinate system is divided into 4 parts. These parts are called quadrants, and are labeled with numbers 1-4 in a counterclockwise direction, starting in the upper right quadrant. Usually, Roman numeral numbers I, II, III, and IV are used to label each quadrant.

An image of the coordinate system is shown below. Notice, the positive x-axis is to the right of the origin and the negative x-axis is to the left. The positive y-axis is above the origin and the negative y-axis is below.

Image of coordinate plane quadrants. Origin at (0,0), y-axis, and x-axis are marked. y-axis ranged from -5 to 5; x-axis ranged from -6 to 6. Quadrant I is top right; quadrant II is top left; quadrant III is bottom left; quadrant four is bottom right.

To represent a point on the coordinate system where -5 is the x-coordinate of a point and 6 is the y-coordinate of a point, we use the ordered pair notation of (-5, 6). The -5 lets us know how far the point is to the left of the origin. The 6 lets us know how far the point is above the origin.

1. Write an ordered pair for each point on the grid.



A: _______

B: _______

C: _______

D: _______

E: _______

2. Plot the ordered pairs on the coordinate plane above. Describe the location of the point, including what quadrant the point lies in.

A. Plot M (-4,1).

B. Which quadrant is (-4,1) in?

C. Plot S (-2,-2).

D. Which quadrant is (-2,-2) in?

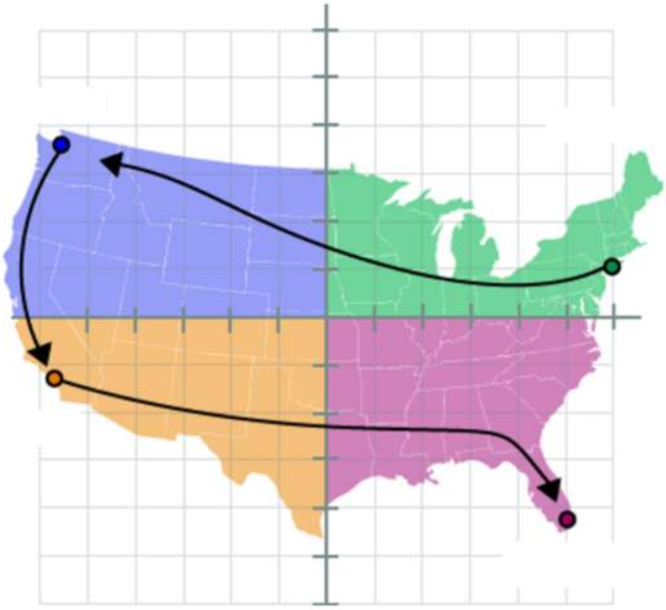
E. Plot U (3,-2).

F. Which Quadrant is (3,-2) in?

TRY THESE 1

What is their location?

A family plans to take a theme park trip to different places around the United States. The family has scheduled to leave New York by plane and fly to Washington. After staying in Washington for three days, the family flies to southern California for a week. From California, the family goes to its final destination, Florida. A path[21] of the trip is shown on the coordinate plane below.



3. The family leaves New York to start their trip.

A. In which quadrant does New York lie?

B. At what point does the trip begin (use ordered pair notation)?

4. Name a state that lies directly on the negative y-axis.

5. A. At what point does the trip end if each tick is a value of one unit?

B. In which quadrant does that point lie?

6. Write two different ordered pairs that represent locations in Texas.

TRY THESE 2

Navigate the map

A map of the rooms in a college hall is fit to the coordinate plane below. Sarah, a college ambassador, has given this map to each freshman student to help them navigate on their first day of classes in this particular building.

Coordinate plane with Exit 1, Physics, Math, Office, Entrance, Psychology, and Biology locations highlighted.

7. A. Savien is in the class that is located at (-5,-1). What class is he in?

B. Describe in words how to get from *the entrance* to this point.

8. A. Savien's next class is 4 units up and 4 units to the right on the map from the class where he is now. What class is Savien going to next?

B. Find the ordered pair that states the location of that class.

9. Adrianna is at Exit 1, but has to go to the office. Give Adrianna directions on how to get to the office.

10. On the map, which **classes** are located in **quadrant IV**? Describe the coordinates of all classes in this quadrant.

FURTHER APPLICATIONS

Your campus is located at (3,-1), which is 3 blocks east and 1 block south of the center of town. To get from your apartment to the campus, you walk 4 blocks west and 2 blocks north. (**Hint**: It may help to draw a Cartesian coordinate system with everything labeled).

11. What ordered pair corresponds to the location of your apartment?

12. Is your apartment or your campus closer to the center of town? Explain.

Questions: Plotting Points

Tell whether the following statement is sometimes, always, or never true. Explain your reasoning.

1. The x-coordinate of points in quadrant IV are negative.

2. The x-coordinate of a point in quadrant III has the same sign as the y-coordinate of a point in quadrant II.

3. Name the ordered pair that is 4 units right and 2 units down from (-3,2).

4. In which quadrant is the point $\left(-\frac{5}{2}, \frac{10}{3}\right)$.

5. Delanie made a map of her running trail through the woods in such a way that each intersection is a point on a coordinate plane. Delanie plans to start at point (3,2). Give the ordered pair of where she will be if she moves 5 units to the right and 7 units down on her map.

6. After Delanie moves 5 units to the right and 7 units down, what quadrant is she in?

7. Say we have a figure with the vertices A (0, 1), B (-4, 1), C (-4, -1), and D (0, -1). Draw the figure in the coordinate plane, and then find the perimeter of the figure.

8. Find the area of the figure.

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[21] Image retrieved from www.magoosh.com.

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5.29: Fundamentals 27 - Technology and Math

LEARNING GOALS

By the end of this lesson, you should understand

- how to use the correct keystrokes on a scientific calculator to get a correct result.

By the end of this lesson, you should be able to

- solve problems involving a variety of operations using a scientific calculator.
- solve problems in different forms, such as scientific notation.

FUNDAMENTALS OF THE LESSON

Basic Calculations

Your scientific calculator is programmed to use *order of operations*. The operations are addition, subtraction, multiplication, division, exponentiation, and grouping. The order in which the operations are performed is established so that we are all in agreement of the result. Your calculator follows the order of operations agreement and will perform the operations in this order: 1) exponents (powers or roots); 2) operations within grouping symbols, such as parentheses, brackets, and absolute value; 3) multiplication and division, as they occur from left to right; and 4) addition and subtraction, as they occur from left to right. When using a scientific calculator, enter basic calculations in exactly the same order as written on paper. The calculator will display the calculation as you enter and when you press equal (=), the answer is displayed.

1. Use your calculator to enter in the calculations as you see them below. Write the answer in the space provided.

A. 1295 + 75 × 3.47 Answer:

B. 2025 ÷ 5 × 29 Answer:

C. 16 × (16,550 - 1,200) Answer:

D. (16,500 + 22,500) × (37,500 - 15,500) Answer:

Decimals and Fractions

When solving problems that involve decimals or fractions, you can convert the fraction to a decimal number (by dividing the numerator by the denominator) or use the fraction key. If a fraction cannot be converted into a decimal number that terminates , the fractional value should be used.

For example, to convert the fraction 3/8 to a decimal, if you use your calculator to divide 3 by 8, you will get 0.375, a decimal number that terminates. If you type the fraction 5/6 in your calculator, you will get 0.833333333, a decimal number that never terminates. If this is the case, it is usually better to use the fraction for operations.

Find the fraction key (ABC key) on the calculator. Directly above the fraction key (ABC key) is the mixed number key.

2. Use your calculator to calculate each of the following. Round your answer to two decimal places when needed.

A. Calculate $\frac{3}{8}$ of $180. Answer:

B. Express $\frac{25}{4}$ as a mixed number. Answer:

C. Simplify: $\frac{2}{3}(37,500 - 15,500)$ Answer:

Powers: Using the Exponent Key

There are two keys on the scientific calculator that enable you to perform calculations involving powers. For a square, which is a power of 2, you can use the x squared key. For all powers greater than 2, you will use the exponent key, Upward caret. For all powers greater than 2, enter the number first, which is called the base, and then the exponent key, Upward caret and finally the power. For example, if you were trying to evaluate , you would enter 10, press the exponent key, Upward caret , press 4, and then press enter, which would result in 10,000.

Reading Exponents

Some numbers have too many digits to display on the calculator. When this happens, the calculator displays numbers in scientific notation. On many scientific calculators, the letter E is used to denote 10 to the power of whatever number comes after it. For example 135 billion appears on the scientific calculator as 1.35E11. Take a few moments and write our 135 billion and 1.35E11 to show that they are the same.

You can add, subtract, multiply, and divide large numbers on the calculator just as you would small ones. The results may appear in scientific notation if they have too many digits to be displayed. The value after the letter E represents how many powers of 10 to multiply the decimal number by and you will use this to be able to write out the number.

3. Use your calculator to evaluate each of the following.

A. 100^2 Answer:

B. 10^6 Answer:

Percents

A percent can be thought of and used in different ways. You can convert any number from its percentage to its decimal form by (1) dividing the percentage given by 100 or (2) by using the percent key.

The percent key is located above the digit 8. To use the percent key, first type in the percentage value given, then keystroke the percent key, %, then multiply by the given number you are taking a percentage of.

4. Use your percent key to evaluate each of the following.

A. Find 6% of 1000 Answer:

B. Find 25% of $250.00 Answer:

C. Find 10% of 238 Answer:

D. Find 0.5% of 1590 Answer:

E. Find 8.5% of $290 Answer:

TRY THESE 1

Caloric Baseline

Sarah wants to lose weight. According to BeachBody, LLC[22], to find your calorie target for weight loss you need to first calculate your Caloric Baseline. To calculate your Caloric Baseline, multiply your current weight (in pounds) by 11, and add 400 calories. Then, subtract 600 calories. This will be your Caloric Baseline.

5. Sarah weighs 200 pounds and wants to calculate her calorie target.

A. Write a mathematical expression for the order in which this calculation should be entered into your calculator.

B. Calculate Sarah’s caloric baseline.

TRY THESE 2

Department of Energy (DOE)

At the end of 1994, the Department of Energy’s inventory of high-level radioactive waste was approximately 378,400 cubic meters.[23] The table below shows the high-level radioactive waste at the end of the years 1990, 1991, 1992, 1993, and 1994.

Table showing the high-level radioactive waste at the end of the years 1990, 1991, 1992, 1993, and 1994. 1990 = 253.6, 12.0, 131.7, 1.2, and a total of 398.5. 1991 = 256.4, 10.4, 127.9, 1.7, and a total of 396.5. 1992 has 258.7, 11.2, 126.9, 1.6, and a total of 398.3. 1993 has 261.7, 10.5, 129.3, 2.0, and a total of 403.5. 1994 has 238.9, 11.0, 126.3, 2.2, and a total of 378.4.

6. What is the total amount of recorded HLW (high-level radioactive waste) in cubic meters in 1990? Write this number in expanded form.

7. What are some benefits of using scientific notation in this table versus expanded form?

8. Calculate the total HLW recorded from 1991 through 1993.

9. In 1994, the HLW was 378,400m^3. If this level went unchanged for the next 1000 years, how much HLW would be added to the Earth?

TRY THESE 3

How many calories is that?

A 175-pound man with a moderate exercise schedule should consume 2,800 calories a day, that is 16 calories per pound.[24]

10. Using the chart below, determine how many calories of each nutrient should be consumed each day to reach a target calorie amount of 2,800. Fill in the Target Amount of Calories column.

Nutrient	Percent of Daily Calories	Target Amount of Calories
Total Fat	30%	A
Saturated Fat	7%	B
Trans Fat	1%	C
Total Carbohydrates	50%	D
Simple Sugars	25%	E
Complex Carbohydrates	25%	F
Protein	20%	G

Questions: Technology and Math

1. Simplify: $\frac{(3-7)(4+6)}{-(2+3)}$

2. Simplify: $\frac{4\times\frac{1}{2}}{3\times\frac{8}{9}}$

3. Write the number in standard form: 2.75 E 10

4. Simplify: $(3 \times 10^5)(4 \times 10^3)$. Write the number in standard form.

5. Simplify: $\frac{80.45\times 10^9}{34,482\ thousand}$. Round to the nearest whole number.

6. Calculate 1.5% of 314 *thousand*.

7. You want to leave a 20% tip on your dinner bill of $120.00. What is the total you will pay for dinner, including the tip?

8. Joe and Shelby go to lunch. The bill before taxes and gratuity (tip) is $38.50. If the sales tax rate is 7%, how much sales tax will be added to the bill? Round to the nearest penny.

9. Joe and Shelby want to leave a 20% gratuity (tip) for the waiter, before sales tax has been added to the bill. How much gratuity will be given to the waiter?

10. What is the total bill, including both taxes and gratuity?

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[22] Beach Body Science. *What is Beachbody Science?*, accessed May 31, 2019, https://www.beachbodyondemand.com/blog/beachbody-experts

[23] https://ieer.org/resource/classroom/scientific-notation/

[24] https://www.health.harvard.edu/staying-healthy/dietary-guidelines-and-caloric-percentages

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CHAPTER OVERVIEW

6: Social Justice Lessons

6.1: Quantway Core 1.5-SJ (Gerrymandering) - Student Lesson
  6.1.1: Preparation 1.5-SJ
  6.1.2: Exercise 1.5-SJ
6.2: Quantway Core 1.6-SJ (US Incarceration Rates) - Student Lesson
  6.2.1: Preparation 1.6-SJ
  6.2.2: Exercise 1.6-SJ
6.3: Quantway Core 1.8-SJ (Race and Incarceration) - Student Lesson
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6.1: Quantway Core 1.5-SJ (Gerrymandering) - Student Lesson

SPECIFIC OBJECTIVES

By the end of this lesson, you should understand that

- quantitative reasoning and math skills can be applied in various contexts.
- different electoral maps can lead to different results in an election.
- reading quantitative information requires filtering out unimportant information (introductory level).

By the end of this lesson, you should be able to

- recognize common mathematical concepts used in different contexts.
- apply skills and concepts from previous lessons in new contexts.
- identify a complete response to a prompt that asks for connections between mathematical concepts and a context.

PROBLEM SITUATION 1: COMPARING ELECTION RESULTS

The United States House of Representatives is the lower chamber of the United States Congress, which, along with the Senate, composes the legislature of the United States. U.S. law fixes the total number of U.S. representatives, also called the number of seats in the House, at 435. Seats in the House of Representatives are apportioned among the states according to their population, as determined by the census conducted every ten years. Each state, however, is entitled to at least one Representative. Since 1967, states that are entitled to more than one representative are divided into single-member districts. This means that these states have to be divided into districts that have approximately the same population and each district elects one representative to the House. Because populations change, states typically change their district boundaries after each census.

(1) The image below is the map of a (rectangular) state.[1] This state has 50 precincts with equal population (represented by squares), and has to be divided into 5 districts. Each district consists of exactly 10 precincts, and elects one representative to Congress. Out of the 50 precincts, 30 usually vote for the Dark Gray Party, and 20 usually vote for the Light Gray Party. Out of the 5 representatives this state will send to Washington, how many, in your opinion, should be from the Dark Gray Party and how many from the Light Gray Party?

10 (length) by 5 (height) grid. The top three rows are shaded dark gray, and the two bottom rows are shaded light gray.

(2) In the previous rectangular state, the legislature decided to divide the state into 5 districts as in the picture below.[2] Remember that because the state has to be divided into 5 equally populated districts, 10 precincts form one district. Each district elects a representative.

10 (length) by 5 (height) grid. The top three rows are shaded dark gray, and the two bottom rows are shaded light gray. The grid is split into 5 equal column of 2 by 5. Each column contains 6 dark squares and 4 light squares.

(a) How many representatives will be elected from the Dark Gray Party? How many will be elected from the Light Gray Party? Are these results similar to what you got in Question 1?

(b) Why do you think Light Gray voters couldn't get anyone elected? Explain as completely as possible.

(3) Can you come up with a division of the state into 5 districts in a way that is more fair than the division in Question 2? How many representatives will each party get? Note that each district should have 10 precincts, and precincts in a district must be adjacent (i.e., they must share a common border).

10 (length) by 5 (height) grid. The top three rows are shaded dark gray, and the two bottom rows are shaded light gray.

(4) Explore the division of the state in the image below.[3]

10 (length) by 5 (height) grid. The top three rows are shaded dark gray, and the two bottom rows are shaded light gray. The grid is divided in 5 divisions containing 10 squares. Division 1 = 9 dark, 1 light, Division 2 = 9 dark, 1 light. Division 3 = 4 dark, 6 light. Division 4 = 4 dark, 6 light. Division 5 = 4 dark, 6 light.

(a) Which party gets more representatives? Is that division fair?

(b) Why do you think DG voters weren't able to elect as many representatives as the LG party? Explain as completely as you can.

In Question 2, LG voters could not elect any representatives because they were dispersed across districts. That practice is called **cracking**, and it is intended to diminish the influence of the group being dispersed.

10 (length) by 5 (height) grid. The top three rows are shaded dark gray, and the two bottom rows are shaded light gray. The grid is split into 5 equal column of 2 by 5. Each column contains 6 dark squares and 4 light squares. Each column has the label "1 DG rep from this district."

In Question 4, DG voters could not elect the number of representatives they deserved because they were highly concentrated in two districts. This practice is called **packing**, and it is intended to reduce the influence of a large group to as few districts as possible.

10 (length) by 5 (height) grid. The top three rows are shaded dark gray, and the two bottom rows are shaded light gray. The grid is divided in 5 divisions containing 10 squares. Division 1 = 9 dark, 1 light, Division 2 = 9 dark, 1 light. Division 3 = 4 dark, 6 light. Division 4 = 4 dark, 6 light. Division 5 = 4 dark, 6 light. Each division is labeled with "1 DG rep from this district".

PROBLEM SITUATION 2: The Efficiency Gap function

Proving in court that an electoral map gives unfair advantage to a political party is not easy. The Supreme Court has never struck down a map on grounds of political gerrymandering. In 2004, Justice Anthony Kennedy wrote that no one had come up with a “workable standard” to decide if a given redistricting is so extreme that it crosses a constitutional line. In 2016, for the first time in 30 years, a federal court rejected a voting map as partisan gerrymandering. The Wisconsin voters who sued claiming that their map was gerrymandered used the so-called Efficiency Gap Function to argue their case. In this lesson, you will learn how to calculate the Efficiency Gap.[4] Given a division of a state into districts, the Efficiency Gap function measures how efficiently a party’s voters performed in an election. There isn’t a perfectly fair electoral map with a gap of 0% but when the efficiency gap of a certain map is higher than 7%, it is argued that the map violates the Constitution.

Let us consider a state that has 500 voters divided into five districts with 100 voters each. This table shows the result of the vote in each district:[5]

District	DG Votes	LG Votes	Result
1	75	25	DG wins
2	60	40	DG wins
3	43	57	LG wins
4	48	52	LG wins
5	49	51	LG wins
Total	275	225	

(5) (a) What percent of the population voted for the DG party, and how many seats should the DG party have?

(b) What percent of the population voted for the LG party and how many seats should the LG party have?

(c) Which party made better use of its votes? Explain.

As you saw in Question 2, “cracking” leads to spreading the votes of one group of people among several districts so their votes almost do not count. Their votes are considered **wasted**. As you saw in Question 4, “packing” leads to having an overwhelming number of votes for one group in a single district, many more than needed to win. In other words, any vote above the 51% needed to win is also considered **wasted**. The **number of net wasted votes** in an election is the difference of the wasted votes between the two parties. **The Efficiency Gap function** calculates the percentage of the total number of votes that these net wasted votes represent.

(6) Complete the following table:

District	DG Votes	LG Votes	Wasted DG Votes	Wasted LG Votes	Net Wasted Votes
1	75	25	24	25	1 for LG
2	60	40			
3	43	57			
4	48	52	48	1	47 for DG
5	49	51			
Total	275	225			

(7) According to your computation, which party wasted more votes? How many more votes did they waste? This last number is called the **number of net wasted votes**.

(8) (a) Compute the Efficiency Gap by finding the percentage of the total number of votes that the net wasted votes represent. Round to the nearest whole percent.

(b) What is the Efficiency Gap telling you? Explain in your own words.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[1] https://commons.wikimedia.org/wiki/File:How_to_Steal_an_Election_-_Gerrymandering.svg

[2] https://commons.wikimedia.org/wiki/File:How_to_Steal_an_Election_-_Gerrymandering.svg

[3] https://commons.wikimedia.org/wiki/File:How_to_Steal_an_Election_-_Gerrymandering.svg

[4] https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2457468

[5] https://www.brennancenter.org/sites/default/files/legal-work/How_the_Efficiency_Gap_Standard_Works.pdf

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6.1.1: Preparation 1.5-SJ

Preparation 1.5-SJ: Gerrymandering

(1) Which expressions are ways to write the number 5,200,000? There may be more than one correct answer.

(i) 52 million

(ii) 5.2 billion

(iii) 5.2 million

(iv) Five million, two hundred thousand

(v) Fifty-two million

(vi) 5,200

(2) Estimate the following percentages without using a calculator.

(a) 10.1% of 7,800

(b) 0.99% of 83,583

(c) 20% of 5,008,340

(3) Which expressions are equivalent to $\frac{32}{9}(53-31+48)$? There may be more than one correct answer.

(i) $\frac{32}{9}\cdot 53-\frac{32}{9}\cdot 31+\frac{32}{9}\cdot 48$

(ii) $\frac{32\cdot 53-32\cdot 31+32\cdot 48}{9}$

(iii) $\frac{32}{9}(53-31-48)$

(iv) $32\left(\frac{53}{9}-\frac{31}{9}+\frac{48}{9}\right)$

(v) $\frac{32}{9}\cdot 53-31+48$

Introduction to Gerrymandering

The United States House of Representatives is the lower chamber of the United States Congress, which, along with the Senate, composes the legislature of the United States. U.S. law fixes the total number of U.S. representatives, also called the number of seats in the House, at 435. Seats in the House of Representatives are apportioned among the states according to their population, as determined by the census conducted every ten years. Each state, however, is entitled to at least one Representative. Since 1967, states that are entitled to more than one representative are divided into single-member districts. This means that these states have to be divided into districts that have approximately the same population and each district elects one representative to the House. Because populations change, states typically change their district boundaries after each census.

A state, county or region can be divided into voting districts in such a way as to give unfair advantage to one group of people. This, in turn, puts other groups at a disadvantage. This practice is called **gerrymandering** and it dates back to the founding of our country. These groups of people can be a political party or an ethnic group. In those cases, we speak of **partisan gerrymandering** or **racial gerrymandering**.

In the past 100 years, both the Democratic Party and Republican Party have engaged in partisan gerrymandering. A recent example occurred in the 2018 congressional election in North Carolina. In 2018, the North Carolina congressional map divided the state into thirteen congressional districts. In the election, Democrats won 3 districts, Republicans won 9 districts, and 1 district (9th district) did not seat a winner due to election problems. The map below shows the thirteen districts. The three districts shaded in blue were won by Democrats.

The North Carolina congressional map divided the state into thirteen congressional districts. The map below shows the thirteen districts. The three districts shaded in blue were won by Democrats.

North Carolina Democrats filed a lawsuit asserting that the congressional map used in the 2018 election was unfair to Democratic voters. The map concentrated Democratic voters into three districts, enabling Democrats to easily win these districts, but split the vote of Democrats across other districts, making these districts easier for Republicans to win. In the 2018 election, 3,380,609 people voted. Of all voters, 50.49% voted for Republicans, and 48.30% voted for Democrats.

(4) What percentage of 2018 congressional districts in North Carolina were won by Democrats? Round to the nearest percent.

(5) Given that about 49% of North Carolina voters in 2018 supported Democrats, how many congressional districts should the Democrats have won if the congressional map did not favor either party? Round to the nearest whole number.

In Collaboration 1.5-SJ, we will explore the strategies behind gerrymandering which produce election results that are unfair to specific groups of voters.

After Preparation 1.5 Survey

You should be able to do the following things for the next class. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Lesson 1.5, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
name and understand large numbers written in different forms.	
use benchmarks to estimate percentages including percentages less than 1%.	
use order of operations and the Distributive Property to write expressions in different forms.	

Self Regulated Learning (SRL): Plan

The next collaboration (1.5) will be a review of concepts in the course so far. After reading the information above about the plan phase, think about what you might do to be well prepared for your next class session. Try to incorporate the ideas of confidence (self-efficacy), time and effort, strategies, and your goals. Write out your planning ideas. Your instructor may ask you to discuss this in class.

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6.1.2: Exercise 1.5-SJ

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the lesson?

(i) The number $1.177 trillion can also be written as $1,177 billion.

(ii) Gerrymandering can occur anywhere and is a major problem in the United States.

(iii) A percentage is always a number greater than one.

(iv) Understanding numbers includes knowing how numbers compare in size, knowing what numbers represent in situations, and using estimation to answer questions about numbers.

DEVELOPING SKILLS AND UNDERSTANDING

(2) Let's return to the 2018 congressional election in the state of North Carolina. The table below displays the approximate voting results for the thirteen districts in the 2018 House of Representatives election. The Republican candidate in District 3 was unchallenged by an opponent, and the election results in District 9 did not count due to election problems.

(a) Complete the table by finding the number of wasted votes for each political party, the net wasted votes for each district, the total net wasted votes for each party, and the difference in net wasted votes. Several rows in the table have been completed for you. Note the calculations present in the row for District 1. In District 1, the total number of votes was 190,500 + 82,200 = **272,700**, so the number of votes needed to win would have been (272,700 *0.50) + 1 = **136,351**.

District	Democratic Party Votes	Republican Party Votes	Wasted Dem. Party Votes	Wasted Rep. Party Votes	Net Wasted Votes
1	190,500	82,200	190,500 - 136,351 = 54,149	82,200	82,200 - 54,149 = 28,051 for Rep.
2	152,000	170,100	152,000	9,049	142,951 for Dem.
3	0	187,900	0	93,949	93,949 for Rep.
4	247,100	82,100			
5	120,500	159,900	120,500	19,699	100,801 for Dem.
6	123,700	160,700			
7	120,800	156,800	120,800	17,999	102,801 for Dem.
8	114,100	141,400	114,100	13,649	10,451 for Dem.
9	…	…	…	…	…
10	113,300	165,000	113,300	25,849	87,451 for Dem.
11	116,500	178,000	116,500	30,749	85,751 for Dem.
12	204,000	75,200	64,399	75,200	10,801 for Rep.
13	130,400	147,600	130,400	8,599	138,999 for Dem.
Total	1,632,900	1,706,900	1,192,347	477,541	714,806 for Dem.

(b) Which party has more wasted votes?

(c) Compute the Efficiency Gap by finding the percentage of the total number of votes that the net wasted votes represent. What does the Efficiency Gap mean? Explain in your own words.

(d) Which districts appear to show "packing" of Democratic voters? Explain.

MAKING CONNECTIONS ACROSS THE COURSE

Big budget movies are tracked by investors and consumers. The following table gives data on the six movies with the largest budgets that had been released as of June 17, 2014.[6] The data includes an estimate of the U.S. gross earnings and gross earnings in other countries. **Gross earnings** is the amount of money that a movie takes in.

Release Date	Movie	Distributor	Budget	U.S. Gross Earnings	Gross Earnings Outside U.S.
5/25/2007	Pirates of the Caribbean: At World's End	Buena Vista	$300,000,000	$309,420,425	$654,000,000
11/24/2010	Tangled	Walt Disney Studios Motion Pictures	$260,000,000	$200,821,936	$390,973,000
5/4/2007	Spider-Man 3	Sony Pictures/ Columbia Pictures	$258,000,000	$336,530,303	$554,341,323
3/9/2012	John Carter	Walt Disney Studios Motion Pictures	$250,000,000	$73,078,100	$211,061,000
7/15/2009	Harry Potter and the Half Blood Prince	Warner Bros.	$250,000,000	$301,959,197	$632,457,290
12/18/2009	Avatar	20th Century Fox	$237,000,000	$760,507,625	$2,021,767,547

(3) Write the number in words for the gross earnings outside the United States for Avatar.

(4) Which of the following calculations shows a correct method to estimate the net earnings for *Pirates of the Caribbean: At World's End*? Net earnings is the total amount the movie makes after expenses (the budget) are taken out. There may be more than one correct answer.

(i) ($310,000,000 + $650,000,000) – $300,000,000 = $660,000,000

(ii) $650,000,000 – ($310,000,000 + $300,000,000) = $660,000,000

(iii) The budget and the U.S. gross earnings are about the same and cancel each other out. The net earnings would be about the same as the gross earnings outside the United States, or about 650 million dollars.

(iv) The gross earnings are about $600 million plus $300 million, or $900 million. The expenses are about $300 million. So, the net earnings are about 600 million dollars.

(5) Refer to the data for *Harry Potter and the Half-Blood Prince*.

(a) Estimate the net earnings.

(b) Write two statements to explain a way to estimate the net earnings. One should be a numeric expression, as in Question 8(i), and the other should be in words, as in Question 8(iv).

(6) The **return on investment** is the net earnings as a percentage of the budget. Which of the following statements best estimates the return on investment for *Pirates of the Caribbean: At World's End*? There may be more than one correct answer.

(i) The net earnings for *Pirates of the Caribbean: At World's End* were more than triple the investment.

(ii) The net earnings for *Pirates of the Caribbean: At World's End* were more than double the investment.

(iii) The return on investment for *Pirates of the Caribbean: At World's End* was more than 300%.

(iv) The return on investment for *Pirates of the Caribbean: At World's End* was more than 200%.

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[6] http://boxofficemojo.com/

6.2: Quantway Core 1.6-SJ (US Incarceration Rates) - Student Lesson

SPECIFIC OBJECTIVES

By the end of this lesson, you should understand that

- the magnitude of large numbers is seen in place value and in scientific notation.
- proportions are one way to compare numbers of varying magnitudes.
- different comparisons may be needed to accurately compare two or more quantities.

By the end of this lesson, you should be able to

- express numbers in scientific notation.
- estimate ratios of large numbers.
- calculate ratios of large numbers.
- use multiple computations to compare quantities.
- compare and rank numbers including those of different magnitudes (millions, billions).

PROBLEM SITUATION 1: COMPARING POPULATIONS

In this problem situation, you are going to compare the populations of China and the United States, and you will compare the population of each country to the population of the world. Since the population of each of these countries is very large, you will use scientific notation to write your answers. Recall that **scientific notation** is a special way of writing very large or very small numbers. A number in scientific notation is written in the form: $a \times 10^n$ where $1 \leq a < 10$; and n is an *integer*. An **integer**, commonly known as a whole number, is a number that can be written without a fractional component. Below are two examples of very large numbers written in scientific notation.

Examples:

- In 2013, the total population of Asia was approximately 4.3 billion people.[2] You can write this as 4,300,000,000 or you can use scientific notation to write it as 4.3×10^9 people.
- 28,930,000 can be written in scientific notation as 2.893×10^7.

(1) In mid-2014, the population of the United States was 318,000,000.[3] In mid-2014, Earth’s population was about 7.2 billion.

(a) Write these numbers in scientific notation.

(b) How does writing the two quantities in scientific notation help you compare them?

(2) A **ratio** is an expression that compares how much there is of one thing to another. Ratios are often written as fractions. Use a ratio to compare the population of the United States in 2014 to the population of Earth in 2014. Write your answer as a fraction and as a percent.

(3) In mid-2014, the population of China was 1.363 billion.[4] Compare China’s population with the population of the United States using a ratio (with the U.S. population as the reference value). Write a sentence that interprets this ratio in the given context.

PROBLEM SITUATION 2: COMPARING PRISON POPULATIONS

The table below shows a portion of the World Prison Population List, developed by The International Centre for Prison Studies.[5] In the table, you can see the overall population and the prison population of five countries in 2015.

Table 2: Prison Populations in 2015

Country	Total Population (in millions)	Prison population (in thousands)	Percent of Total Population in Prison
China	1393.8	1,658	
United States	317.76	2,217	
Indonesia	253.18	162	
Brazil	202.03	608	

cc by nc

Russia	144.4	642	

(4) Looking at the numbers in the table (but **without doing any calculations yet**), what do you notice about the prison population in the U.S. and in China? Can you think of some reasons for this?

(5) Notice that in Table 1, the total populations are listed in *millions*, and the prison populations are listed in *thousands*. This means the total population of China in 2015 was 1,393,800,000 people.

(a) Write the prison population in China in 2015 as a number in standard notation.

(b) Create a ratio comparing the prison population to the total population for China. Use a calculator to convert the fraction into a decimal and a percentage, then enter your percentage into Table 1 on the row for China.

(6) (a) The countries in Table 1 are listed from highest to lowest population. Calculate the percentages for the remaining countries and enter them in the table.

(b) Which country has the lowest percentage of its population in prison? How do the U.S. and China compare in terms of the percentage of their population in prison?

(7) In question 2, you used a ratio and a percentage to compare the U.S. population to the earth's population. Now do the same calculation to compare the U.S. *prison* population to the earth's *prison* population. Use this fact: In 2015, there were about 10.35 million people in prison worldwide.

(8) Write a sentence to explain the meaning of your answers to questions 2 and 7. Remember the **Writing Principle**: Use specific and complete information. Someone who reads your sentence should understand what you are trying to say, even if he/she has not read the question or writing prompt.

FURTHER APPLICATIONS

(9) In 1961, there were 220,149 prisoners held in state and federal prisons in the United States.[6] That same year, the U.S. population was 183.7 million people.

(a) Calculate the percentage of the U.S. population that was in prison in 1961.

(b) Compare the U.S. population in 1961 to the U.S. population in 2015. How many times larger was the population in 2015?

(c) Compare the U.S. prison population in 1961 to the U.S. prison population in 2015. How many times larger was the prison population in 2015?

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[2] http://www.prb.org/DataFinder/Geography/Data.aspx?loc=355

[3] http://www.census.gov/popclock/

[4] http://www.stats.gov.cn/english/statisticaldata/AnnualData/

[5] https://www.prisonstudies.org/sites/default/files/resources/downloads/world_prison_population_list_11th_edition_0.pdf

[6] https://www.ncjrs.gov/pdffiles1/digitization/111098ncjrs.pdf\

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6.2.1: Preparation 1.6-SJ

Preparation 1.6-SJ: Incarceration Rates

(1) Which of the following can be used to represent 5 out of 20? There may be more than one correct answer.

- (i) $\frac{1}{2}$
- (ii) 0.25
- (iii) 25%
- (iv) 0.25%
- (v) 25

(2) What is another way to represent $\frac{3}{5}$? There may be more than one correct answer.

- (i) 1.66
- (ii) 166%
- (iii) 60%
- (iv) 0.6
- (v) $\frac{6}{10}$
- (vi) 3 out of 5

(3) Which of the following is the standard form of 1.23×10^{11}?

- (i) 1,230,000,000,000
- (ii) 123,000,000,000
- (iii) 12,300,000,000
- (iv) 1,230,000,000

Scientific Notation and Calculators

Scientific notation is useful because it is easy to make mistakes when working with numbers that contain a lot of zeros. You can use scientific notation with a calculator that has an exponent feature. Instructions for using exponents with two different types of calculators are given as follows.

Scientific Calculator: These calculators have a key that looks like one of the following:

y^x.png

or

x^y.png

The keystrokes for entering 10^8 are

keys 1.png keys 0.png x^y.png keys 8.png

Graphing Calculators and Computers: Graphing calculators and computers have a key that looks like an upside down “v” (see picture below). This is called a *caret* symbol. It is also used for exponents in computer programs, including spreadsheets.

keys ^.png

The keystrokes for entering 10^8 are

keys 1.png keys 0.png keys ^.png keys 8.png

Calculators automatically display results to calculations in scientific notation when the numbers have too many digits to be displayed on the screen. The way that these are displayed varies slightly with different calculators. One common display is 2.34 E9, which represents 2.34×10^9.

(4) As of 2014, the world population is estimated to be about 7.17×10^9. (Recall the discussion of scientific notation from OCE 1.4.) About 4.4% of the world's population lives in the United States. Approximately how many people live in the United States?

(i) 1.63×10^7

(ii) 3.15×10^7

(iii) 1.63×10^8

(iv) 3.15×10^8

(v) 1.63×10^9

(vi) 3.15×10^9

Comparing Prison Populations

The International Centre for Prison Studies regularly publishes a report called the World Prison Population List. The last report came out in 2015. The report compares the numbers of prisoners held in 223 different countries across the world. According to the authors of the report, "The data may prompt fresh thought among policy-makers and other criminal justice experts about the size of the prison population in their country, given the high costs and disputed efficacy of imprisonment." Researchers, policy makers, and citizens of countries throughout the world use the report to understand how imprisonment varies in countries throughout the world.

(5) The table below shows a portion of the World Prison Population List.[1] In the table, you can see the overall population and the prison population of three countries in 2015.

Table 1: Prison Populations in 2015

	Total Population	**Prison Population**
India	1.275 billion	418,536
France	64.39 million	60,896

(a) Write the prison population of France in scientific notation.

(b) Write the population of India in scientific notation.

(c) Write the population of France in standard notation (as a number).

In Collaboration 1.6-SJ we will explore data from the 2015 World Prison Population List to examine the size of prison populations in countries around the world.

After Preparation 1.6 Survey

You should be able to do the following things for the next class. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Lesson 1.6, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
calculate a quotient (one number divided by another).	
use a calculator to divide numbers.	
use scientific notation.	
convert between fractions, percentages, and decimals.	

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[1] https://www.prisonstudies.org/sites/default/files/resources/downloads/world_prison_population_list_11th_edition_0.pdf

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6.2.2: Exercise 1.6-SJ

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the lesson?

(i) Ratios are a way to compare measurements in different situations.

(ii) The ratio of incarcerated people in a country to the population of a country varies from country to country.

(iii) The number 318,000,000 can be written in scientific notation as 3.18×10^8.

(iv) A number in scientific notation has the form $a \times 10^n$ where $1 \leq a < 10$; and n is an integer.

DEVELOPING SKILLS AND UNDERSTANDING

(2) The World Prison Population List defines a *prison population rate* as the number of individuals in prison per 100,000 people in a population. Use the table below to answer the following questions.

	Total Population (in millions)	**Prison population (in thousands)**
China	1393.8	1,658
United States	317.76	2217

(a) Find the prison population rate of the United States. Round to the nearest whole number.

(b) Find the prison population rate of China. Round to the nearest whole number.

(c) The prison population rate of Canada is 106. In 2015, Canada's prison population was 37,864 people. What is an estimate of Canada's total population? Round to the nearest ten thousand.

(d) How do you write the number found in (c) in scientific notation?

(i) $3{,}572 \times 10^4$ adults

(ii) 357.2×10^5 adults

(iii) 35.72×10^6 adults

(iv) 3.572×10^7 adults

(e) The **prison population rate** of Japan is 48. In 2015, Japan's population was approximately 127 million people. What is an estimate of Japan's prison population? Write the exact result.

(f) Write the number found in (e) in scientific notation?

(3) The World Prison Population List reported that the typical prison population rate for western European countries is 84, and the typical prison population rate for countries spanning Europe and Asia (e.g. Russia and Turkey) is 236.

(a) Which statement below best compares the levels of imprisonment of people in these two regions?

(i) People in western European countries are less likely to be imprisoned than people in countries spanning Europe and Asia.

(ii) People in western European countries are more likely to be imprisoned than people in countries spanning Europe and Asia.

(iii) People in countries spanning Europe and Asia are about twice as likely to be imprisoned than people in western European countries.

(iv) People in countries spanning Europe and Asia are about three times as likely to be imprisoned than people in western European countries.

(b) In 2015, what percentage of people in western European countries were imprisoned? Express your answer as a percentage.

(c) In 2015, Germany (located in western Europe) had a population of 81.29 million people. Estimate the number of people that were imprisoned in Germany in 2015. Round to the nearest hundred.

(4) The website for the nonprofit organization Charity: Water discusses the need for clean water around the world.

(a) The website states that worldwide “90% of the 30,000 deaths that occur every week from unsafe water and unhygienic living conditions are of children under five years old.”[7] The following statements are all correct interpretations of this statistic. Which gives the most complete information?

(i) 27,000 children die every week from unsafe water and unhygienic living conditions.

(ii) 27,000 out of the 30,000 deaths that occur every week from unsafe water and unhygienic living conditions are of children under five years old.

(iii) 90% of deaths from unsafe water and unhygienic living conditions are of children under five years old.

(iv) 90 out of 100 deaths that occur every week from unhealthy living conditions are of children under five years old.

(b) On their website, the organization Action Against Hunger claims: “Almost a billion people on the planet don’t have access to clean drinking water.”[8] If there are 7.2×10^9 people in the world, then approximately what percent of them live without clean drinking water? Round to the nearest tenth of a percent.

MAKING CONNECTIONS ACROSS THE COURSE

(5) Tannika has a health insurance plan that will reimburse her for 60% of her family’s health care expenses after she pays a $2,000 deductible. A **deductible** is the amount a person pays (to a hospital, for example) before an insurance company will begin to pay for a percentage of the remaining expenses. Tannika has to pay the deductible and the percentage not covered by the insurance company. These are called “**out-of-pocket expenses**” because they are paid for by the person who owns the policy.

Tannika records the total of her health care expenses in the spreadsheet below.

Screen capture showing Tanika's spreadsheet, indicating the following: A1 = Service B1 = Cost D1 = Amount Paid by Insurance A2 = Doctors visits B2 = $1,235 D2 = Out-of-pocket expenses A3 = Prescriptions B3 = $1,044 A4 = Hospital bills B4 = $8,270 A5 = Lab tests B5 = $600

(a) Which of the following formulas could Tannika use in Cell E1 to calculate the amount paid by her insurance?

(i) = 0.6(B2 + B3 + B4 + B5) – 2000

(ii) = 0.6 * B2 + B3 + B4 + B5 – 2000

(iii) = 2000 − 0.6(B2 + B3 + B4 + B5)

(iv) = 0.6(B2 + B3 + B4 + B5 – 2000)

(b) Which of the following formulas could Tannika use in Cell E2 to calculate her out-of-pocket expenses? There may be more than one correct answer.

(i) = 0.4(B2 + B3 + B4 + B5 – 2000) + 2000

(ii) = 0.4 * B2 + B3 + B4 + B5 + 2000

(iii) = 2000 + 0.4(B2 + B3 + B4 + B5)

(iv) = (B2 + B3 + B4 + B5) – E1

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[7] http://www.charitywater.org/whywater/

[8] http://www.actionagainsthunger.org/impact/water-sanitation-hygiene

6.3: Quantway Core 1.8-SJ (Race and Incarceration) - Student Lesson

SPECIFIC OBJECTIVES

By the end of this lesson, you should understand that

- percentages involve a numerator (comparison value) and a denominator (reference value).

By the end of this lesson, you should be able to

- correctly identify the quantities involved in a verbal statement about percentages.
- convert between ratios and percentages.
- convert between the decimal representation of a number and a percent.
- read and use information presented in a two-way table.

PROBLEM SITUATION: THE LANGUAGE OF PERCENTAGES

In this activity we will examine percentages related to incarceration rates in the United States from the “Mass Incarceration: The Whole Pie” report. Percentages are useful when comparing different groups. The language used to describe percentages can be difficult to understand; therefore, pay close attention to the language used to describe a percent at the beginning of this lesson. This will help you to understand the findings stated in the report.

Consider the following four quantities:

- Quantity 1 (P1): The percentage of Black Americans who are incarcerated.
- Quantity 2 (P2): The percentage of White Americans who are incarcerated.
- Quantity 3 (P3): The percentage of incarcerated Americans who are Black.
- Quantity 4 (P4): The percentage of incarcerated Americans who are White.

(1) (a) Which do you think is larger, P1 or P2? Discuss your reasoning with your group.

(b) Which do you think is larger, P1 or P3? Discuss your reasoning with your group.

INCARCERATION RATES

The bar graph below presents data about incarcerated populations in the U.S in 2019.[2] With the aid of this graph, you will be able to test your ideas from Question (1).

Bar chart showing U.S. incarceration rates by race in 2019. Y-axis is percentage ranged from 0 to 70 in increments of 10. x-axis is white, Latino, and Black, each with two columns: gray = % of U.S. population and white = % of incarcerated population. White = 60% (gray) just under 40% (white). Latino = just under 20% (gray), just over 20% (white). Black = about 12% (gray), just under 40% (white).

(2) The six bars on the graph tell us about six different quantities. Fill in the blanks to write one sentence for each bar identifying what that bar tells us. The first one is done for you.

About ___60______ % of ____the U.S. population____________________ is ____white_____________.

About ___________ % of ____ is ______________________.

About ___________ % of ____ is ______________________.

About ___________ % of ____ is ______________________.

About ___________ % of ____ is ______________________.

About ___________ % of ____ is ______________________.

(3) Use the percentages from Question 2 to answer the following questions.

(a) In 2019, there were approximately 328 million people living in the United States. How many people were Black, and how many people were White? Show your calculations.

(b) In 2019, there were approximately 1.9 million people incarcerated in the United States. How many people incarcerated were Black, and how many people incarcerated were White? Show your calculations.

Write your answers to (a) and (b) in this table. Round to the nearest thousand people.

	White	**Black**

Incarcerated		
Total		

(c) From the table:

(i) Calculate the percentage of Black Americans who are incarcerated. Round to the nearest hundredth of a percent.

(ii) calculate the percentage of White Americans who are incarcerated. Round to the nearest hundredth of a percent.

(4) Summarize the results by filling in the blanks below.

- The percentage of Black Americans who are incarcerated (P1): ___________
- The percentage of White Americans who are incarcerated (P2): ___________
- The percentage of incarcerated Americans who are Black (P3): ___________
- The percentage of incarcerated Americans who are White (P4): ___________

(5) Explain why it is possible that P1 is different from P2 but P3 is about equal to P4.

(6) Which pair of ratios would be more relevant to compare in order to determine whether or not there is a racial disparity in incarceration?

- P1 versus P2 or
- P3 versus P4

(7) Compare the two quantities you chose by calculating their ratio. Then write a quantitative statement interpreting the result. (Remember the Writing Principle: Include both quantitative information and information about context. Someone who reads your sentence should understand what you are trying to say, even if he/she has not read the question or writing prompt.)

(8) What are some social factors that might have contributed to the differences in the percentages you found?

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[2] Numbers from https://www.prisonpolicy.org/profiles/US.html

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6.3.1: Preparation 1.8-SJ

Preparation 1.8-SJ: Race and Incarceration

(1) Which of the following represents 0.02%? There may be more than one correct answer.

(i) 2

(ii) 0.02

(iii) 0.0002

(iv) 2 out of 100

(v) 0.2 out of 100

(vi) 0.02 out of 100

(vii) 2 out of 1,000

(viii) 2 out of 10,000

(2) Which of the following is equivalent to 4%? There may be more than one correct answer.

(i) 0.04

(ii) $\frac{1}{25}$

(iii) $\frac{4}{100}$

(iv) $\frac{40}{100}$

(v) 2 out of 5

(vi) 4 out of 100

(3) Which of the following is correct? There may be more than one correct answer.

(i) A percent is one part out of 100 parts.

(ii) A percent can be converted into a decimal number by dividing that percent number by 100.

(iii) A percent can be converted into a decimal number by moving the decimal point two places to the left and removing the percent sign (%).

(iv) 50% means 50 per 100 or $\frac{50}{100} = 0.5$.

(4) Which of the following is the percent estimate of 1/3, rounded to the nearest hundredth of a percent?

(i) 0.33%

(ii) 3.3%

(iii) 33.3%

(iv) 33.33%

Criminal Justice Reform: Investigating Mass Incarceration

In March 2017, the nonpartisan Prison Policy Initiative published a report called “Mass Incarceration: The Whole Pie”.[1] The goal of the report was to spark a national conversation about criminal justice reform. The report includes information about percentages of people in the U.S. who are *incarcerated*, meaning that they are held in prisons, jails, or detention centers.

(5) The United States has one of the highest per capita incarceration rates in the world. There are many factors that contribute to this, including the types of ways in which people are incarcerated. More recently, in 2019, the U.S. incarcerated about 1.9 million people, of which 1,042,000 were in state prisons, 547,000 were in local jails, and 208,000 were in federal prisons and jails.

(a) What percent of people who are incarcerated are detained in state prisons? Round to the nearest percent?

(b) As a result of the Covid-19 pandemic, prison populations decreased, on average, by 15% in 2020. Estimate the number of people in state prisons in the year 2020 after the decline due to the Covid-19 pandemic.

After Preparation 1.8 Survey

You will be expected to do the following things for the next class. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Lesson 1.8, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
understand the word percent and the notation used to describe percentages (%).	
use a calculator to divide two numbers and interpret the resulting decimal representation as a percent.	
calculate and estimate percentages.	

Self-Regulating Your Learning: The Work Phase

In an earlier lesson, you read in detail about what it means to effectively *plan* for your learning. That involved accounting for time and effort, your confidence (self-efficacy), study strategies, and learning goals. In this lesson, we will discuss the second phase of self-regulated learning (SRL), the ***work*** phase.

As the name implies, the work phase of SRL is where you are actually working on the problem or assignment. However, it is more than just getting the assignment done. In this phase, you **monitor** or pay attention to a variety of things. For example:

- What you are or are not **understanding** (and when)
- Which **strategies** you are using; which ones are working, and which ones are not
- What **emotions** and feelings you are experiencing, both positive and negative
- When you should **seek help** from others

Let’s explore each of these in a little more detail.

Understanding: Self-regulated learners monitor what they understand and what they do not. This is done by frequently asking yourself: “Do I understand this?” or “Could I explain this to someone?” The goal is to monitor your understanding so that you may adapt your strategies, especially if you get stuck. Being honest about your understanding is important because it can help you progress successfully on a problem, or make you aware of your learning strengths and weaknesses. Sometimes, people talk about this as “thinking about your thinking.” Researchers call it *metacognition*.

Strategies: In learning about the SRL *plan* phase, you discovered that it can be useful to think about multiple strategies *before* you start working on a problem. In the *work* phase of SRL, having multiple strategies in mind (both those you have used before and those you plan to try) can help when you get stuck. You can stop, think about how the problem is progressing, and try another strategy that you think might work. Self-regulated learners often make mental notes about which strategies work in which situations, and which ones are easiest to use. Evaluating strategies allows you to become better at solving a variety of problems.

Emotions: Self-regulated learners know how to monitor their emotions—especially negatives ones such as frustration or anger—so that these emotions do not cause them to give up on a problem. When they start feeling frustrated, self-regulated learners often do things such as try new strategies, seek help, or engage in *positive self-talk*. This is saying things to yourself such as: “I know I can do this if I choose the right strategies and put in the effort, even if it is challenging.” The opposite is called *negative self-talk*, which involves saying things such as: “I am never going to get this! What is the point?” Monitoring and controlling your emotions, especially the negative ones, can be challenging and may require a lot of practice, but the benefits are worth it.

Seeking Help: With practice and experience, self-regulated learners know when it is beneficial to stop working and find someone else with whom they can discuss the problem. There is nothing wrong with getting help when you are learning something new. Some people think that asking for help means you do not have ability, but the truth is that knowing when to seek help is part of being an effective learner. Seeking help can save you time because you avoid the added frustration of making a lot of effort without making any progress. If you are spending a lot of time on a problem, you have tried several strategies without success, or you are not able to control negative emotions, stop and write down your questions. Bring the written questions with you and discuss the problem with someone else as soon as you can. Help could come from your instructor during his or her office hours, a campus learning center, or other classmates.

During the *work* phase, you are required to juggle two things at once: (1) working on the problem or assignment, and (2) monitoring your progress (e.g., *thinking about your thinking*). This process takes practice; however, it is important to master if you want to become a self-regulated learner. Thinking about how you are working makes the work easier and gives you information for the SRL *reflect* phase. You will explore the *reflect* phase in an upcoming lesson. Until then, practice these work strategies while you are working on problems and class assignments.

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[1] https://www.prisonpolicy.org/reports/pie2017.html

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6.3.2: Exercise 1.8-SJ

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the lesson?

(i) A percentage is calculated by dividing one number by another number.

(ii) The demographics of people who are incarcerated are not the same as the general population.

(iii) A percentage is a comparison of two numbers. To understand the meaning of a percentage, it is important to know what two quantities are being compared.

(iv) The percentage of students who are parents is the same as the percentage of parents who are students.

DEVELOPING SKILLS AND UNDERSTANDING

(2) Data from the March 2022 “Mass Incarceration: The Whole Pie”[3] report details the offenses which result in people being incarcerated and the type of incarceration. After being arrested for an offense, a person can be incarcerated for a lengthy amount of time before they are convicted, and sometimes they are never convicted.

About 1 in 5 people who are incarcerated in the U.S. are incarcerated due to a drug offense. Many people are incarcerated for drug offenses prior to being convicted, and there are several factors that impact whether this occurs. These factors include the location of the offense, the type of offense, a person’s income, and a person’s access to a lawyer. On a given day in 2022, about 373,000 people are incarcerated in the U.S. due to a drug offense. The table below provides a breakdown of the detention locations where people are incarcerated due to drug offenses.

Detention Location	Number of People Incarcerated	Percentage of People Incarcerated
State prisons	146,000	
Local jails (not convicted)	113,000	
Local jails (convicted)	24,000	
Federal prisons	88,000	
Youth detention	1,600	
Indian Country jails	220	
Military detention	80	
Total	**372,900**	

(a) Complete the table by filling in the *percentages* of people incarcerated on a drug offense who are at each detention location. Round to the nearest tenth of a percent. For Indian Country jails and Military detention, round to the nearest hundredth of a percent. Use the total provided in the table.

(b) The table details the number of people incarcerated in local jails due to a drug offense. What percentage of individuals who are incarcerated at local jails have not been convicted of a crime?

(c) Fill in the blanks to complete the following statements.

About _______ out of every 100 individuals incarcerated due to a drug offense are detained at a state prison.

About _______ out of every 100 individuals incarcerated due to a drug offense are detained at a local jail.

(d) One factor that has contributed to the large number of people incarcerated in the U.S. is *pretrial detention* - people incarcerated in a jail or prison after being arrested but prior to a trial and prior to being convicted of a crime. In 2018, there were approximately 490,000 people detained in local jails in the U.S. who were not convicted of crimes. If we want to examine if there is a difference in the ways in which Black Americans and White Americans are incarcerated prior to being convicted of crimes, what percentages should be compared? Explain.

(3) Select the answer(s) that correctly complete the statement from the list below: A *New York Times* story reported that 10% of male high school dropouts are in jail or detention centers.[4] According to this statistic, about ________ in every ________ male high school dropouts is (are) in jail or juvenile detention. There may be more than one correct answer.

(i) 1 in every 10

(ii) 1 in every 10%

(iii) 0.1 in every 100

(iv) 1 in every 100

(v) 10 in every 100

(4) A teacher has collected data on the grades his students received in his morning and afternoon classes. The following tables show two different ways to represent the same data as percentages.

Table 1

	Grade A	Grade B	Grade C	Grade D	Grade F
Morning Class	12.5%	25.0%	37.5%	6.3%	18.7%
Afternoon Class	14.3%	20.0%	37.1%	8.6%	20.0%

Table 2

	Grade A	Grade B	Grade C	Grade D	Grade F
Morning Class	44.4%	53.3%	48.0%	40.0%	46.2%
Afternoon Class	55.6%	46.7%	52.0%	60.0%	53.8%

(a) Which table could be used to answer the following question: What percentage of the students who received an “A” is in the morning class?

(b) Which table could be used to answer the following question: What percentage of the students in the morning class received an “A”?

(c) What are the reference values in Table 1?

(i) The number of students in a certain class.

(ii) The number of students who earned a certain grade.

(iii) The number of students in a certain class who earned a certain grade.

(iv) The total number of students in both classes.

(d) What are the reference values in Table 2?

(i) The number of students in a certain class.

(ii) The number of students who earned a certain grade.

(iii) The number of students in a certain class who earned a certain grade.

(iv) The total number of students in both classes.

MAKING CONNECTIONS ACROSS THE COURSE

(5) Congratulations! You won a lottery prize and have a taxable income of $1,025,400. Use the table below to answer the following questions.

Taxable Income	Tax
$0–$9,075	10% of taxable income

$9,076–$36,900	$907.50 plus 15% of excess over $9,075
$36,901–$89,350	$5081.25 plus 25% of excess over $36,900
$89,351–$186,350	$18,193.75 plus 28% of excess over $89,350
$186,351–$405,100	$45,353.75 plus 33% of excess over $186,350
$405,101–$406,750	$117,541.25 plus 35% of excess over $405,100
$406,751 plus	$118,118.75 plus 39.6% of excess over $406,750

(a) How much tax will you pay on your winnings? Round to the nearest dollar.

(b) Which of the following expressions can be used to calculate the tax?

- (i) $1,025,400 – 0.396($406,750 – $118,118.75)
- (ii) 0.396 + $406,750($1,025,400 – $118,118.75)
- (iii) ($1,025,400 + $118,118.75) – ($406,751 × 0.396)
- (iv) $118,118.75 + 0.396($1,025,400 – $406,750)
- (v) $1,025,400 – 0.396($1,025,400 – $406,750)

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[3] https://www.prisonpolicy.org/reports/pie2017.html

[4] http://www.nytimes.com/2009/10/09/education/09dropout.html?_r=0

CHAPTER OVERVIEW

7: TCU Lesson

7.1: Quantway Core 3.7-TCU - Student Lesson
  7.1.1: Preparation 3.7-TCU
  7.1.2: Exercise 3.7-TCU

7.1: Quantway Core 3.7-TCU - Student Lesson

SPECIFIC OBJECTIVES

By the end of this collaboration , you should understand that

- proportional relationships are based on a constant ratio.
- rules for solving equations can be applied in unfamiliar situations.

By the end of this collaboration, you should be able to

- set up a proportion based on a contextual situation.
- solve a proportion using algebraic methods.

BACKGROUND

In response to the construction of Mt. Rushmore, Lakota Chief Henry Standing Bear wanted to create a memorial honoring North American Indians. He wrote to a sculptor named Korczak Ziolkowski and asked him to create a memorial of a great hero, Crazy Horse.

Crazy Horse or *Tasunke Witco* was born as a member of the Oglala Lakota on Rapid Creek about 40 miles northeast of Thunderhead Mt. (now Crazy Horse Mountain) in c. 1840. He was a Oglala Lakota warrior famous for defeating Custer and the Seventh Cavalry at the Battle of Big Horn.

It was a time when cultures clashed, and land became an issue of deadly contention. Traditional Native ways were threatened and oppressed. Crazy Horse responded by putting the needs of his people above his own, which would forever embed him and his legacy in American History. He is a beloved symbol for the Lakota people because “he never conceded to the white man.” He was killed at Fort Robinson, Nebraska, by a soldier around midnight on September 5, 1877.[1]

On May 3, 1947 Korczak began work on the Crazy Horse Memorial. He worked for over 35 years and his work is continued by his children and grandchildren to this day. This memorial carved from Thunderhead Mt. is much larger than Mt. Rushmore. If the President’s heads were stacked one on top of the other, they would only reach a little more than half way. The scale of the Crazy Horse Memorial when complete will be grand, becoming the largest sculpture in the history of the world.[2]

Photograph of the unfinished Crazy Horse memorial. [3]

There are many Lakota who praise the memorial, grateful that it offers an unmissable reminder of the frequently ignored Native history of the hills, and a counterpoint to the four white faces on Mt. Rushmore. But others argue that a mountain-sized sculpture is an ill-chosen tribute. When Crazy Horse was alive, he was known for his humility, which is considered a key virtue in Lakota culture.

PROBLEM SITUATION 1: CRAZY HORSE MEMORIAL

As of Fall 2020, the Crazy Horse Memorial is far from complete. To begin construction on the Crazy Horse Memorial, two sculptures were created as scale models. Scale models are smaller versions, but proportionally equal, to the final model.

Photograph of sculpture of Crazy Horse memorial. [4] Photograph of sculpture of Crazy Horse memorial. [5]

(1) These models are at a scale of 1:34 and 1:300. Which scale model do you think is larger? 1:34 or 1:300? Explain your thinking.

(2) Crazy Horse Memorial is 641 feet long and stands 563 feet high.[6]

(a) Complete the table by finding the **length** for each scale model shown above. Round your answers to one decimal place.

Scale Model	Length
1:34	
1:300	

(b) Complete the table by finding the **height** for each scale model shown above. Round your answers to one decimal place.

Scale Model	Height

1:34	
1:300	

(3) On the Crazy Horse Memorial, Crazy Horse’s outstretched arm is 263 feet. Here again, is the photo of the original model with the artist:

Photograph of sculpture of Crazy Horse memorial with the artist.

(a) What is a more appropriate measurement for Crazy Horse’s arm on the model?

(i) Inches

(ii) Feet

(b) Using the scale factor of 1:300, what is the length of Crazy Horse’s outstretched arm on the model? Round to two decimal places.

(c) On the Crazy Horse Memorial, Crazy Horse’s face is 87 feet, 6 inches. Using the scale factor of 1:300, what is the height of Crazy Horse’s face on the model? Write your answer rounded to one decimal place.

(d) On the original model, Crazy Horse’s pointing finger is 1.18 inches long. How many feet and inches long is that finger on the Memorial?

PROBLEM SITUATION 2: DEVIL’S TOWER

A scale factor can be used to enlarge or reduce any shape.

One large rectangle with corners ABCD; height AD is 14 feet. A smaller rectangle has corners EFGH; length EF is 8 feet.

(4) The scale factor of ABCD to EFGH is 3:1. Determine the missing lengths of EFGH.

(a) The length of $\overline{AB}$ (the distance from A to B) is ___________ feet.

(b) The length of $\overline{EH}$ (the distance from E to H) is ____________ feet. Round your answer to two decimal places.

(5) Devil’s Tower is 867 feet tall, but the length across the base is unknown. There is a scale model of Devil’s Tower that is 5 inches tall, and it has a base length of 9.7 inches. Round to the nearest foot.

Image of Devil's Tower.

(a) What is the base length of the actual Devil’s Tower? Round to the nearest foot.

(b) What is the scale factor for the model? Round to one decimal place.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[1] https://crazyhorsememorial.org/story/the-history/about-crazy-horse-the-man/

[2] https://tinyurl.com/y5ebcjrg

[3] https://images.app.goo.gl/LLsRrMRtff4tJaPLA

[4] https://crazyhorsememorial.org/story/pictorial-timeline.

[5] https://www.flickr.com/photos/checco/4823835826.

[6] https://crazyhorsememorial.org/story/the-mountain/

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7.1.1: Preparation 3.7-TCU

Preparation 3.7-TCU

(1) Ben has $75 in his savings account. He plans to deposit $35 per week to build his account balance.

(a) Complete the following equation to represent the amount of money (A) Ben will have in his account after any number of weeks. Use x as your input variable.

A =

(b) What does the input variable (x) represent in this problem?

(c) Which of the following values could be the value of the input variable (x) in this context? There may be more than one correct answer.

(i) -5

(ii) 3

(iii) 4.2

(iv) 18

(d) Ben wants to use his savings to buy a computer for $740. Use the equation from part (a) to determine the number of weeks it will take him to save enough money to buy the computer.

PROPORTIONALITY

Length-to-width and width-to-length ratios

The dimensions of a figure can be written as a ratio. Imagine a rectangle with a length of ten inches and a width of three inches.

Diagram showing a rectangle, indicating the following: Width = 3 inches Length = 10 inches

You can say that the ratio of the length to width is 10:3 or 10/3. It is also correct to say that the ratio of the width to length is 3:10 or 3/10.

In Collaboration 3.7, when we compare multiple figures using their respective ratios, it will be important to be consistent.

Equivalent Fractions (review)

As you recall from Unit 3.1, equivalent fractions (ratios) can be written in many equivalent forms. For example, all of the following fractions simplify to 10/3 in lowest terms: 10/3 = 20/6 = 30/9 = 100/30

Proportional Figures

Now, we can define what it means for two figures to be proportional. Consider the following figures and compare them using their length-to-width ratios.

Large rectangle: Width = 6 inches Length = 20 inches

Small rectangle: Width = 3 inches Length = 10 inches

The length-to-width ratio of the figure on the left is 10/3.

The length-to-width ratio of the figure on the right is 20/6 = 10/3 in lowest terms. Note that 10/3 and 20/6 are equivalent fractions.

While these figures are not *equivalent*, they are *proportional* to each other because their dimensions have the same ratio.

(2) Use the figure below to answer the following questions.

Diagram showing the cylinder, indicating the following: Height = 9 feet Width = 5 feet

(a) Write the ratio of the dimensions of the cylinder shown above in the form of diameter to height.

(b) Give the dimensions of a cylinder that would be proportional to the one shown.

Diameter:

Height:

(3) Which of the following fractions has a ratio of 4:3? There may be more than one correct answer.

- (i) 2418
- (ii) 169
- (iii) 912
- (iv) 2015
- (v) 8.86.6

After Preparation 3.7 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 3.7, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
interpret the meaning of ratios, including when written as fractions.	
understand the use of a variable to represent an unknown.	
solve a two-step equation such as 2x + 9 = 13.	

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7.1.2: Exercise 3.7-TCU

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Graphic artists have to be very aware of proportionality and know how to solve proportions.

(ii) The rules for solving equations are the same for all types of equations.

(iii) The rules for solving equations depend on the type of equation.

(iv) To solve for a variable in the denominator of a fraction, multiply both sides of the equation by the variable.

(2) In Module 1, you learned that a statement such as “30% of voters support Candidate A” can be interpreted as 30 out of 100.

(a) How many voters out of 1,000 support Candidate A?

(b) How many voters out of 1,500 support Candidate A?

(c) Do the ratios from parts (a) and (b) represent a proportional relationship?

DEVELOPING SKILLS AND UNDERSTANDING

(3) The tables below each give the dimensions of four different rectangles. Write in the blank if the four rectangles in each set are “Proportional” or “Not Proportional”.

(a) ____________

	Width	**Length**
Rectangle 1	17	110.5
Rectangle 2	23.4	152.1
Rectangle 3	33	214.5
Rectangle 4	52.2	339.3

(b) ____________

	Width	**Length**
Rectangle 1	7.4	15.5
Rectangle 2	17	40.8
Rectangle 3	23.4	68.5
Rectangle 4	33	72.6

(4) A marine biologist would like to feed some dolphins a mix of fish that consists of 9 parts cod to 4 parts mackerel. List a combination that would be an acceptable mixture of these fish.

(5) Identify the proportions that have the same solution as the one below. There may be more than one correct answer.

$$\frac{3}{x} = \frac{24}{56}$$

(i) $\frac{x}{3} = \frac{24}{56}$

(ii) $\frac{3}{24} = \frac{x}{56}$

(iii) $\frac{24}{3} = \frac{56}{x}$

(iv) $\frac{x}{24} = \frac{3}{56}$

(v) $4\frac{x}{3} = \frac{56}{26}$

(6) Solve the following proportions (round to one decimal place):

(a) $\frac{x}{5} = \frac{10}{13}$

(b) $\frac{3}{4} = \frac{15}{2x}$

(7) Erica would like to bake an 8-pound roast for a family gathering. The cookbook tells her to bake a 5-pound roast for 135 minutes. Create and solve a proportion that would allow Erica to cook her 8-pound roast.

(8) Cefaclor is a medication used for infections. It is often given in liquid form by mixing the powdered medication with a fluid. A pharmacist is mixing a dosage for a child. The instructions indicate that 125 mg of the medication should be mixed with 5 mL of fluid. If the child only requires a dosage of 100 mg of Cefaclor, how much fluid should the pharmacist use?

MAKING CONNECTIONS ACROSS THE COURSE

(9) A company is making pennants or flags for a sports team. The team wants small versions for fans and large versions that will fly over the stadium. The dimensions of the small version are shown below.

Diagram showing the small version indicating the following: Height = 8 inches Width = 20 inches

(a) The large version needs to be 12.5 feet across the base (the short side of the triangle). How long should it be?

(b) How much material will be used for the large version of the flag? Round up to the nearest tenth of a square foot to ensure the company has enough material.

(10) A staircase is made up of individual steps that should be consistent in height and width. The height of each step is called the rise, and the width of the step is called the run.

(a) The staircase below is made up of four steps each with a rise of 6.5" and a run of 8.25". Find the (i) height and (ii) depth of the entire staircase.

Image of a staircase made up of four steps each with a rise of 6.5" and a run of 8.25".

(b) Builders have to follow guidelines on the rise and run of stairs when building a staircase. One acceptable ratio is a rise of 7-3/4 inches for a run of 9-3/4 inches. If a builder is using this ratio to build a staircase that is 15 feet high, how deep will the staircase need to be (d in the drawing below)? Note that the drawing does not show the correct number of steps. Round to the nearest tenth of a foot.

Image of a staircase that is 15 feet high with a run of d

GRAPHING ON A COORDINATE PLANE

Now, you will once again return to graphing on a coordinate plane. You may have noticed that the two axes split the coordinate plane into four sections. These are called **quadrants** and are numbered using Roman numerals as shown below.

For practical reasons, only a small part of the coordinate plane can be shown, but understand that the axes can go on infinitely in all four directions. The **scale** of the grid tells you which numbers are included in the portion of the plane that is shown. You can change the scale to make graphs with very large or very small numbers. The scale on a single axis must be consistent. In other words, if the distance between the gridlines represents five units on one part of the horizontal axis, then that same distance must always represent five units on that axis. However, the vertical and horizontal axes can have different scales as in the example below. As you have seen with other types of graphs, it is important to pay close attention to the scale.

Image showing a blank graph indicating the following: Top-right corner = Quadrant I Top-left corner = Quadrant II Bottom-left corner = Quadrant III Bottom-right corner = Quadrant IV

Blank coordinate plane graph with x and y axes ranged from -14 to 14.

(11) Place the following points on the graph. Label each with its ordered pair.

(a) (–14, 7)

(b) (0, 9)

(c) (13, –6)

(12) Indicate if each statement is true or false.

(a) The point (–7, –5) is in Quadrant II.

i) True ii) False

(b) The point (0, 5) lies on the vertical axis.

i) True ii) False

(c) All the points in Quadrant IV have a positive horizontal coordinate and a negative vertical coordinate.

i) True ii) False

(d) The points (20, 12) and (20, 200) lie on the same horizontal line.

i) True ii) False

Many applications use only positive numbers. In these cases, only Quadrant I of the graph is usually shown because that is the only quadrant that is used. An example of this is given below.

(13) You learned about the *golden ratio* in Exercise 3.5. A rectangle whose dimensions match the golden ratio is called a *golden* rectangle. The graph below shows the widths and lengths of golden rectangles.

Line graph showing the Golden Rectangles. Y-axis represents the length, ranging from 0 to 35. X-axis represents the width, ranging from 0 to 25. 0 = 0 5 = in between 5 and 10 10 = just over 15 15 = just under 25 20 = in between 30 and 35 25 = over 35

(a) Based on the graph, is a rectangle with a width of 17 inches and a length of 30 inches a golden rectangle?

i) Yes ii) No

(b) Use the graph to complete the table of values below. Estimate to the nearest whole number.

Width	Length
5	(i)
8	(ii)
(iii)	20
(iv)	31

CHAPTER OVERVIEW

8: Workforce Lessons

- 8.1: Quantway Core 1.4 Workforce (Health Care) - Student Lesson
  - 8.1.1: Preparation 1.4-W Affordable Care Act
  - 8.1.2: Exercise 1.4 (Workforce) Affordable Care Act
- 8.2: Quantway Core 1.8 Workforce (Environment) - Student Lesson
  - 8.2.1: Preparation 1.8-W Waste and Recycling
  - 8.2.2: Exercise 1.8-C Waste and Recycling
- 8.3: Quantway Core 1.9 Workforce (Health Care) - Student Lesson
  - 8.3.1: Preparation 1.9-W Medical Testing
  - 8.3.2: Exercise 1.9-W Medical Testing
- 8.4: Quantway Core 2.3 Workforce (Environment) - Student Lesson
  - 8.4.1: Preparation 2.3-W Renewable Energy
  - 8.4.2: Exercise 2.3-W Renewable Energy
- 8.5: Quantway Core 2.4 Workforce (Health Care) - Student Lesson
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  - 8.9.1: Preparation 3.7-W Medical Dosages
  - 8.9.2: Exercise 3.7-W Medical Dosages
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- 8.11: Quantway Core 4.3 Workforce (Healthcare) - Student Lesson
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8.1: Quantway Core 1.4 Workforce (Health Care) - Student Lesson

INTRODUCTION

Comprehension and Synthesis (CaS) Charts

You may consider using the Comprehension and Synthesis (CaS) Chart in this collaboration and in future collaborations. Remember, using the CaS Chart can help you have a deeper understanding of the problem situation. CaS Charts can help you understand the main issue(s) that need(s) to be resolved and will help you to recognize the quantitative information that is available in the problem situation that can help you to solve the problem. You can use CaS Charts in some of the Quantway collaborations to "unpack" problem situations and support problem-solving. First, on your own, recall the steps for completing the CaS Chart below. As you read the problem situation, you may complete the CaS Chart to supplement your thinking.

As you are reading a Problem Situation:

- Complete Column A: What issues in the problem situation do you need to understand in order to solve the problem? Is there contextual information that you need to know in order to understand the problem situation?
- Complete Column B: What quantitative information is provided in the problem situation that will help you solve the problem? Hint: Quantitative information is often a number, but also could be a number word, like "two".
- Complete Column C: It is not necessary to solve the problem or use calculations right now. In this column, brainstorm ways you might address the issues presented in the problem situation (Column A) using the quantitative information in Column B. There are no wrong answers.

Column A	Column B	Column C
What is/are the main issue(s) in this problem situation?	**What is the key quantitative information you need to solve the issue(s) in the problem situation?**	**Describe in writing how the information in Column B will help you address the issue(s) in Column A later in the lesson.**

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- flexibility with calculations is an important quantitative skill.
- different methods of calculation are often possible and helpful.

By the end of this collaboration, you should be able to

- write a calculation in at least two different ways based on
    - equivalent forms of fractions/decimals/percentages.
    - relation of multiplication and division.
    - the *Commutative Property* (knowing when the order of numbers can be reversed, such as 3 + 4 = 4 + 3, but 3 – 4 ≠ 4 – 3).
    - order of operations.
    - the Distributive Property (5(3 + 4) = 5 x 3 + 5 x 4).

SPECIFIC LANGUAGE AND LITERACY OBJECTIVES

By the end of this collaboration, you should be able to

- read and comprehend the problem situation and "Premium Contribution" chart.
- complete the CaS Chart with quantitative and health care information from the problem situation about the Affordable Care Act.
- demonstrate an understanding of mathematics by writing complete and correct responses to questions.
- demonstrate the ability to describe, analyze, and synthesize information about the Affordable Care Act.
- use appropriate quantitative and health care vocabulary to discuss mathematics in this lesson.

PROBLEM SITUATION 1: THE AFFORDABLE CARE ACT

In preparing for this collaboration, you learned about the Affordable Care Act and the Marketplace. The government offers financial aid to low-income individuals and families who enroll in the Marketplace. The amount of aid depends on many factors such as income, family size, age, and location. Federal poverty lines (FPL) are used to see if an individual or family qualifies for aid when buying insurance through the Marketplace.

The problem for many individuals and families is figuring out how much health care coverage will cost. The federal poverty line in 2014 was $11,670 for an individual, and $23,850 for a family of four.[2] Individuals and families making 100-400% of the poverty line are considered "low-income". The government uses this information to figure out how much a family can pay for health care coverage.

The government offers financial assistance to cover part of health care costs. How much the government covers depends on how much money an individual or family can pay from their own income (the amount the family pays is commonly called an out-of-pocket expense). The government offers financial assistance to cover the rest of the cost of health care coverage beyond what the family or individual pays out-of-pocket.

For example, imagine a household income is between 100% and 133% of the federal poverty line. That individual or family would not have to pay more than 2% of their annual income to cover their health insurance costs. In this situation, a family would pay 2% of their income as an out-of-pocket expense for health care coverage. The government would pay for the remaining cost of coverage through a tax credit.

The table below summarizes expected contributions for the health care cost for a family of four in 2014. The information is listed for families having an income between 100% and 400% of the poverty line. An individual or family making more than 400% of the federal poverty line would not be eligible for financial aid from the government.

A premium is the amount of money a family or individual pays per month for insurance.[3] The problem is that many low-income families are having trouble understanding how to calculate the cost of health care coverage under the Affordable Care Act. The purpose of this collaboration is to explore this problem.

Figure 1: Premium Contribution by Income under the Health Care Reform for a Family of Four[4]

Percentage of Poverty Line	Premium Contribution as Percentage of Income
100-133%	2%
133-150%	3-4%
150-200%	4-6.3%
200-250%	6.3-8.1%
250-300%	8.1-9.5%

300-350%	9.5%
350-400%	9.5%

(1) What are the factors that determine how much money low-income individuals or families will pay for health care coverage? Brainstorm possible answers with your group. Write your answer in **1-2 complete sentences**. (It is important to write complete sentences because it helps your instructor better understand your mathematical thinking.)

(2) **Estimate** the income of an individual who has a yearly income that is 150% of the poverty line. Try to do the estimation without writing it down or using a calculator if you can. Explain your strategy for your estimation. (It is okay if people in your group use different strategies, and for your estimations to be different).

Performing Calculations in Multiple Ways

The ability to solve problems in multiple ways is an important quantitative reasoning skill. Today's lesson asks you to brainstorm different ways to find the answer to a question. Different strategies are often useful in different situations.

You saw in a previous collaboration that estimation strategies often depend on specific numbers. This can also be true in calculations. Sometimes changing the order of operations or grouping operations can be helpful. It is important to know when you can make changes (that is, which changes are okay because you will still be able to make the correct calculations after making the changes).

(3) How would you write a mathematical expression to find the exact income that is 150% of the poverty line? (**Hint**: 2.5 x 130 is an example of a mathematical expression.) Try to find as many different mathematical expressions as possible. Input your expression into your calculator to find this income.

(4) William, Vanessa, and their two children have a yearly income of $31,200. Help this family understand their expected **monthly health care costs**.

(a) Calculate this family's income as a percentage of the poverty line. Remember, the poverty line for a family of four is $23,850.

(b) Calculate the maximum amount of money this family would have to spend on health care coverage **per month**.

PROBLEM SITUATION 2: THE AFFORDABLE CARE ACT AND MONTHLY COSTS OF HEALTH CARE COVERAGE

Families have options when selecting a plan in the Marketplace. There are five categories of plans that determine health care coverage costs: Catastrophic, Bronze, Silver, Gold, and Platinum. The Silver Plan is considered the standard plan. Individuals or families that have a yearly income between 100% to 400% of the poverty line are considered low or moderate income. These families will receive financial assistance from the government to cover additional insurance costs. The amount of this financial assistance is calculated by finding the difference between the cost of the standard plan (the Silver Plan) and the individual or family's expected cost for health care coverage.[5] If you buy the gold plan, you have to pay the additional cost since the government does not pay more.

(5) Let's return to William, Vanessa, and their two children. In their community, the total yearly cost of the standard plan for this particular family is $11,064. Imagine that William and Vanessa have decided to enroll their family in this plan. Write a mathematical expression that can be used to calculate how much this family will receive in the form of government monthly aid to cover insurance.

(6) Write **1-2 sentences** explaining the amount of money that William and Vanessa would have to spend for health care coverage under the Affordable Care Act. **Write complete sentences and include this information in your answer**: the amount of money William and Vanessa pay for health care coverage each month, and how much they receive as aid from the government per month.

(7) William and Vanessa's family have the following average monthly expenses. Calculate how much the family should expect to spend in **one year**.

Rent	$1,250
Utilities	$200
Food	$400

Health Care	$52
Total	

(8) William and Vanessa have neighbors who are trying to figure out which health care plan is best for them. The Aziz family is a family of four, with a household income of $72,000. This family is interested in the standard plan that costs $11,064 per year.

(a) What is the maximum percentage of the Aziz's household income that would have to go toward health care coverage?

(b) Calculate the maximum amount of money the Aziz family would have to spend on health care coverage per month.

(c) If the Aziz family chooses the same standard as William and Vanessa, what is the monthly aid from the government that this family will receive to help cover insurance costs?

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[2] Based on 2014 numbers.

[3] http://www.cbpp.org/files/QA-on-Premium-Credits.pdf.

[4] http://www.cbpp.org/sites/default/files/atoms/files/QA-on-Premium-Credits.pdf

[5] https://www.healthcare.gov/how-do-i-choose-marketplace-insurance/.

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8.1.1: Preparation 1.4-W Affordable Care Act

(1) Match the fractions to their equivalent percent form (rounded to the nearest percent).

(a) $\frac{1}{5}$	(i) 10%
(b) $\frac{2}{3}$	(ii) 20%
(c) $\frac{3}{4}$	(iii) 25%
(d) $\frac{1}{2}$	(iv) 33%
(e) $\frac{1}{3}$	(v) 50%
(f) $\frac{1}{4}$	(vi) 67%
(g) $\frac{1}{10}$	(vii) 75%

(2) In Exercise 1.1, you used a place-value chart for place values greater than one. Place value also extends to the right of the decimal to represent numbers less than one.

(a) Complete the missing names in the chart below.

Hundred thousands
Ten thousands
Thousands
Hundreds
Tens
Ones
Tenths
(i)
(ii)
(iii)
Hundred thousandths

(b) What is the name in words for the number 0.035?

(c) What is the name in words for the percent 0.02%?

(3) Round 54,927.2382 to the specified place value.

(a) Hundredths

(b) Hundreds

(c) Tens

(d) Thousands

(e) Tenths

(f) Thousandths

(4) Convert each fraction to a decimal. Round to the nearest thousandth.

(a) $\frac{2}{5}$

(b) $\frac{12}{7}$

(c) $\frac{25}{3}$

(d) $\frac{25}{6}$

(e) $\frac{254}{100}$

(f) $\frac{6}{25}$

(5) Collaboration 1.4 focuses on skills needed to be a “flexible quantitative thinker.” One of these skills is recognizing when calculations can be done in different ways. In the following question, you will be asked if two number expressions are *equivalent*. A **number expression** is a statement that uses numbers and operations, such as 3 + 4. An equivalent number expression is indicated by an equal sign (=), such as 3 + 4 = 7. Also note 3 + 4 is equivalent to 4 + 3 because it does not matter in what order you add the numbers.

Complete the table by marking whether the first and second expressions are equivalent.

First Expression	Second Expression	Equivalent? Yes/No
5 × 7	7 × 5	(a)
8 – 4	4 – 8	(b)
10 ÷ 2	2 ÷ 10	(c)
20 ÷ 2	$20 \times \frac{1}{2}$	(d)

Multiplying Fractions

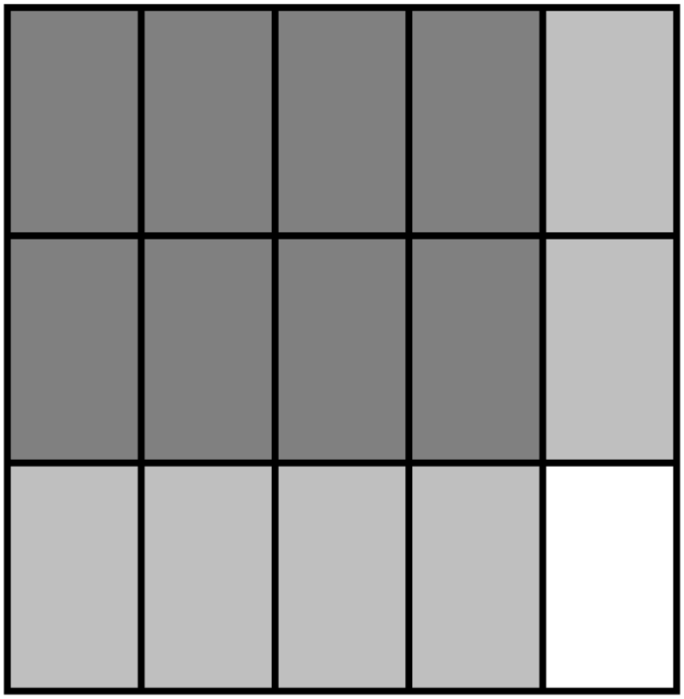
You can think of multiplying fractions in terms of area. Look at the squares below.

Square split into three equal parts horizontally. The top two parts are shaded grey. Square split into 5 equal parts vertically. 4 parts of shaded grey.

The square on the left is divided into thirds. Two of the thirds are shaded gray, that is, the shaded part is $\frac{2}{3}$ of the whole square.

The square on the right is divided into fifths. Four of the fifths are shaded gray, that is, the shaded part is $\frac{4}{5}$ of the whole square.

Now, see what happens when we overlap the two squares.



The region that is shaded twice is darker than the rest. The dark gray area represents the product $\frac{2}{3} \times \frac{4}{5}$.

Notice that the square is now divided into 15 regions (15 = 3 × 5), and the number of those regions that are dark gray is 8 (8 = 2 × 4). So 8 out of 15 pieces of the square are dark gray, or $\frac{8}{15}$.

Recall the rule for multiplying fractions: Multiply the numerators (2 × 4); multiply the denominators (3 × 5); then simplify if possible. Therefore, $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$.

(6) Multiply the following fractions. Write your answer in simplified form.

(a) $\frac{3}{4} \times \frac{2}{5} =$

(b) $\frac{1}{8} \times \frac{3}{4} =$

(7) The United States government offers many different programs for people living in the U.S. These programs provide aid for people living in poverty. The “Federal Poverty Line”, or FPL for short, is a measure that determines who qualifies to receive this aid.

The “Federal Poverty Line” (FPL) is a minimum amount of income a person needs for basic necessities. Some basic necessities are food, clothing, transportation, and shelter. The government calculates this measure by looking at how much an individual person or family would need for these basic necessities.

Household size (how many people live in a home) helps the government set the rate for the federal poverty line. The Federal Poverty Line rate is adjusted each year. The Federal Poverty Line also uses information about the amount of income that people in a household earn each year.

In the United States, the FPL is used to determine if an individual or family is eligible for services. These services include health care coverage or assistance with the cost of housing. The government analyzes household income in terms of percentage of the federal poverty line. Here are some examples:

- For 2014, the federal poverty line for a household of two is $15,730 per year.
- A household of two making 100% of the poverty line has an income of $15,730.

- A household of two making 200% of the poverty line has an income of $31,460.

Using the information about households and the federal poverty line above, answer the following questions.

(a) What is the yearly household income that is 300% of the poverty line?

(b) What is yearly household income that is 400% of the poverty line?

(8) Many people have difficulties knowing how the Affordable Care Act, also known as Obamacare, affects them. Low-income families, in particular, do not know how much financial aid they can get to help pay for healthcare coverage. Today you will learn how to help them!

In 2013, approximately 46 million U.S. citizens did not have health insurance. This was about 15% of the U.S. population. Without health insurance, a person who gets seriously ill or injured may not be able to afford the health care he or she needs to get better. The person may go into serious debt to pay medical expenses.

The Affordable Care Act, also known as Obamacare, was created to restructure America’s health care system. Obamacare requires that most Americans have health insurance by 2014. By 2015, the law requires that businesses with 50 or more employees must offer health coverage to employees. Individuals and businesses that do not meet these requirements risk having to pay a fine.

The way individuals and businesses get health coverage is to join the new Health Insurance Marketplace (Marketplace). The Marketplace is a place where individuals can find health care coverage from private companies. The federal and state governments run different websites for the Marketplace.[1]

Watch this video about “Obamacare” in preparation for the next lesson on the Affordable Care Act:

https://carnegiemathpathways.org/go/ytobamacare

What do you know from this video, the media or personal experience about the Affordable Care Act, or health care coverage in general?

After Preparation 1.4 (survey)

You should be able to do the following things for the next class. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 1.4, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	Rating from 1 to 5
recognize common fraction benchmarks and equivalent percent forms.	
round a whole number to a given place value.	
perform calculations using a calculator.	
understand the relationship of multiplication and division (dividing by 3 is the same as multiplying by 1/3).	
convert between a fraction and its decimal form.	
multiply fractions.	

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[1] https://www.healthcare.gov/do-i-qualify-for-medicaid/

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8.1.2: Exercise 1.4 (Workforce) Affordable Care Act

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) You can change a calculation in any way that you think will make it easier to do.

(ii) Calculations can often be performed in different ways based on mathematical rules.

(iii) Multiplying by 2/3 is the same as multiplying by 2 and then dividing by 3.

(iv) The new Affordable Care Act would allow families with income up to 400% of the poverty line to receive government aid to cover their health care cost.

DEVELOPING SKILLS AND UNDERSTANDING

In Collaboration 1.4, you used several important mathematical rules and relationships to perform calculations in different ways. Those rules are summarized for you here so you can refer back to them. The rules also have formal names. You do not have to memorize these names for this course, but you may use them in other math classes. If you want more help with any of the rules, use their formal names to find resources on the Internet.

Mathematical rules are defined in terms of **variables**. The variables are symbols, usually letters that represent numbers. You use variables to show that the rule can apply to multiple numbers. This is called **generalizing** because it shows that a rule can be used in *general* and not just in specific cases. The mathematical rules below are shown using both variables and numbers.

While mathematical rules are very important, in this course the authors emphasize *reasoning* over memorizing rules. As you review the rules, try to make sense of the rules so that they will become a part of your thinking.

Commutative Property

The **Commutative Property** states that the order of addition and multiplication can be changed.

General Rule	Example
$a + b = b + a$	$8 + 3 = 3 + 8$
$a \times b = b \times a$	$5 \times 6 = 6 \times 5$

Note: It is important to remember that the Commutative Property does not apply to subtraction and division.

Order of Operations

The **order of operations** defines the order in which operations are performed.

General Rule	Example
1. Operations within grouping symbols, innermost first. Grouping symbols include: • Parentheses ( ) • Brackets [ ] • Fraction Bar – the line between the top number and the bottom number in a fraction as shown to the right	$15 + [12 - (3 + 2)] - 2 \times 3^2 \div 6$ $15 + [12 - (5)] - 2 \times 3^2 \div 6$ $15 + [7] - 2 \times 3^2 \div 6$
2. Exponents	$15 + [7] - 2 \times 9 \div 6$
3. Multiplication and division, left to right	$15 + [7] - 18 \div 6$ $15 + [7] - 3$
4. Addition and subtraction, left to right	$22 - 3$ 19

Distributive Property

The **Distributive Property** general rule can be written as:

a (b + c) = a × b + a × c

Note about subtraction: Subtraction is related to addition. The Distributive Property is shown using addition, but it also works with subtraction, such as 8 (5 – 1) = 8 × 5 – 8 × 1.

Notation: The operation of multiplication is shown in many ways. You have already seen the use of the multiplication symbol (×). Another way to indicate multiplication is a number or variable in front of parentheses with no other symbol. For example 6(2) = 6 × 2, or a(b) = a × b. You will learn other symbols for multiplication later in the course.

The Distributive Property is easiest to understand by looking at examples.

Example: 4 (3 + 1) = 4 × 3 + 4 × 1.

To demonstrate that these two calculations are equivalent, each side is done separately.

Left side: Using order of operations, the operation inside the parentheses is done first.

4 (3 + 1)

4 (4)

16

Right side: Using the Distributive Property, the multiplication is distributed over the addition.

4 (3 + 1)

4 × 3 + 4 × 1

Order of operations tells you to multiply first.

12 + 4

Division

Division is the same as multiplication by the **reciprocal**. You get the reciprocal of a number when you write the number as a fraction and reverse the numerator (the top number) and the denominator (bottom number).

General Rule	Example
$a \div b = a \times \frac{1}{b}$	$15 \div 5 = 15 \times \frac{1}{5}$
$a \div \frac{b}{c} = a \times \frac{c}{b}$	$10 \div \frac{3}{5} = 10 \times \frac{5}{3}$

(2) In Collaboration 1.4, you saw that there was a relationship between multiplication and division. Refer back to this work to complete the following statement. (Fill in the blank with a fraction.)

65,596 ÷ 4 is the same as 65,596 × __________.

(3) Using the concept from the previous question, fill in the blanks in the table below to create equivalent statements.

Multiplication	Division
$85 \times \frac{1}{5}$	(a) 85 ÷ ___
(b) 1.23 × ___	1.23 ÷ 7
(c) 1.23 × ___	$1.23 \div \frac{2}{3}$

(4) Which expressions are equivalent to $16 \times \frac{3}{4}$? There may be more than one correct answer.

(i) 16 × 3 ÷ 4

(ii) 16 ÷ 0.75

(iii) 3 × 16 ÷ 4

(iv) 3 ÷ 4 × 16

(v) 16 × 0.75

(vi) 16 ÷ 4 × 3

(vii) 0.75 × 16

(viii) 16 × 4 ÷ 3

(5) According to the Consumer Expenditure Survey, the average American household spent $6,599 on food in 2012. About two-fifths of that was spent on eating out at restaurants. Calculate two-fifths of $6,599.

(6) The Gomez family, a family of four, is considering two different healthcare plans: the standard “Silver Plan” or the Gold Plan.

In the standard plan, the Gomez family will need to pay $250 monthly for their portion of their healthcare coverage costs. The government will cover $650 per month no matter which plan they choose.

If the Gomez family decides to upgrade to the Gold Plan, they will pay an additional $340 per month. They are not sure of the yearly cost for the Gold Plan.

Calculate the yearly cost, including government aid, of the Gold Plan for the Gomez family.

Introduction to Spreadsheets

A spreadsheet is a computer program used to organize and analyze data. In the example below, Lisa has created a spreadsheet for her monthly budget. Data is entered into **cells**, like the boxes in a table. The cells are named by the letter of the column along the top and numbered rows down the side. **Note**: The cell that contains the word income is labeled as A2, not 2A.

Monthly income spreadsheet. Column A is Monthly Budget. A2 = income, A3 = rent, A4 = utilities, A5 = food, A6 = Insurance, A7 = gas for car. B2 = 1677, B3 = 750, B4 = 230, B5 = 160, B6 = 89, and B7 = 80

Use this spreadsheet to answer Questions 8–10.

(7) What is in Cell B4?

(8) What does the number in Cell B4 represent in Lisa’s budget?

(i) The money she plans to spend on rent each month.

(ii) The money she plans to spend on utilities each month.

(iii) The money she plans to spend on food each month.

(iv) The money she plans to spend on insurance each month.

(v) The money she plans to spend on gas for her car each month.

Formulas and Spreadsheets

Formulas are algebraic expressions that show important and non-changing relationships. They can be used to perform calculations in spreadsheets. The formulas use the cell name as a *variable* that represents the value in that cell. For example, in the spreadsheet above, the formula “=B3+B4” would result in the calculation $750 + $230, and “$980” would be displayed. Spreadsheets are a valuable tool because once a formula is written, the result will change when the values change. So if Lisa’s rent increases, she can change the number in Cell B3 and the formula will calculate the new result(s) automatically.

(9) Lisa put the following formula in her spreadsheet:

$$= B2 - B3 - B4 - B5 - B6 - B7$$

(a) Calculate the result of this formula.

(b) What does this value represent for Lisa?

(i) The amount of money she expects to lose each month.

(ii) The amount of money she expects to have left after paying bills each month.

(iii) The percentage of her income that she will be able to save each month.

(iv) The value has no meaning for Lisa.

(c) Which of the following expressions would give the same result as Lisa’s formula?

(i) = B2 – B3 + B4 + B5 + B6 + B7

(ii) = B2 – (B3 + B4 + B5 + B6 + B7)

(iii) = (B3 + B4 + B5 + B6 + B7) – B2

(iv) = B3 + B4 + B5 + B6 + B7 - B2

MAKING CONNECTIONS ACROSS THE COURSE

(10) Which of these expressions shows how to calculate 25% of 2,310? There may be more than one correct answer.

(i) 2,310 ÷ 4

(ii) 2,310 × 4

(iii) 2,310 ÷ 25

(iv) 2,310 × 25

(v) 2,310 × 0.25

(vi) 2,310 ÷ 0.25

(vii) $\frac{1}{4} \times 2{,}310$

(viii) $\frac{1}{4} \div 2{,}310$

(ix) 0.25 × 2,310

(x) 0.25 ÷ 2,310

(11) Which expression is the same as 20% of a billion? There may be more than one correct answer.

(i) 0.2 × 1,000,000,000

(ii) 0.2 × 1,000,000

(iii) $10^9 \div 5$

(iv) $10^9 \div 20$

(v) $10^6 \div 5$

(vi) One-fifth of 1,000 million

(vii) 20,000,000

(viii) 20 ÷ 100 × 1,000,000,000

Scientific Notation

In Exercise 1.3, you saw that a large number can be written as a number times a power of 10 in many different ways. For example, the number 124,000 can be written as 1.24 × 105 or 12.4 × 104. These different forms are all equivalent.

Scientific notation is a very specific way to write a large number as a power of 10. The purpose of scientific notation is to make it easier for people to use and communicate with large numbers. It would be confusing if two people working together on one project wrote the same number in two different ways. To avoid this, people decided that numbers in scientific notation would always be written in the same way: a number between 1 and 10 times a power of 10.

From the previous example:

- 1.24×10^5 is in scientific notation because 1.24 is a number between 1 and 10.
- 12.4×10^4 is not in scientific notation because 12.4 is larger than 10.

(12) Write 16,900,000 in scientific notation.

(13) Write 4,275,000,000 in scientific notation.

Self-Regulating Your Learning: The Plan Phase

At the start of this module, the authors briefly described what it means to be a “self-regulated learner.” As you already learned, being a self-regulated learner involves going through three phases when you are working on a problem or an assignment. The phases are:

1. Plan

2. Work

3. Reflect

Let’s look at what you should be doing during the ***plan*** phase. As you might imagine, the planning phase involves thinking about all the things you need to do to successfully complete a problem or assignment *before* you begin working on it. As was said previously, researchers who study how people learn found that experts often spend a lot more time planning how they are going to finish a task than they spend actually doing the task.

The planning phase involves several important aspects. The following are some that will be explored in this course:

- How much *confidence* you have that you can successfully complete the problem.
- The amount of *time and effort* you think it will take to understand and work on the problem.
- The *strategies* you might use to solve the problem.
- The *goals* you have as you try to work on the problem.

The authors will now describe each aspect in a little more detail. You will also continue to revisit them throughout the rest of the course.

Confidence: People who study how we learn have found that our beliefs regarding our ability to do a given task, like work on a particular math problem, often predicts how well we will actually do. Here is one way to think about it: If you really believe you can succeed at a problem, you are more likely to keep trying and keep working on that problem even if you get stuck. Because you invest more effort, you are more likely to be successful. On the other hand, if you look at a problem and immediately think “I cannot do this,” then when you do get stuck or confused, you might be more likely to give up and not be successful. Researchers call your beliefs about your abilities your **self-efficacy**.

In this course, you will be asked to rate your self-efficacy on certain problems. If you rate yourself low, then you might want to allow more time to do that problem, plan to go get help, or try being more patient than you might normally be. Thinking about your confidence can help you plan your time and effort when you work on a problem or task.

Time and Effort: Obviously, some problems or assignments take more time than others. Some assignments require more effort than others. It can be frustrating to jump into an assignment thinking you can finish it easily or quickly only to discover it is harder or takes more time than you thought it would. You can avoid some or all of that frustration if you have a realistic idea of how hard the assignment will be. Also, having a good idea of how much time and effort will be needed helps you manage your time. For example, you might need to allocate time to discuss the assignment with your instructor, classmates, or tutors. For these reasons, approximating the time and effort needed before starting work on an assignment is a good planning tool.

Strategies: When you start working on a problem or assignment, you often have to try several different strategies before you find an approach that will help you complete it successfully. Sometimes, it is the first strategy you think of, but often it is not. If you think about possible strategies *before* you begin working, you immediately have another one to try if your first one does not work. Self-regulated learners think about many different possible strategies before trying to solve a problem.

Goals: Education researchers have shown that students who have **learning goals** are more likely to succeed than students who have what are called **performance goals**. If you have *learning goals*, you are trying to understand what you are learning and trying to make connections between ideas and concepts. If you have *performance goals*, you care most about finishing an assignment to get points or have it done; you are not focused on understanding the material. Self-regulated learners try to have learning goals more than performance goals. This helps them stay focused and motivated to learn when the problems are challenging. Good planning means making an effort to change your thinking so you have learning goals as often as possible.

In future collaborations and assignments, you will have opportunities to practice the planning ideas presented here. Before then, start incorporating the planning phase whenever you start an assignment. If you do, you will be better prepared and more likely to succeed.

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8.2: Quantway Core 1.8 Workforce (Environment) - Student Lesson

INTRODUCTION

Mathematicians like to shorten things. Sometimes a capital letter (like C or Q) is used as a common variable to indicate that you are talking about two similar quantities. The numbers 1 and 2 indicate that the quantities represent two different quantities, but their values might still be the same. For example, if C1 represents students in one class and C2 represents students in a second class, it is possible that C1 = C2, even though they represent different things.

Note: Remember, an inequality means that two quantities are not equal. Also, remember the inequality symbol (> or <) always points to the smaller of the two quantities.

Quick Check-in:

Please discuss the following questions in your group:

- What specific concept from Unit 1.7 do you feel fairly confident about?
- Are there any specific concepts in Unit 1.7 that you find challenging?

Please take turns, and make sure that every group member answers both questions. If possible, please help your group members understand the concepts that they find challenging.

Completing the CaS Chart

You can use the Comprehension and Synthesis (CaS) Chart in this collaboration. Remember, using the CaS Chart helps you have a deeper understanding of the problem situation. CaS Charts will help you understand the main issue(s) that need to be resolved, and will help you to recognize the quantitative information that is available in the problem situation that can help you to solve the problem. You will use CaS Charts in some of the Quantway collaborations to “unpack” problem situations and support problem-solving.

As you read the problem situation, you may complete the CaS Chart. You may wish to return to these steps as you complete the CaS Chart. You can use the CaS Chart in the following way:

As you are reading through the **Problem Situation: FCC's Trash Problem**:

1. Complete Column A: What issues in the problem situation do you need to understand in order to solve the problem? Is there contextual information that you need to know in order to understand the problem situation?
2. Complete Column B: What quantitative information is provided in the problem situation that will help you solve the problem? Hint: Quantitative information is often a number, but also could be a number word, like “two”.
3. Complete Column C: It is not necessary to solve the problem or use calculations right now. In this column, brainstorm ways you might address the issues presented in the problem situation (Column A) using the quantitative information in Column B. There are no wrong answers.

Column A	Column B	Column C
What is/are the main issue(s) in this problem situation?	**What is the key quantitative information you need to solve the issue(s) in the problem situation?**	**Describe in writing how the information in Column B will help you address the issue(s) in Column A later in the lesson.**

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- percentages involve a numerator (comparison value) and a denominator (reference value).

By the end of this collaboration, you should be able to

- correctly identify the quantities involved in a verbal statement about percentages.
- convert between ratios and percentages.
- convert between the decimal representation of a number and a percent.
- read and use information presented in a two-way table.

SPECIFIC LANGUAGE AND LITERACY OBJECTIVES

By the end of this collaboration, you should be able to

- read and comprehend the problem situation about waste and recycling.
- complete the Comprehension and Synthesis (CaS) Chart with contextual and quantitative information from the problem situation about waste and recycling.
- demonstrate an understanding of mathematics by writing complete and correct responses to questions.
- demonstrate the ability to interpret, predict, analyze, and revise answers about the waste and recycling.
- use appropriate quantitative and environmental science vocabulary to discuss mathematics in this collaboration.

PROBLEM SITUATION: FCC'S TRASH PROBLEM!

You are an environmental specialist in the town of Fairview. The administration at Fairview Community College (FCC) has hired you as an environmental consultant. You will help FCC fix its growing trash problem. Fairview Community College has a very low recycling rate and does not recycle very much of its trash. The college produces much more trash than another nearby college with a similar number of students.

As an environmental consultant, your first job is to understand what makes up the largest percentage of trash at FCC. This is one way to reduce the amount of trash produced by FCC. The reason for doing this is to see what parts of their trash could be recycled. You spend several months analyzing the trash problem at FCC in 2014.

Recycling is one method of dealing with trash. Recycling allows for garbage to be processed and eventually reused. Glass, plastic, metals, cardboard, paper, electronics, and batteries are materials that can be recycled. When we use items made from recycled materials, it means that we are using fewer raw resources (for example, trees to make paper).

During this analysis, you discover that FCC only has a 45% recycling rate. A "recycling rate" is the percentage of waste that gets recycled. At FCC, the recycling rate is the amount of total trash that was recycled. The US Environmental Protection Agency released a report that estimates that nearly 75% of all trash is recyclable.[1] Now you know that FCC's recycling rate can be improved.

You do a further analysis of recycled materials at FCC. Table 1 shows sources of total solid waste or trash at Fairview Community College in 2014. All values are in tons. Much of the paper waste produced included newspaper, copy paper, and paper cups and napkins from the cafeteria. Most of the glass, plastic and metal waste came from drink and beverage containers. Knowing which products have the lowest recycling rates will help determine where to aim recycling efforts.

Table 1: Amount of Paper and Glass, Plastics & Metals Recycled at FCC in 2014 (measured in tons)

	Paper	**Glass, Plastic, and Metals**
Recycled	46	82
Non-recycled	16	139

As an environmental consultant, your job is to figure out goals to increase FCC's recycling rate. The first step is to use the data you collected to determine what materials should be recycled.

(1) Describe one of the values in Table 1 using a complete sentence.

The next question may be challenging. Remember, it's okay to be wrong. This will help you understand the types of mistakes people make when calculating percentages.

(2) (a) Suppose that the percentage of paper that is recycled is called P1 and the percentage of recycled material that is paper is called P2. Which of the following statements do you predict to be true?

P1 = P2

P1 > P2

P1 < P2

(b) How did you come to select your answer in Question 2(a)? Explain.

(c) What percentage of paper is recycled? Round to the nearest tenth.

(d) What percentage of recycled material is paper? Round to the nearest tenth.

(e) Reflect on your answers to questions (c) and (d). Would you change your answer to (a) now? Will this always be the case?

(3) (a) For this question, P1 and P2 will be a new set of quantities.

P1 is now the percentage of glass, metals, and plastic that is recycled. P2 is the percentage of recycled materials that is glass, metals, and plastic.

Which of the following statements do you predict to be true? Explain your reasoning.

P1 = P2

P1 > P2

P1 < P2

(b) What is the percentage of glass, metals, and plastic that is recycled? Round to the nearest tenth.

(c) What is the percentage of recycled material that is glass, metals, and plastic? Round to the nearest tenth.

(d) Reflect on your answers to Questions 4(b) and 4(c). Would you change your answer to 4(a)? Will this always be the case?

(e) Remember, your goal is: to improve the overall percentage of material that is recycled.

Reflect on the analysis you have done. Write a 2-3 sentence proposal to FCC outlining your recommendation based on your analysis. Decide if FCC's recycling efforts should focus on Paper or Glass, Metals & Plastic material. Use your calculations above to support your answer. Provide at least one possible solution.

Writing Tip!

Writing a proposal with multiple parts can be challenging. Below is an example format you could use to write your answer:

FCC should focus their recycling efforts on (write your answer here) because (write your answer here). One way they could do this is to (write your answer here).

Your answer:

(4) Based on your analysis, you know that FCC recycles 45% of its waste. A neighboring school, Clear View Community College (CVCC), also conducts a similar report. The neighboring school recycles 77% of its trash.

(a) Out of 100 pounds of garbage, how many pounds get recycled at FCC?

(b) Out of 1,000 pounds of garbage, how many pounds get recycled at FCC?

(c) Out of 100 pounds of garbage, how many pounds get recycled at CVCC?

(d) Out of 1,000 pounds of garbage, how many pounds get recycled at CVCC?

(e) Which school produced a greater total quantity of non-recycled trash?

FURTHER APPLICATIONS

(5) Consider again your strategy to improve recycling at Fairview, as determined in Question 4(e). Suppose you want to make your argument for focusing on improving this particular category of waste to the college administration. Are there any other percentages that you might use to make your case?

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[1] http://www.greenwaste.com/recycling-stats

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8.2.1: Preparation 1.8-W Waste and Recycling

(1) Which of the following represents 0.02%? There may be more than one correct answer.

(i) 2

(ii) 0.02

(iii) 0.0002

(iv) 2 out of 100

(v) 0.2 out of 100

(vi) 0.02 out of 100

(vii) 2 out of 1,000

(viii) 2 out of 10,000

(2) Which of the following is equivalent to 4%? There may be more than one correct answer.

(i) 0.04

(ii) $\frac{1}{25}$

(iii) $\frac{4}{100}$

(iv) $\frac{40}{100}$

(v) 2 out of 5

(vi) 4 out of 100

(3) Which of the following is correct? There may be more than one correct answer.

(i) A percent is one part out of 100 parts.

(ii) A percent can be converted into a decimal number by dividing that percent number by 100.

(iii) A percent can be converted into a decimal number by moving the decimal point two places to the left and removing the percent sign (%).

(iv) 50% means 50 per 100 or $\frac{50}{100} = 0.5$.

(4) Which of the following is the percent estimate of 1/3, rounded to the nearest hundredth of a percent?

(i) 0.33%

(ii) 3.3%

(iii) 33.3%

(iv) 33.33%

(5) The quantity 1,352 is what percent of 40,929? Round to the nearest tenth of a percent.

In the next collaboration, we will look at an important issue in environmental science: Waste and Recycling. The short video below “I Want to be a Bench” and “How a Recycling Center Works” from the I Want to be Recycled campaign from Keep America Beautiful, shows the process and rewards of recycling for communities. Please watch these two short videos:

https://carnegiemathpathways.org/go/ytbench

https://carnegiemathpathways.org/go/ytrecycle

Recommended: If you are interested, you may also want to explore the “I Want to be Recycled” website: https://carnegiemathpathways.org/go/wantrecycle

Self-Regulating Your Learning: The Work Phase

Earlier, you read in detail about what it means to effectively *plan* for your learning. That involved accounting for time and effort, your confidence (self-efficacy), study strategies, and learning goals. In this lesson, we will discuss the second phase of self-regulated learning (SRL), the ***work*** phase.

As the name implies, the *work* phase of SRL is where you are actually working on the problem or assignment. However, it is more than just getting the assignment done. In this phase, you **monitor** or pay attention to a variety of things. For example:

- What you are or are not **understanding** (and when);
- Which **strategies** you are using;
- Which ones are working, and which ones are not;
- What **emotions** and feelings you are experiencing, both positive and negative;
- When you should **seek help** from others.

Let's explore each of these in a little more detail.

Understanding: Self-regulated learners monitor what they understand and what they do not. This is done by frequently asking yourself: "Do I understand this?" or "Could I explain this to someone?" The goal is to monitor your understanding so that you may adapt your strategies, especially if you get stuck. Being honest about your understanding is important because it can help you progress successfully on a problem, or make you aware of your learning strengths and weaknesses. Sometimes, people talk about this as "thinking about your thinking." Researchers call it *metacognition*.

Strategies: In learning about the SRL *plan* phase, you discovered that it can be useful to think about multiple strategies before you start working on a problem. In the *work* phase of SRL, having multiple strategies in mind (both those you have used before and those you plan to try) can help when you get stuck. You can stop, think about how the problem is progressing, and try another strategy that you think might work. Self-regulated learners often make mental notes about which strategies work in which situations, and which ones are easiest to use. Evaluating strategies allows you to become better at solving a variety of problems.

Emotions: Self-regulated learners know how to monitor their emotions—especially negatives ones such as frustration or anger—so that these emotions do not cause them to give up on a problem. When they start feeling frustrated, self-regulated learners often do things such as try new strategies, seek help, or engage in *positive self-talk*. This is saying things to yourself such as: "I know I can do this if I choose the right strategies and put in the effort, even if it is challenging." The opposite is called *negative self-talk*, which involves saying things such as: "I am never going to get this! What is the point?" Monitoring and controlling your emotions, especially the negative ones, can be challenging and may require a lot of practice, but the benefits are worth it.

Seeking Help: With practice and experience, self-regulated learners know when it is beneficial to stop working and find someone else with whom they can discuss the problem. There is nothing wrong with getting help when you are learning something new. Some people think that asking for help means you do not have ability, but the truth is that knowing when to seek help is part of being an effective learner. Seeking help can save you time because you avoid the added frustration of making a lot of effort without making any progress. If you are spending a lot of time on a problem, you have tried several strategies without success, or you are not able to control negative emotions, stop and write down your questions. Bring the written questions with you and discuss the problem with someone else as soon as you can. Help could come from your instructor during his or her office and/or virtual office hours, a campus learning center, or other classmates.

During the *work* phase, you are required to juggle two things at once: (1) working on the problem or assignment, and (2) monitoring your progress (e.g., *thinking about your thinking*). This process takes practice; however, it is important to master if you want to become a self-regulated learner. Thinking about how you are working makes the work easier and gives you information for the SRL *reflect* phase. You will explore the *reflect* phase later. Until then, practice these work strategies while you are working on problems and class assignments.

After Preparation 1.8 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 1.8, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	Rating from 1 to 5

understand the word percent and the notation used to describe percentages (%).	
use a calculator to divide two numbers and interpret the resulting decimal representation as a percent.	
calculate and estimate percentages.	

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8.2.2: Exercise 1.8-C Waste and Recycling

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) A percentage is calculated by dividing one number by another number.

(ii) Recycling glass is more important than recycling paper.

(iii) A percentage is a comparison of two numbers. To understand the meaning of a percentage, it is important to know what two quantities are being compared.

(iv) The percentage of students who are parents is the same as the percentage of parents who are students.

DEVELOPING SKILLS AND UNDERSTANDING

(2) Data from the *National Postsecondary Student Aid Study* (NPSAS)[2] provides a statistical snapshot of the proportion of community college students who majored in different fields of study in 2003–04. A total of 25,000 community college students were included in the study. The table below displays the total number of students who majored in each of the following fields of study in 2003–04.

(a) Complete the table by filling in the *percentages* of community college students who majored in each field of study. Round to the nearest one percent.

Field of Study	Number of Students who Majored in Field	Percentage of Students who Majored in Field
Humanities	3,700	(i)
Social/Behavioral Sciences	1,250	(ii)
Mathematics and Science	900	(iii)
Computer/Information Science	1,525	(iv)
Engineering	1,025	(v)
Education	2,025	(vi)
Business/Management	4,600	(vii)
Health	5,975	(viii)
Vocational/Technical	1,225	(ix)
Other Technical/Professional	2,775	(x)

(b) What was the *most popular* major in 2003–04?

(i) Humanities

(ii) Business/Management

(iii) Health

(iv) Education

(c) Fill in the blanks to complete the following statements.

About _______ out of every 100 community college students in 2003–04 majored in the most popular field.

About ________ out of every 1,000 community college students in 2003–04 majored in the most popular field.

(3) Select the answer(s) that correctly complete the statement from the list below: A *New York Times* story reported that 10% of male high school dropouts are in jail or detention centers.[3] According to this statistic, about ________ in every ________ male high school dropouts is (are) in jail or juvenile detention. There may be more than one correct answer.

(i) 1 in every 10

(ii) 1 in every 10%

(iii) 0.1 in every 100

(iv) 1 in every 100

(v) 10 in every 100

(4) A teacher has collected data on the grades his students received in his morning and afternoon classes. The following tables show two different ways to represent the same data as percentages.

Table 1

	Grade A	Grade B	Grade C	Grade D	Grade F
Morning Class	12.5%	25.0%	37.5%	6.3%	18.7%
Afternoon Class	14.3%	20.0%	37.1%	8.6%	20.0%

Table 2

	Grade A	Grade B	Grade C	Grade D	Grade F
Morning Class	44.4%	53.3%	48.0%	40.0%	46.2%
Afternoon Class	55.6%	46.7%	52.0%	60.0%	53.8%

(a) Which table could be used to answer the following question: What percentage of the students who received an “A” is in the morning class?

(b) Which table could be used to answer the following question: What percentage of the students in the morning class received an “A”?

(c) What are the reference values in Table 1?

(i) The number of students in a certain class.

(ii) The number of students who earned a certain grade.

(iii) The number of students in a certain class who earned a certain grade.

(iv) The total number of students in both classes.

(d) What are the reference values in Table 2?

(i) The number of students in a certain class.

(ii) The number of students who earned a certain grade.

(iii) The number of students in a certain class who earned a certain grade.

(iv) The total number of students in both classes.

MAKING CONNECTIONS ACROSS THE COURSE

(5) Congratulations! You won a lottery prize and have a taxable income of $1,025,400. Use the table below to answer the following questions.

Taxable Income	Tax
$0–$9,075	10% of taxable income
$9,076–$36,900	$907.50 plus 15% of excess over $9,075
$36,901–$89,350	$5081.25 plus 25% of excess over $36,900
$89,351–$186,350	$18,193.75 plus 28% of excess over $89,350

$186,351–$405,100	$45,353.75 plus 33% of excess over $186,350
$405,101–$406,750	$117,541.25 plus 35% of excess over $405,100
$406,751 plus	$118,118.75 plus 39.6% of excess over $406,750

(a) How much tax will you pay on your winnings? Round to the nearest dollar.

(b) Which of the following expressions can be used to calculate the tax?

- (i) $1,025,400 – 0.396($406,750 – $118,118.75)
- (ii) 0.396 + $406,750($1,025,400 – $118,118.75)
- (iii) ($1,025,400 + $118,118.75) – ($406,751 × 0.396)
- (iv) $118,118.75 + 0.396($1,025,400 – $406,750)
- (v) $1,025,400 – 0.396($1,025,400 – $406,750)

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[2] http://nces.ed.gov/pubsearch/pubsinf...?pubid=2006184

[3] http://www.nytimes.com/2009/10/09/education/09dropout.html?_r=0

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8.3: Quantway Core 1.9 Workforce (Health Care) - Student Lesson

INTRODUCTION

In this collaboration, we will be making quantitative statements about data that can be displayed in two-way frequency tables. Two-way frequency tables are used to display data based on two categorical variables, where each cell shows how many observations fall in each category.

Consider the following example. Out of 170 students who took a Chemistry 201 test,

- 20 students passed it even though they didn't complete the corresponding Test Review assignment provided by the instructor
- 80 students passed after completing the corresponding Test Review assignment provided by the instructor
- 60 students didn't pass it, and they didn't complete the corresponding Test Review assignment provided by the instructor
- 10 students didn't pass it even though they completed the corresponding Test Review assignment provided by the instructor

The given dataset of 170 students can be displayed in the following two-way frequency table:

	Completed the Review Assignment	Didn't Complete the Review Assignment	Total
Passed the test	80	20	100
Didn't pass the test	10	60	70
Total	90	80	170

In your group, can you think of other examples of data that can be displayed in a two-way frequency table like the one above?

Double-Entry Journal for Problem Situation

In this collaboration, you can use a Double-Entry Journal (DEJ) to help you better understand the problem situation. The DEJ is designed to give you a better understanding of the reading. Sometimes a problem situation has additional information about a situation. A DEJ helps you focus on the key ideas in a problem situation. As you read through **Problem Situation: HIV Testing**, you may want to complete the Double-Entry Journal using the steps below:

1. In the Left Column, based on information in the problem situation, list the opportunities and challenges associated with the ELISA test. (It might be helpful for you to think about "opportunities" as the positive benefits of using the ELISA test. Then the "challenges" are the problems with the test.)
2. In the Right Column, based on information in the problem situation, write supporting details about the opportunities and challenges associated with the ELISA test.

Opportunities and Challenges Associated with the ELISA Test	Supporting Details about the Opportunities and Challenges Associated with the ELISA Test

1.	1.

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- a percent has different uses, including being used to express the likelihood (or probability) of a certain event.
- it is important to select the correct comparison value and reference value when calculating percentages.

By the end of this collaboration, you should be able to

- extract relevant information from a table.
- select the appropriate values to calculate probabilities.

SPECIFIC LANGUAGE AND LITERACY OBJECTIVES

By the end of this collaboration, you should be able to

- read and comprehend the problem situation about HIV testing.
- complete a Double-Entry Journal (DEJ) with contextual and quantitative information from the problem situation about HIV testing.
- demonstrate an understanding of mathematics by writing complete and correct responses to questions.
- demonstrate the ability to interpret, predict, analyze, and revise answers about HIV testing.
- use appropriate quantitative and health care vocabulary to discuss mathematics in this lesson.

PROBLEM SITUATION: HIV TESTING

Lilia is a screening counselor in an HIV testing clinic called the ScreenMe Clinic. The Human Immunodeficiency Virus (HIV) is a virus that damages the body's ability to defend against sicknesses. There are effective treatments for people with HIV. But, there is no cure for HIV. This means that once you have HIV, you have it for life. HIV can, but doesn't always lead to AIDS.[3] 1.2 million people in the United States are infected with the HIV virus. 1 in 7 people with HIV are unaware of their infection.

If you think you may have been exposed, it is important to get tested for HIV. Knowing if you have HIV can help prevent spreading the virus. Unfortunately, HIV cannot be cured by medicine. But, there are medicines available to help prevent it from turning into AIDS.

As a screening counselor, Lilia gives HIV tests and counseling to clients about results. There are several HIV tests available. The ScreenMe Clinic uses the ELISA test. Often clinics use the ELISA test because it can be done using saliva or blood. It can provide quick results. This is important because sometimes people who get tested do not come back to get their results. About 30-39% of people who get tested for HIV do not return for the results. With the ELISA test, almost all clients receive their results because clinics provide the results during the testing visit.[4]

The problem is the ELISA test is not always accurate. Sometimes the test returns a positive result even though the person does not have HIV. Lilia has to share results with patients. This can be complicated and difficult. Here is an example: Imagine a person takes the ELISA test. He tests positive for HIV. Lilia has to tell him: "Your preliminary test result is positive. But, we do not know for sure if you are infected with HIV. We need to get the results from a confirmation test. In the meantime, you should take precautions to avoid passing the virus to other people."[5]

An example of a confirmation test is the Western Blot test. The patient is tested using the Western Blot test to confirm that the ELISA result is correct. Results from the Western Blot test can take over two weeks to come back, but are accurate.

Patients in this situation get very upset. Often they suffer from extreme anxiety while waiting for the results from the Western Blot test. Lilia wants the clinic to consider a different type of screening test. She needs you to analyze ELISA test data. You must help Lilia determine if ScreenME should continue to use the test, or look for a new one. Data will include information about how often the ELISA test provides results that are not accurate.

You will analyze the likelihood or chance of getting accurate or inaccurate results. You will study "probability." The probability of an event is the likelihood that the event occurs. Probabilities are often represented by percentages. For example, if a fair coin is flipped, there is a 50% chance it will land heads. This is because there are only two equally likely outcomes: heads or tails. "Heads" is one of those outcomes: 1 out of 2 = 1/2 = 0.5 = 50%.

(1) We know from the problem situation that 1 in 7 people infected with HIV in the U.S. do not know they have it. What is the probability that a person who is infected with HIV is unaware they have the virus?

(2) Lilia finds data on test results of a million people who were screened for HIV with ELISA, including the actual HIV status of those who were screened. Some of what she finds is presented in the table below. Fill in the empty cells in the table.

Table 1

	Carry HIV	No HIV	Total
Positive ELISA result	4885	73630	(iii)
Negative ELISA result	115	(ii)	(iv)
Total	(i)	995,000	1,000,000

Use the numbers in Table 1 to answer the questions below. You will use the numbers in the table to decide on the probability that this test gives correct and incorrect results. Report probabilities in percentages (%). Be careful about which numbers you use for the numerator and denominator in your calculations.

(3) If a person has HIV, what is the probability that their ELISA test will come back positive? Round to the nearest percent.

(4) If a person does not have HIV, what is the probability that their ELISA test will come back negative? Round to the nearest percent.

(5) Based on the above answers, would you say that ELISA is an accurate screening test?

(6) (a) Given that the ELISA test is not always accurate, it is possible that the test can fail to identify HIV in someone who carries HIV. This is called a false negative. Suppose that a person carries HIV. What is the probability that this person's test will receive a negative test result? Round your answer to the nearest one-hundredth of a percent.

(b) If a person tests negative using ELISA, should they feel confident that they don't have HIV?

(7) (a) Is there also the possibility of a false positive? Explain your answer.

(b) What is the probability that a person who does not carry HIV will receive a positive test result?

(8) After seeing the answer for Question 7(b), reconsider Question 6. Do you think ELISA is an accurate test? Write Lilia a short note explaining why you think ScreenMe should continue to use the ELISA test or replace it. Be sure to use your calculations. Write your note using complete sentences.

(9) At ScreenMe, if someone does get a positive result for HIV using the ELISA screening test, they are given the Western Blot Test. This test is 100 percent accurate in identifying HIV. However, the Western Blot test costs $200, whereas the ELISA test costs $75 and is more readily available to healthcare workers.

Do you think the ELISA screening test is a cost-effective way to identify HIV-positive status? Why or why not? Write your answer in **two or three complete sentences**.

FURTHER APPLICATIONS

Some athletes use performance-enhancing drugs (PEDs) to improve their performance in sports. Schools, sports leagues, and other sports organizations usually do not allow the use of PEDs. These groups can administer or give athletes a blood or urine test to determine whether the athletes are using drugs.

Imagine a situation in which 500 athletes have undergone a test to determine if they use PEDs. A positive (+) test result indicates or shows that the athlete is using a PED. A negative (–) test result indicates the athlete is not using these drugs. However, this test is not 100% accurate. This means that some errors may have occurred in the test results. The table below shows how often the test correctly determined if athletes used PEDs.

	Athletes Using PEDs	Athletes Not Using PEDs	Total
Positive Test Result	9	5	(a)
Negative Test Result	1	485	**486**
Total	**10**	(b)	**500**

(10) The table is missing one row total and one column total. Discuss with your group what the missing numbers should be. Complete (a) and (b) in the table.

(11) You might want to know: "If an athlete is using PEDs, what is the probability that this test gives a positive result?" Use the completed table to work through the following questions.

(a) How many athletes are using PEDs?

(b) How many of the athletes using PEDs received a positive test result?

(c) If an athlete is using PEDs, what is the probability that this test gives a positive result?

(12) The table shows that one athlete who was using PEDs received a negative test result. This means the test incorrectly identified this single athlete. What percentage of athletes who are using PEDs incorrectly test negative?

(13) You might want to know: "If an athlete is not using PEDs, what is the probability that this test gives a negative result?" Work through (a) – (c) below to answer this question.

(a) How many athletes are not using PEDs?

(b) How many of the athletes not using PEDs received a negative test result?

(c) If an athlete is not using PEDs, what is the probability that this test gives a negative result? Discuss in your group. Round to the nearest whole percent.

(14) Think about a situation in which a school principal receives a positive result on a PED test for a student athlete. Answer these questions:

(a) What is the chance the result is a false positive?

(b) How should the principal think about this percentage? What should the principal do with this information?

STRESS REAPPRAISAL

(to be completed before the Module 1 exam, as instructed)

As you might expect, taking a math test can be a very stressful experience. Before taking the Module 1 exam, we are going to complete a short reading exercise designed to help you perform well.

In the following pages, you will be presented with scientific research. Because scientific research articles can be long, complex, and difficult to understand we had students like you summarize the research articles discussed below.

The research presented in the following materials was conducted with people with anxiety disorders to help them cope with feelings of overwhelming anxiety. More specifically, materials provide you with information about how stress can help improve performance.

While you read each summary, please try to think about how stress can help you. Then, after each one, you will be asked to write a short summary using your own words describing how the information presented can help maximize your performance on the Module 1 exam. The information you write here will be included in messages to future Quantway and Statway students designed to provide students with the confidence to thrive in testing situations.

The following summary is of Jamieson and Mendes' 2010 study that appeared in *the Journal of Experimental Psychology*:

> In stressful situations, people experience changes in their body. They might experience these changes as "unsettled feelings" or "butterflies in their stomach," and conclude that they are nervous. However, the bodily changes that happen during stress can be good, not just bad. For instance, scientists have found that feelings of "butterflies" or nervousness indicate that the body is gathering resources to meet the demands of the situation. In other words, the body needs energy to perform and stress helps deliver this energy to where it is needed.
>
> Stress can be "good" or "bad," and the type of stress we experience depends on our perceptions and beliefs. To demonstrate how the body and mind work together, imagine you are a skier staring down a steep, icy slope with no other way off the mountain than going down this dangerous trail. Regardless of whether you like skiing, this situation would cause stress. Expert skiers may experience the stress as "excitement" if they believe they can handle the difficult trail, whereas novices are more likely to experience the stress as "fear" if the difficulty of the trail exceeds their skill level. Thus, the skier's response (excitement vs. fear) depends on how they perceive the situation and their body's responses.
>
> The negative consequences of stress are the direct result of people's beliefs that feeling anxious will make them perform poorly. However, our research from people with anxiety disorders indicates that **stress does not hurt performance, but can actually help** because our brain releases chemicals that help us think quickly. So, during the Module 1 exam, try and view stress as a coping tool.

In your own words please briefly describe how this information can help you perform well on the Module 1 exam:

____

____

____

____

The following summary is of Nock et al.'s (2011) study that appeared in the *Journal of Clinical Psychology*:

> Stress is a normal physical reaction that helps you face the challenges in your life. It is not harmful. In fact, if we did not have stress reactions we could not survive. If stress is helpful, then why do most people see it as a negative experience?
>
> Research from patients coping with anxiety disorders indicates that negative reactions to stressful situations like taking an exam in class or going on a job interview are the result of how we think about stress (also known as "cognitive appraisals"). When the "*fight or flight*" system activates, our brain searches for potential sources of harm. However, in modern society

> there is often no physical threat. When no explanation can be found, the search turns inwards and the brain invents explanations such as, “There must be something wrong with me.” Nothing could be further from the truth.
>
> During stressful situations remember that your body’s responses are good. Increased heart rate, sweating, and heavy breathing all help deliver oxygen where it is needed. Remember, if you find yourself experiencing stress during tests, focus on the benefits of your body’s responses.

In your own words please briefly describe how this information can help you perform well on the Module 1 exam:

____

____

____

____

The following is an illustrative diagram that shows the biological changes that happen when we experience stress. Please take a minute to note where the changes occur and how these help us do well.

Illustration showing how stress affects the brain, heart and breathing, indicating the following: Brain: Stress releases chemicals that help you think more quickly & solve problems better. Maybe you’ve even heard of one of these chemicals before: Adrenaline. Heart: Stress increases your heart rate so it can deliver fuel to your brain to help you think. Your heart sends oxygen to fuel your brain. Breathing: Stress speeds up your breathing. You might even feel short of breath. This is okay because it means your body has lots of oxygen to help you do well.

In your own words please briefly describe how this information can help you perform well on the Module 1 exam:

____

____

____

____

Great job! You have finished the reading exercise.

Remember during the Module 1 exam, we ask that you try to remind yourself that your body’s responses to the stressful testing situation will help you to perform well.

Good luck!

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[3] http://www.cdc.gov/hiv/basics/index.html

[4] http://www.cdc.gov/hiv/testing/lab/clia/rtcounseling.html

[5] http://www.cdc.gov/hiv/testing/lab/clia/rtcounseling.html

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8.3.1: Preparation 1.9-W Medical Testing

According to the World Health Organization (WHO), “Every person is at risk of foodborne illnesses.”[1] A foodborne illness is an illness a person gets from eating food that has spoiled or been contaminated in some way.

(1) In industrialized countries, such as the United States, up to 30% of the population suffers from foodborne diseases each year. This means that 30 out of _________ people living in industrialized countries will likely suffer from foodborne diseases each year.

(2) In 1994, an outbreak of illness due to ice cream contaminated with the bacteria salmonella occurred in the U.S. The outbreak affected an estimated 224,000 people.

(a) If the total population in the United States at that time was 260,000,000, which is the best estimate for the percentage of people who were affected?

- (i) About 0.1%
- (ii) About 1%
- (iii) About 5%
- (iv) About 10%
- (v) About 25%

(b) Complete the following statement: _________ out of every 100,000 people in the United States were affected by the 1994 salmonella outbreak. Round to the nearest whole person.

(3) A report about foodborne illnesses indicated “About 1 egg out of every 20,000 contains salmonella inside the shell.” This means that

- (i) 0.005% of eggs contain salmonella.
- (ii) about 1% of eggs contain salmonella.
- (iii) more than 1% of eggs contain salmonella.
- (iv) approximately 50 out of every 100,000 eggs contain salmonella.

(4) In 2011, Germany had an outbreak of illness caused by the bacteria called E. coli. As of June 15 of that year, 3,235 people in Germany had become sick and 36 had died.[2] What percentage of those who got sick also died? Round to the nearest tenth of percent.

In the next collaboration, you will be looking at probability related to HIV testing. Please read the following article about HIV testing in preparation: https://carnegiemathpathways.org/go/slatehivtest

After Preparation 1.9 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 1.9, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
understand the word percent and the notation used to describe percentages (%).	
use a calculator to divide two numbers and interpret the resulting decimal representation as a percent.	
calculate percentages.	

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[1] http://www.who.int/foodsafety/foodborne_disease/en/

[2] http://health.usnews.com/health-news/managing-your-healthcare/articles/2011/06/15/health-highlights-june-15-2011

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8.3.2: Exercise 1.9-W Medical Testing

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) A probability is a percentage in which the chance of an event is measured as a ratio out of 100.

(ii) Medical tests are far less accurate than most people think.

(iii) To calculate a percentage, divide the comparison value by the reference value.

(iv) Probabilities are not related to percentages.

DEVELOPING SKILLS AND UNDERSTANDING

(2) About 14% of drivers in the United States are uninsured.[6] There were approximately 212 million drivers in the United States in 2011.[7] How many of these drivers were likely uninsured?

(i) 29,680

(ii) 29,680,000

(iii) 296,800,000

(iv) 2,968,000,000

(5) A test is administered to 500 athletes to determine if they are using performance-enhancing drugs (PEDs). It is assumed that a positive test result indicates that the athlete is using PEDs; and a negative test result indicates that the athlete is not using PEDs. However, this test is not 100% accurate, so some errors occur. The following table shows the test results for a group of athletes.

	Athletes Using PEDs	**Athletes Not Using PEDs**	**Total**
Positive Test Result	90	4	**94**
Negative Test Result	10	396	**406**
Total	**100**	**400**	**500**

Use the information in the table to answer the following questions.

(a) How many athletes are using PEDs?

(b) If an athlete is using PEDs, which of the following describes the chance that this test will give a positive result? There may be more than one correct answer.

(i) 9%

(ii) 90%

(iii) 9 out of 100

(iv) 90 out of 100

(c) What is the chance of the test giving a negative result for an athlete using PEDs (false negative)?

(d) If an athlete is not using PEDs, what is the chance that this test will give a negative result?

(i) 39.6%

(ii) 99%

(iii) 4 out of 396

(iv) 4 out of 400

(e) What is the chance of the test giving a positive result for an athlete not using PEDs (false positive)?

(4) A hospital tracks the number of cases (patients) that come into its Emergency Room during each eight-hour shift. The cases are listed in categories based on the severity of the illness or injury. The categories from least severe to most severe are: stable, serious,

and critical. The following table gives the data for a week.

(a) Complete the missing blanks in the table.

Shift	Stable	Serious	Critical	Total
8:00 a.m. – 3:59 p.m.	250	120	45	**415**
4:00 p.m. – 11:59 p.m.	270	230	105	(i)
12:00 a.m. – 7:59 a.m.	(ii)	175	95	**460**
Total	**710**	(iii)	**245**	**1480**

(b) A nursing supervisor ranks the shifts based on two different criteria. Which shift received the highest percentage of the total critical cases? For each shift, find the percentage of **critical cases compared to total critical cases**. Round to the nearest one percent.

Shift	Percentage of Total Critical Cases	Rank
(i) 8:00 a.m. - 3:59 p.m.		
(ii) 4:00 p.p. - 11:59 p.m		
(iii) 12:00 a.m. - 7:59 a.m.		

(c) Rank the shifts from highest to lowest.

(d) Which shift has the highest percentage of **critical cases compared to that shift's total cases**? Round to the nearest one percent.

Shift	Percentage ol Cases Compared to Shift's Total Cases	Rank
(i) 8:00 a.m. - 3:59 p.m.		
(ii) 4:00 p.p. - 11:59 p.m		
(iii) 12:00 a.m. - 7:59 a.m.		

(e) Rank the shifts from highest to lowest.

(f) The hospital schedules nursing staff based on the number and severity of expected cases. One goal of scheduling is to have more experienced nurses work on critical cases. The nursing supervisor considers the overall number of experienced nurses and the ratio of experienced nurses to the less experienced nurses. Using these data and the tables you created in the questions above, which of the following conclusions could be drawn?

(i) The highest *number* and the highest *ratio* of experienced nurses should be scheduled during the 12:00 a.m. to 7:59 a.m. shift.

(ii) The highest *number* and the highest *ratio* of experienced nurses should be scheduled during the 4:00 p.m. to 11:59 p.m. shift.

(iii) The highest *number* of experienced nurses should be scheduled during the 4:00 p.m. to 11:59 p.m. shift. The highest *ratio* of experienced nurses should be scheduled during the 12:00 a.m. to 7:59 a.m. shift.

(iv) The highest *number* of experienced nurses should be scheduled during the 12:00 a.m. to 7:59 a.m. shift. The highest *ratio* of experienced nurses should be scheduled during the 4:00 p.m. to 11:59 p.m. shift.

MAKING CONNECTIONS ACROSS THE COURSE

(5) You can save time and work by using spreadsheets to perform calculations. To do this, you use formulas (as seen in Exercise 1.4). In this question, you will use a spreadsheet to calculate Marianne Lopez's self-employment taxes, which were discussed in Collaboration 1.7.

(a) Use the information from Collaboration 1.7 to fill in the blanks for cells B2–B5 in the spreadsheet below.

Screenshot of spreadsheet. A1 = Marianne Lopez. A2 = Revenue. A3 = Expenses. A4 = % of income that is taxed (as a decimal). A5 = Tax rate (as a decimal). A7 = self employment tax. B2 = i, B3 = ii, B4 = iii, and B4 = iv.

(b) Which of the following is a formula for cell B7 that will calculate the self-employment tax. Remember that the formula will start with an equal sign (=). In a spreadsheet, you use an asterisk (*) to indicate multiplication. So B3 × B4 would be written as B3*B4.

(i) = B4*B5*(B2-B3)

(ii) = B4*B3*(B2-B5)

(iii) = (B5-B4)*B2*B3

(iv) = B5*B3*(B4-B3)

(c) Suppose Marianne made a mistake in calculating her expenses. The expenses should be $4,371. Enter this new value into the spreadsheet. What is the new amount for the self-employment tax? Remember that since we are working with monetary values here, include both dollars and cents in your answer.

Self-Regulating Your Learning: The Reflect Phase

The last phase in regulating your learning is the ***reflect*** phase. In this phase, after you finish a problem or assignment, you intentionally reflect on how your learning and problem solving went. You gather information about yourself, about studying, and about learning in general. This information is then used to improve future learning situations. Here are some things that self-regulated learners reflect upon:

- Confidence (self-efficacy)
- Strategy selection
- Time and effort
- Emotions
- Causes of success and setbacks

Confidence (Self-Efficacy): Regulating your learning means you continuously pay attention to how much you believe that you can succeed at what you are trying to accomplish. This is important after you complete a problem because it allows you to plan for what you will need to do as you move forward. If you rate your confidence low, then you would benefit from spending more time practicing and studying, and you may decide to get additional help. If you do not stop to think about how things went, it is easy to just move on to the next concept, thinking that you understand something you do not. Also, it is important to give yourself credit for your successes. You will feel better about the time and effort you spent, and you will be more motivated to continue to work hard.

Strategy Selection: When regulating your learning, look back at the strategies you used when working on a problem or assignment. Make note of what seemed to work well for certain problems and what strategies seemed easier. This information will guide your *plan* phase in preparing for a new assignment. You can practice this by asking yourself: “What worked well and what did not?”

Time and Effort: Pay attention to the time and effort it took to complete an assignment. Reflect on whether you planned accurately. When doing this, try to identify the types of things you may need more time for in the future, or what might be more of a challenge. This information will be used during the plan phase.

Emotions: For successful learning, it is important to manage your emotions, especially your frustration and anxiety. One way to do this is to ask yourself what caused you to become frustrated or anxious and think about what helped you overcome those feelings. This gives you tools to deal with those feelings when they come up again.

Causes of Success and Failure: One of the most common challenges for students is correctly identifying the reasons for both their successes and their setbacks. Students often blame *external* factors for shortcomings. When faced with challenges, students often blame the teacher (“She does not explain well”); or maybe the book (“It is hard to read and it is confusing”); or the test itself (“It was full of trick questions”). The problem with blaming external factors is that it gives you little control over your own learning outcomes. On the other hand, by considering *internal* factors—ones you can control—then you are in charge of your learning outcomes. For example, when facing a setback, ask yourself:

- Did I spend enough time on this assignment?
- Did I use the right strategies?

- Did I seek help when I needed it?
- Did I put in the work and effort that was really required?

These types of self-reflection questions help you to understand yourself better and assist you in becoming a more effective learner.

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[6] http://online.wsj.com/news/articles/SB10001424052702303281504579221972801700610

[7] http://www.fhwa.dot.gov/policyinformation/statistics/2011/dv1c.cfm

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8.4: Quantway Core 2.3 Workforce (Environment) - Student Lesson

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- a relative change is different from an absolute change.
- a relative measure is always a comparison of two numbers.

By the end of this collaboration, you should be able to

- calculate a relative change.
- explain the difference between relative change and absolute change.

SPECIFIC LANGUAGE AND LITERACY OBJECTIVES

By the end of this collaboration, you should be able to

- read and comprehend the problem situation and the “renewable energy” chart.
- complete the Double-Entry Journal with using challenging vocabulary words from the problem situation.
- demonstrate an understanding of mathematics by writing complete and correct responses to questions.
- demonstrate the ability to interpret, predict, analyze, and compare information about energy use and renewable energy generation.
- use appropriate quantitative and environmental science vocabulary to discuss mathematics in this collaboration.

INTRODUCTION

Double-Entry Journal for Problem Situation

In this collaboration, you can use a Double-Entry Journal (DEJ) to help you better understand the problem situation.

The DEJ is designed to give you a better understanding of the reading. Sometimes a problem situation has additional information about a situation. A DEJ helps you focus on the key ideas in a problem situation. Sometimes DEJs help to outline key ideas in a reading. Other times, as in this collaboration, DEJs can help to organize important vocabulary or phrases. As you read through **Problem Situation: Renewable Energy**, you may want to complete the Double-Entry Journal using the steps below:

- In the Left Column, list the **key vocabulary words, key phrases, or key concepts** in the problem situation. These words include words you think are important in the problem situation. Or, these words can be ones that you are not familiar with and want to understand better.
- In the Right Column, **write your definition of the words, phrases, or concepts. You don't need to write a "dictionary definition.” Your definition should be based on what the words, phrases, or concepts seem to mean** in the problem situation and your own ideas and understandings.

Key Vocabulary or Concept in the Problem Situation	My Definition
1.	1.

PROBLEM SITUATION: RENEWABLE ENERGY GENERATION

Congratulations! You just started a new job as an environmental researcher. As an environmental researcher, you study new environmental policies and technologies used around the world. You create reports and presentations to talk about policies and technologies that could be used in your state. You work for a non-profit organization called the S&G Energy Association. The S&G

Energy Association advocates for clean and sustainable sources of energy. Sustainable means that the source is continuously available. That is, it is a source that we will not run out of quickly. S&G is a local leader in your state on environmental issues.

Nearly 85% of the energy produced in the United States comes from fossil fuels. Fossil fuels were formed millions of years ago from the remains of living organisms. Fossil fuels include oil, natural gas, and coal. There are three main environmental concerns about using fossil fuels as a source of energy. First, burning fossil fuels creates air pollution. Second, burning fossil fuels is a source of a large amount of carbon dioxide (CO2), which is a colorless and odorless gas that is produced when we breathe, and when we burn fossil fuels. CO2 contributes to global warming. Third, fossil fuels take millions of years to form and so they are limited resources. We will run out of fossil fuels eventually.

For these reasons, governments, organizations, and individuals are interested in finding alternative sources of energy. Renewable energy comes from resources that can naturally be replenished at a rate equal to their consumption.[1] In other words, renewable resources can be renewed or replenished at the same rate at which they are consumed. Some renewable sources of energy are sunlight, wind, water, and plant matter.

In 2010, the world used approximately 18,051 billion kilowatt hours (kWh) of electricity. But countries consume energy at very different rates. For example, the United States used 3,884 billion kWh of electricity in 2010. Germany used only 538 billion kWh of electricity in 2010.[2]

Countries also produce energy at different rates. This is especially true for the amount of renewable energy that countries produce. Some countries produce more renewable energy than others. Other countries rely more on using fossil fuels. It is important to know which countries are able to produce the most renewable energy and their policies. Knowing their policies will help you determine what policies might work in your state.

Your first task is to find out which countries produced the most renewable energy between 2000-2010. These countries are leaders in using renewable energy in the world. Table 1 shows a list of some countries' renewable energy generation in 2000 and 2010.

Table 1: Renewable Energy Generation of Electrical Power in 2000 and 2010

	Renewable Energy Generation 2000 (measured in billion kWh)	Renewable Energy Generation 2010 (measured in billion kWh)
United States	361	440
Russia	165	168
Germany	41	110
Norway	140	117
Brazil	309	433
Canada	363	366
Saudi Arabia	0	0
World	2872	4177

(1) Based on the data in Table 1, predict which nation has the technologies and policies that best support renewable energy development? Explain why you chose this nation. **Write your answer in 1-2 complete sentences**.

One way to compare which countries supported renewable energy best is to look at absolute and relative change. **Absolute change** is the total amount of change. **Relative change** is change that is relative to another point in time. In our example of renewable energy,

- The **absolute change** in a country's renewable energy generation tells us the total amount of change in a country's renewable energy generation. For example, the absolute change in the world's renewable energy generation between 2000 and 2010 is 1,305 billion kWh.

Absolute change has the same units as the original measurement.

- The **relative change** is the change from earlier renewable energy generation. In other words, we need to compare the change to what a country produced in an earlier time. Relative change is often given as a percentage. For example, the relative change in the world's renewable energy generation between 2000 and 2010 is 45% (1305/2872).

Relative change is the absolute change expressed as a percent of the original amount.

(2) Use the information in Table 1 to help you calculate answers to the following questions about the changes in renewable energy generation in Germany and Brazil.

(a) Calculate the absolute and relative change of Brazil's renewable energy generation from 2000-2010.

(b) Calculate the absolute and relative change of Germany's renewable energy generation from 2000-2010.

(3) In the chart below, predict how much renewable energy Germany will generate in 2020 using absolute and relative change.

- First, make a prediction using absolute change. Put this in the (a) box in the table below.
- Remember, the relative increase in generating renewable energy in Germany is 168% between 2000 and 2010. Assume that it will be the same between 2010 and 2020. Now, predict how many billion kWh Germany will generate by 2020. Put this in the (b) box in the table below.

Germany's Renewable Energy Generation	Actual Value of Renewable Energy Generation in 2000 (Billion kWh)	Actual Value of Renewable Energy Generation in 2010 (Billion kWh)	Predicted Value of Renewable Energy Generation in 2020 (Billion kWh)
Absolute Change: 69 Billion kWh	41	110	(i) ____________
Relative Change: 168%	41	110	(ii) ____________

(b) Now, think about and discuss the calculations you made above. When do you think it's important to use absolute change? When do you think it's important to use relative change?

(4) In Table 1, notice that Russia and Canada both increased their renewable energy generation by 3 billion kWh.

(a) Does 3 billion, in this statement, represent absolute change or relative change?

(b) Compare Russia's relative change to Canada's relative change from 2000 to 2010. Which country do you think has better renewable energy generation policies? Why?

Table 2 shows the percentage of each country's total energy generation that is renewable energy.

Table 2: Percent of Total Energy Generation that comes from Renewable Energy

	Percent of Total Energy Generation that is Renewable Energy in 2000	Percent of Total Energy Generation that is Renewable Energy in 2010
United States	9.5 %	11.7 %
Russia	19.9 %	17.1 %
Germany	7.6 %	18.7 %
Norway	99.3 %	96.7 %
Brazil	90.4 %	85.4 %
Canada	61.7 %	62.3 %
Saudi Arabia	0 %	0 %
World	19.6 %	20.6 %

(5) The table shows that in 2010, 20.6% of the world's energy generation was from renewable energy generation. **Using 1-2 complete sentences**, describe what this means.

(7) Using the information in Table 2, and using absolute and relative change, compare Norway to the other countries in terms of renewable energy generation. Would you be interested in researching Norway's renewable energy policies and technologies? Explain your answer in 1-2 complete sentences.

(7) (a) Brazil showed a decrease in percent of energy generation from renewable sources between 2000 and 2010. Calculate the relative change for Brazil. Round your answer to two decimal places.

(b) How is the answer to Question 7(a) possible given the data discussed in Table 1??

FURTHER APPLICATIONS

The table below shows total solar energy generation for several countries in 2000 and 2010.

Table 3: Total Solar Energy Generation by Country

	Total Solar Energy Generation in 2000 (measured in billion kWh)	Total Solar Energy Generation in 2010 (measured in billion kWh)
United States	0.49	1.21
Germany	0.06	11.68
Norway	0.01	0.02
Japan	0.35	3.80

(8) Which of the above countries most actively pursued a policy of developing solar energy? Explain.

(9) Compare Germany's relative change in total solar energy generation over the decade to Japan's, using Japan's as a reference value.

(10) Suppose, as a U.S. politician, you wish to use the example of Germany to set goals for solar energy development. Would the absolute or relative change in Germany's total solar energy generation be a more realistic goal?

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[1] http://www.ucsusa.org/sites/default/files/legacy/assets/documents/clean_energy/renewablesready_fullreport.pdf

[2] http://www.eia.gov/cfapps/ipdbproject/iedindex3.cfm?tid=2&pid=2&aid=2&cid=regions&syid=2000&eyid=2010&unit=BKWH

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8.4.1: Preparation 2.3-W Renewable Energy

(1) Calculate the following values.

(a) What percent of 20 is 2?

(b) 28 is what percent of 80?

(c) What percent is 18/24?

(2) A school principal of 810 students needs to determine what percent of students passed and did not pass a statewide examination. Round to the nearest percent.

(a) If 550 students passed the exam, what percent passed the test?

(b) What percent did not pass the test?

(3) A laptop computer that you want to purchase was originally priced at $1,225. You will receive a 20% student discount, and the sales tax rate is 8%. How much money will you pay for the laptop?

In Collaboration 2.3C, we will look at an important issue in environmental science: energy use and alternative energy sources. The video, “The Future of Energy?”, from Cambridge University, provides a brief overview of the dilemma we are facing with energy and alternative sources of energy. Watching this will help you understand some of the concepts in the problem situation. It will introduce different sources of energy, their benefits and drawbacks, and how energy is produced.

Please watch this video here before Collaboration 2.3C: https://carnegiemathpathways.org/go/pbsenergy

After Preparation 2.3 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 2.3 you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can...	Rating from 1 to 5
find a percent of a number.	

Self-Regulated Learning: Reflect

How much time and effort per week is this course taking? Is it what you expected? Is there anything you need to adjust in your weekly schedule to make sure you are successful?

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8.4.2: Exercise 2.3-W Renewable Energy

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Absolute change is measured as a quantity (for example, an increase of $3). Relative change is measured as a percentage compared to the reference value (for example, an increase of 3%).

(ii) To find relative change, subtract the original number from the new number and divide by the original.

(iii) Consider this situation: Quantity 1 increases by 15%. Quantity 2 increases by 20%. Quantity 2 must have increased by a larger amount than Quantity 1.

(iv) The need for more renewable energy requires that all countries use more solar power.

DEVELOPING SKILLS AND UNDERSTANDING

(2) The following headlines all refer to change. Identify the change as *absolute* or *relative*.

(a) “Enrollments at Northeastern University are expected to increase by 1,500!”

(i) Absolute change

(ii) Relative change

(b) “Another 14% tuition increase is expected.”

(i) Absolute change

(ii) Relative change

(c) “A new proposal has sales tax rates dropping from 3% to 1%, a drop of only 2%.”

(i) Absolute change

(ii) Relative change

(d) “A new proposal has sales tax rates dropping from 3% to 1%, a 67% drop!”

(i) Absolute change

(ii) Relative change

(3) The two questions below refer to data taken from the U.S. Census.[3] The dollar values take into account the changes in the economy over the years (i.e., inflation).

(a) A typical high-income household in 1980 earned $125,556. A similar household in 2009 earned $180,001. What was the relative increase in income for these households from 1980 to 2009? Round to the nearest one percent.

(b) A typical middle-income household in 1980 earned $34,757. A similar household in 2009 earned $38,550. What was the relative increase in income for these households from 1980 to 2009? Round to the nearest one percent.

(4) After the U.S. Congress passed temporary tax cuts in 2010, persons in the U.S. who earned $50,000 per year had a lower tax rate. On average, these persons retained 2% more of their income than before the cuts. The 2010 law also gave the same persons a separate $850 credit on their federal tax return.

(a) How much extra money did a typical person earning $50,000 receive due to the federal tax cuts?

(b) This extra money represents what percent of the typical person’s salary? Round to the nearest tenth of a percent.

(5) Due to the same 2010 law, persons earning $500,000 per year were taxed at a lower rate on the first $106,800 they earned, saving them 2% of this portion of their salary. The same persons also received $14,250 more on their federal tax return. Fill in the blanks to complete the statements below.

(a) A person earning $500,000 a year received $___________ more of their income. Round to the nearest dollar.

(b) A person earning $500,000 a year received ___________% more of their income. Round to the nearest tenth of a percent.

MAKING CONNECTIONS ACROSS THE COURSE

(6) In Collaboration 1.4, you used results from the 2012 Consumer Expenditure Survey on how Americans spend their income. A summary of this information is given in Table 1.[4]

Table 1: Percentages of Average Annual Household Expenditures

Housing	32.83%
Food	12.83%
Transportation	17.49%
Everything Else	36.85%

Which pie chart best represents the data given in Table 1?

(i) Circle graph showing the average annual household expenditures, shown from highest to lowest: - Everything else - Housing - Transportation - Food	(iii) Circle graph showing the average annual household expenditures, shown from highest to lowest: - Housing - Everything else - Transportation - Food
(ii) Circle graph showing the average annual household expenditures, shown from highest to lowest: - Everything else - Housing - Food - Transportation	(iv) Circle graph showing the average annual household expenditures, shown from highest to lowest: - Housing - Everything else - Food - Transportation

(7) Many egg producers keep chickens in small cages that do not allow the chickens to move. Some people believe that this is unhealthy, so they buy eggs from chickens that are not caged 24-hours a day. These are sometimes called “free-range” chickens. The United States Department of Agriculture (USDA) allows chickens to be called free-range as long as the chickens spend some of their time outside. The European Union (EU), however, has several additional restrictions. One of these is that the farmers must provide enough outside area so that if all the chickens were outside, the density of chickens would be no more than 0.25 chickens/sq meter.[5]

(a) How many square meters does the EU require for one chicken?

(b) A farmer in the United States wants to meet the EU guidelines. She measures her area in square yards (1 sq meter = 1.196 sq yards). How many square yards does she need for 1,100 chickens?

(i) 301.1 square yards

(ii) 4,400.0 square yards

(iii) 4,812.2 square yards

(iv) 5,262.4 square yards

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[3] https://www.census.gov/data/tables/time-series/demo/income-poverty/historical-income-households.html

[4] http://www.bls.gov/cex/csxann12.pdf

[5] http://en.Wikipedia.org/wiki/Free_range_eggs

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8.5: Quantway Core 2.4 Workforce (Health Care) - Student Lesson

INTRODUCTION

You saw the graph below in Preparation 2.4:

Line graph showing the average household income from 2001 to 2012. Y-axis represents the average household income, ranging from $50,000 to $56,000. X-axis represents the years, ranging from 2001 to 2012. 2002 = just over $54,000 2003 = just over $54,000 2004 = just under $54,000 2005 = in between $54,000 and $55,000 2006 = just under $55,000 2007 = in between $55,000 and $56,000 2008 = in between $53,000 and $54,000

Based on the graph, discuss the following questions in your group:

- How much did the Average Household Income increase from 2002 to 2007?
- How much did the Average Household Income decrease from 2007 to 2012?

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- the scale on graphs can change the perception of the information they represent.
- to fully understand a pie chart, the reference value must be known.

By the end of this collaboration, you should be able to

- calculate relative change from a line graph.
- estimate the absolute size of the portions of a pie chart given its reference value.
- use data displayed on two graphs to estimate a third quantity.

SPECIFIC LANGUAGE AND LITERACY OBJECTIVES

By the end of this collaboration, you should be able to

- read and comprehend the problem situation.
- read, interpret, and explain the data about trends in obesity in the line graphs.
- complete the Double-Entry Journal using information from the problem situation.
- demonstrate an understanding of mathematics by writing complete and correct responses to questions.
- demonstrate ability to describe, interpret, synthesize, and predict information using the lesson text about diabetes and nutrition.
- use appropriate quantitative and health care vocabulary to discuss mathematics in this lesson.

Annotating Problem Situation 1

For this collaboration, you will use a skill called annotation. Annotation is a reading comprehension tool that helps you pull out the most important information in a text. In this lesson you will use two types of annotation:

- **Double-underline the main ideas** in the problem situation, and
- **Draw a triangle around key words, phrases, or quantitative information** that you think are important in the problem situation.

Use the annotation symbols in the problem situation below. Your instructor will annotate the first sentence or two with you, as a whole class. Then you will be asked to annotate the remainder of the problem situation on your own as you read.

Image showing the annotation symbols, indicating the following: - Double-underline key concepts in the problem situation (shown with double lines under this sentence) - Key words, phrases, or quantitative in formation that you think are important to the problem (shown in a triangle).

PROBLEM SITUATION 1: UNDERSTANDING DIABETES

You are a nurse practitioner and have a new patient named Jane. Jane makes an appointment for a physical examination. You notice in her chart that Jane has gained over 100 pounds (lbs) in the past five years. You run and review some tests on Jane. These tests show that Jane has elevated blood sugar and high blood pressure. She doesn't have diabetes right now, but you are concerned that she may develop the disease. These tests indicate that she is at high risk for developing diabetes. In addition to diabetes, obesity increases risks of having high blood pressure, heart disease, strokes, cancer, and many other health problems.

Medical professionals think about *risk factors* that might put patients in danger of having diabetes. Risk factors for diabetes include age, family history of diabetes, high blood pressure, level of physical activity and lifestyle choices, being overweight, and ethnicity.[2]

You are concerned about Jane because obesity significantly increases the chance a person will develop diabetes. 85.2% of people with type 2 diabetes are overweight. Obesity is increasing in the United States. Approximately 33% of adults and 17% of teens and children in the United States are overweight or obese.[3] Figures 1 and 2 below show trends of adult obesity in the United States.

You suggest that Jane should try to lose weight to lower her risk of developing diabetes. Your task is to help Jane understand the link between weight and the high risk of developing diabetes, and to help her make better food and exercise choices to promote weight loss.

Figure 1: Line Graph Depicting Trends of Obesity in the United States

Line graph showing the trends in the prevalence of being overweight among adults aged 20 and over, by gender: United States 1999-2010 (Charts are based on data from the Centers for Disease Control and Prevention: http://www.cdc.gov/nchs/data/hestat/obese/obese99.htm). Y-axis represents the percentage, ranging from 0 to 50. X-axis represents the years, ranging from 1999-2000 to 2009-2010. Men data line is shown starting in between 20 and 30 in 1999-2000, slowly increasing to in between 30 and 40 in 2009-2010. Women data line is shown starting in between 30 and 40 in 1999-2000, remaining there until 2009-2010.

Figure 2: Line Graph Depicting Trends of Obesity in the United States[4]

Line graph showing the trends in the prevalence of being overweight among adults aged 20 and over, by gender: United States 1999-2010 (Charts are based on data from the Centers for Disease Control and Prevention: http://www.cdc.gov/nchs/data/hestat/obese/obese99.htm). Y-axis represents the percentage, ranging from 25 to 41. X-axis represents the years, ranging from 1999-2000 to 2009-2010. Men data line is shown starting in between 27 and 29 in 1999-2000, increasing to in between 35 and 37 in 2009-2010. Women data line is shown starting in between 33 and 35 in 1999-2000, increasing to in between 35 and 37 in 2009-2010.

(1) You've just read about the link between weight and diabetes. Using Figure 1 and Figure 2, answer the questions below.

(a) Compare the two line graphs about the prevalence of obesity in the United States. To make your comparison, describe how the two graphs are similar and different. What do you notice?

(b) Using the graphs, calculate the relative change in:

(i) the percentage of men who were obese from 1999 to 2010. Round to the nearest whole percent.

(ii) the percentage of women who were obese during the same time period. Round to the nearest whole percent.

(c) Based on your calculations, would you conclude that, in the future, men are more likely to be at risk for developing diabetes than women? **Provide support for your answer by connecting quantitative information from the line graphs to the problem situation**. Write your answer in 2-3 sentences.

Double-Entry Journal for Problem Situation 2

For the next problem situation, you can use a Double-Entry Journal (DEJ) to help you better understand the details. The DEJ is designed to give you a better understanding of the reading. As you read through Problem Situation 2: Nutrition and Weight Loss, complete the Double-Entry Journal using the steps below.

- In the Left Column, list the key ideas or issues in the problem situation.
- In the Right Column, write down the key quantitative information in the problem situation.

Key Ideas or Issues in the Problem Situation	Key Quantitative Information In the Problem Situation
1.	1.

PROBLEM SITUATION 2: NUTRITION AND WEIGHT LOSS

As a medical professional, you understand that losing weight can be difficult. You help Jane develop a plan to lose weight. You explain to her that she must monitor her calorie intake. Also, Jane needs to balance the amount of fat, carbohydrates, and proteins in her diet.

For patients trying to lose or maintain weight, doctors recommend that fat should not be more than 35% of a person's total calorie intake. One way to monitor fat intake is to choose foods with less than 35% of calories from fat. You explain to Jane that 1 gram of fat has 9 calories; 1 gram of protein has 4 calories; and, 1 gram of carbohydrates has 4 calories. Fat contributes the most number of calories per gram. You suggest to Jane that she should limit the amount of fat she eats. Monitoring her fat intake will also help Jane limit the number of calories she consumes.[5] The problem is that Jane must lose weight. What information does Jane need to make better food choices?

(2) Jane follows a 2000-calorie diet per day.

(a) Use your **estimation** skills to find Jane's recommended calorie intake from fat per day.

(b) **Calculate** the number of grams of fat Jane can consume per day if she follows her diet.

Circle graph showing the Calories that come from Carbohydrates, Protein and Fat in macaroni and cheese. Fat = 108 Carbs = 124 Protein = 20	Circle graph showing the grams of Carbohydrates, Protein and Fat in macaroni and cheese. Fat = 12 Carbs = 31 Protein = 5

(3) Use the two pie charts above (Figures 3 and 4) to help Jane better understand how to make better food decisions to aid in her weight loss plan. Explain to Jane how the two pie charts look different. Write this explanation in **1-2 complete sentences. It is important to write complete sentences because it helps your instructor better understand your mathematical thinking.**

(4) (a) According to the pie graphs shown above, does the percent of calories from fat in macaroni and cheese exceed the recommended amount of 35%?

(b) Based on your answer above, can Jane eat one serving of macaroni and cheese per day and still satisfy the overall daily guidelines for fat content? Explain why or why not?

(c) How many servings of macaroni and cheese could Jane eat per day and still stay within the recommended guidelines for the amount of dietary fat she should consume?

PROBLEM SITUATION 3: INTERPRETING BAR GRAPHS—MAKING HEALTHY FOOD CHOICES

Your task is to help Jane understand how to make healthier food choices. Jane must understand why monitoring fat intake is important when trying to lose weight. Each gram of fat contains more calories than each gram of carbohydrates or protein. Therefore, eating a lot of fat can increase calorie intake significantly.

The following two bar graphs below represent four food products and the fat content in each product. Help Jane understand which products would best support weight loss.

Figure 5: Bar Graph of Fat Grams in Food Products[6]	**Figure 6: Bar Graph of Percent of Calories from Fat in Food Products**[7]
Vertical bar graph showing the Fat Grams in Food Products. Y-axis represents the amount of fat in grams, ranging from 0 to 5. X-axis represents the products. A = 3 B = 3 C = in between 3 and 4 D = 3 E = in between 3 and 4 F = N/A	Vertical bar graph showing the Percent of Calories from Fat in Food Products. Y-axis represents the Percent of Calories from Fat, ranging from 0 to 50. X-axis represents the products. A = in between 15 and 20 B = in between 20 and 25 C = in between 25 and 30 D = 30 E = 45 F = N/A

(5) Think about the statement, "Food items A, B, and D have the same number of grams of fat, but the products have different percentages of fat content."

(a) How can this be true? Write your answer in **1-2 complete sentences**.

(b) Which products would you recommend to someone who is trying to lose weight, and why? Remember, one way to monitor fat intake is to choose foods with less than 35% of calories from fat.

(c) Using both bar graphs (Figures 5 and 6), compare products C and E. How are they similar? How are they different?

(6) Let's imagine a new product is released: Product F. Product F has 3.7 grams of fat and it contains 140 calories.

(a) Calculate the percent of calories in Product F that come from fat. Round to the nearest whole percent.

(b) Would you recommend this product for someone trying to lose weight? Explain.

FURTHER APPLICATIONS

(7) Using the information from the bar graphs in Figures 5 and 6, determine how many calories are in Product D? (Remember: There are 9 calories per 1 gram of fat).

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion. When thinking about what the important ideas were from the discussion, think about today's objectives for the collaboration. Also think about the problem context.

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[2] http://www.diabetes.org/diabetes-basics/?loc=db-slabnav

[3] http://professional.diabetes.org/admin/UserFiles/0%20-%20Sean/FastFacts%20March%202013.pdf

[4] http://www.cdc.gov/nchs/data/hestat/obese/obese99.htm

[5] http://www.mckinley.illinois.edu/handouts/macronutrients.htm

[6] http://nutritiondata.self.com/

[7] http://nutritiondata.self.com/

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8.5.1: Preparation 2.4-W Picturing Health Care Data with Graphics

Data are increasingly presented in a variety of forms intended to interest you and invite you to think about the importance of the data and how they might affect your lives. The following are some of the types of common displays:

- Pie charts
- Scatterplots
- Histograms and bar graphs
- Line graphs
- Tables
- Pictographs

What questions should you ask yourself when you study a visual display of information?

- What is the title of the chart or graph?
- What question is the data supposed to answer? (For example: How many males versus females exercise daily?)
- How are the columns and rows labeled? How are the vertical and horizontal axes labeled?
- Select one number or data point and ask, “What does this mean?”

Use the following chart to help you understand what some basic types of visual displays of information tell you and what questions they usually answer.

This looks like a …	**This visual display is usually used to …**	**For example, it can be used to show …**
pie chart	• show the relationships between different parts compared to a whole.	• how time is used in a 24-hour cycle. • how money is distributed. • how something is divided up.
line graph	• show trends over time. • compare trends of two different items or measurements.	• what is increasing. • what is decreasing. • how the cost of gas has increased in the last 10 years. • which of these foods (milk, steak, cookies, eggs) has risen most rapidly in price compared to the others.
histogram or bar graph	• compare data in different categories. • show changes over time.	• how a population is broken up into different age categories. • how college tuition rates are changing over time.
table	• organize data to make specific values easy to read. • break data up into overlapping categories.	• the inflation rates over a period of years. • how a population is broken into males and females of different age categories.

You will use the following information and your responses to Questions 9 - 11 in your next class. Be sure to take the work to class.

(1) Use Graph 1[1] to answer the following questions. (**Note**: The vertical axis starts at $50,000 instead of $0.)

If you need to review how to read line graphs like this one, you may want to watch this video: https://carnegiemathpathways.org/go/khanline

Graph 1

Line graph showing the average household income from 2001 to 2012. Y-axis represents the average household income, ranging from $50,000 to $56,000. X-axis represents the years, ranging from 2001 to 2012. 2002 = just over $54,000 2003 = just over $54,000 2004 = just under $54,000 2005 = in between $54,000 and $55,000 2006 = just under $55,000 2007 = in between $55,000 and $56,000 2008 = in between $53,000 and $54,000

(a) Select the best phrase to complete this sentence: The numbers on the **horizontal axis** (the line across the bottom of the graph) represent

(i) income from $2,001 to $2,013.

(ii) income from $2,001,000 to $2,013,000.

(iii) years from 2001 to 2013.

(iv) years and income.

(b) Select the best phrase to complete this sentence: The numbers on the **vertical axis** (the line along the left of the graph) represent

(i) years from $50,000 to $56,000.

(ii) income for a household or family from $50,000 to $56,000.

(iii) income for one person from $50,000 to $56,000.

(iv) income in 2012 from $50,000 to $56,000.

(c) A good estimate of the average household income in 2002 is

(i) $54,100.

(ii) $54,500.

(iii) $54,600.

(iv) $55,000.

(d) A good estimate of the average household income in 2012 is

(i) $50,000.

(ii) $50,250.

(iii) $50,750.

(iv) $51,000.

(e) Use the estimates from (c) and (d) to calculate the relative change in the average household income from 2002 to 2012. Round to the nearest one percent. Indicate if the change is an increase or decrease.

(2) Jane's Weight Gain: Two sets of statements are given below. How can both sets be true?

In 2005 and 2010, Jane gained 25 pounds.	In 2005, Jane gained 20% of her body weight. In 2010, Jane gained 10% of her body weight.

(3) Decide if the following statement is true or false based on the two graphs below. Be prepared to discuss your answer in class.

Circle graph showing Projected obesity in the U.S. for 2050. Obese = 42% Non-obese = 58% Circle graph showing obesity in the U.S. for 2005. Obese = 23% Non-obese = 77%

This pair of graphs **cannot** be used to predict that the number of non-Hispanics in the United States is expected to decline between 2010 and 2050. In 1-2 sentences, explain why.

After Preparation 2.4 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 2.4, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
read a line graph.	
read a bar graph.	
read a pie graph.	

calculate relative change.	

Self-Regulated Learning: Reflect

Which problems from this lesson do you feel you understood well? Which ones might you find it beneficial to talk to your teacher or someone else about?

When planning to do this homework assignment, how accurately did you predict how long the assignment was going to take?

Name two strategies you used when solving problems in this assignment.

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[1] http://www.census.gov/hhes/www/income/data/historical/household/index.html

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8.5.2: Exercise 2.4-W Picturing Health Care Data with Graphs

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

Circle graph showing Graph A, indicating the following: Part A1 = Smaller portion Part A2 = Larger portion

Circle graph showing Graph B, indicating the following: Part B1 = Smaller portion Part B2 = Larger portion

(i) It is important to monitor your calorie intake for a healthy diet.

(ii) In Graph A above, you can find the quantity represented by “Part A1” by multiplying the percentage for the section times the total quantity represented by the circle.

(iii) In the graphs above, you know that “Part A2” represents a larger quantity than “Part B2” because the piece of the graph is larger.

(iv) In the graphs above, you cannot compare the quantities represented by the parts because you do not know the reference values.

DEVELOPING SKILLS AND UNDERSTANDING

(2) Use the graph[8] below to answer the following questions. (Note: The median is the middle number of a data set when the numeric values are arranged in order of size.)

Line graph showing the Median New House Cost in the United States annual data from 2002 to 2014. Y-axis represents the prices, ranging from $180,000 to $280,000. X-axis represents the years, ranging from 2002 to 2014. 2003 = in between $180,000 and $200,000 2004 = in between $200,000 and $220,000 2005 = in between $220,000 and $240,000 2006 = in between $240,000 and $260,000 2007 = in between $240,000 and $260,000 2008 = in between $220,000 and $240,000 2009 = in between $200,000 and $220,000 2010 = in between $220,000 and $240,000 2011 = in between $220,000 and $240,000 2012 = in between $240,000 and $260,000 2013 = in between $260,000 and $280,000 2014 = in between $260,000 and $280,000

(a) Estimate the percentage increase in new housing prices from 2003 to 2007. (Choose the best answer.)

(i) The prices increased from around $190,000 to around $245,000, about a 29% increase.

(ii) The prices increased from around $190,000 to around $220,000, about a 16% increase.

(iii) The prices increased from around $190,000 to around $245,000, about a 22% increase.

(iv) The prices increased from around $190,000 to around $220,000, about a 14% increase.

(b) Estimate the percentage increase in new housing prices from 2011 to 2013. (Fill in the blanks.)

The prices increased from ___________ to ___________. This is about a ________% increase.

(c) Estimate the percentage decrease from 2007 to 2009. (Fill in the blanks.)

The prices decreased from ___________ to __________. This is about a ________% decrease.

(3) South Central Bank has a policy that limits the amount of debt customers may have in order to receive a loan. The following pie chart shows the *highest* percentage of debt that the bank will allow.

Circle graph showing the South Central Bank has a policy that limits the amount of debt customers. Debt = 36% Other Expenses = 64%

(a) What is the reference value in this situation?

(i) Other expenses

(ii) Debt

(iii) Total of other expenses and debt

(iv) Other expenses minus debt

(b) Three graphs are given below. Each graph represents a loan customer. Each customer’s debt is broken into three categories: car, credit card, and mortgage (the loan on a house). Which customer(s) meet the bank policy on the limit of the amount of debt? There may be more than one correct answer.

(i)	(ii)

Circle graph showing the Customer A. Car = 4% Credit Card = 2% Mortgage = 30% Other Expenses = 64%	Circle graph showing the Customer B. Car = 8% Credit Card = 4% Mortgage = 35% Other Expenses = 53%

(iii)

Circle graph showing the Customer C. Car = 5% Credit Card = 5% Mortgage = 10% Other Expenses = 80%

(4) The pie charts below present data about obesity in the U.S.[9] The population of the United States in 2005 was approximately 296 million people. In 2050, the U.S. population is predicted to be 439 million. Use this information to answer the questions below.

Circle graph showing Projected obesity in the U.S. for 2050. Obese = 42% Non-obese = 58%	Circle graph showing obesity in the U.S. for 2005. Obese = 23% Non-obese = 77%

(a) Calculate the number of non-obese people in the U.S. in 2005. Round your answer to the nearest million.

(b) Calculate the projected number of the non-obese people in the U.S. 2050. Round your answer to the nearest million.

(c) What is the change in the number of non-obese people in the U.S. from 2005 to 2050 in relative terms?

(d) What is the change in the number of non-obese people in the U.S. from 2005 to 2050 in absolute terms?

MAKING CONNECTIONS ACROSS THE COURSE

(5) From 2000 to 2006, a total of 19,076 teens (ages 15–19) were killed in car crashes in the United States. The number of teen males who were killed was 12,479 and the number of teen females who were killed was 6,597.[10]

A reporter used this information to write, “About 34%, or one out of every three girls will be killed in a car crash.” Which of the following statements is the best critique of this statement?

(i) The statement is correct because 6,597 is 34% of 19,076, which is very close to one-third. The ratio of 1 out of 3 is a good estimate of the percentage.

(ii) The statement is incorrect because 6,597 is 34% of 19,076, which is very close to three-tenths. The ratio of 1 out of 3 is not a good estimate of the percentage.

(iii) The statement is incorrect because the reference value is the number of teens killed, not all teen girls. It would be correct to say that 34% of teens killed in car crashes are female.

(iv) The statement is incorrect because 6,597 is about 50% of 12,479. The ratio of 1 out of 3 is not a good estimate of the percentage.

(6) The following graphs show how much two families spend on tickets to professional baseball games each year.

Vertical bar graph showing how much two families spend on tickets to professional baseball games in 2010. Y-axis represents the amount, ranging from $0 to $7,000. X-axis represents each family. Cobb family = $6,000 Wagner family = in between $1,000 and $2,000

Vertical bar graph showing how much two families spend on tickets to professional baseball games in 2010 (percent of income). Y-axis represents the percentage, ranging from 0% to 6%. X-axis represents each family. Cobb family = 3% Wagner family = 5%

Choose the true statement based on these graphs.

(i) The Wagners must have a higher income than the Cobbs. They spend less on baseball tickets, but it is a higher portion of their total income.

(ii) The Cobbs must have a higher income than the Wagners. They spend more on baseball tickets, but it is a smaller portion of their total income.

(iii) The Wagners must have a higher income than the Cobbs. They spend more on baseball tickets, but it is a smaller portion of their total income.

(iv) The Cobbs must have a lower income than the Wagners. They spend more on baseball tickets, but it is a smaller portion of their total income.

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[8] https://www.census.gov/construction/nrs/pdf/uspricemon.pdf

[9] http://news.harvard.edu/gazette/story/2010/11/obesity-rate-will-reach-at-least-42/

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[10] http://www.cdc.gov/MotorVehicleSafety/Teen_Drivers/data.html

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8.6: Quantway Core 2.7 Workforce (IT) - Student Lesson

INTRODUCTION

Annotating the Problem Situation

For this collaboration, you use a skill called annotation. Annotation is a reading comprehension tool that helps you highlight the most important information in a text. In this collaboration, you will use two types of annotation:

- **Double-underline the main ideas** in the problem situation, and
- **Draw a triangle around the key words, phrases, or quantitative information** that you think are important in the problem situation.

Use the annotation symbols in the problem situation below. Your instructor will annotate the first sentence or two with you, as a whole class. Then you will be asked to annotate the rest of the problem situation on your own as you read.

Image showing the annotation symbols, indicating the following: - Double-underline key concepts in the problem situation (shown with double lines under this sentence) - Key words, phrases, or quantitative information that you think are important to the problem (shown in a triangle).

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- each statistic—the mean, median, and mode—is a different summary of numerical data.
- conclusions derived from statistical summaries are subject to error.
- measures of central tendency can be used to make decisions.

By the end of this collaboration, you should be able to

- make good decisions using information about data.
- interpret the mean, median, or mode in terms of the context of the problem.
- match data sets with appropriate statistics.

SPECIFIC LANGUAGE AND LITERACY OBJECTIVES

By the end of this collaboration, you should be able to

- read and comprehend the problem situation.
- complete annotation and summarization with quantitative and information technology information from the problem situation.
- demonstrate an understanding of mathematics by writing complete and correct responses to questions.
- demonstrate ability to describe, analyze, and synthesize information using measures of central tendency to look at patterns of data.
- use appropriate quantitative and information technology vocabulary to discuss mathematics in this collaboration.

PROBLEM SITUATION: HOW DO STUDENTS TEXT?

You are the Director of Information Technology at White Oak Community College (WOCC). Recently, students and faculty have been complaining to you about the online campus platform. It can be difficult to communicate about assignments and school events on the platform.

Most students at WOCC do not have laptops or computers at home. Instead they use smartphones or tablets to do school work. Students also complain that it takes instructors too long to answer messages about homework assignments. Instructors complain that students do not check their messages often enough, or do not receive information about course work.

The college has a group of staff members who talk about technology issues on campus. This group is called the Committee for Virtual Learning (the Committee). The Committee proposes to install a mass text messaging system on campus. You believe that this is a good idea. This system is called EducText. The system would allow students and faculty to text about assignments, course work, and anything school-related for free. It would help instructors provide quick responses to students, and send text messages to the entire class at one time. EducText would allow the school to communicate time-sensitive information, using text messages, to students and staff about urgent safety issues, school closings, and weather alerts.

EducText is a very expensive system. Given the significant costs of implementing EducText, the Committee needs to decide if it is cost-effective. The Committee determines that implementation is only cost-effective if most students send or receive text messages at least 20 times a day. You assume that most students are active texters, but you must conduct research to understand how students at WOCC use text messaging in their daily lives. This will help you determine if EducText is worth the cost. You conduct this research during the 2014–2015 school year.

(1) Please answer, in one to two sentences, the following questions about the problem situation.

(a) What problem are the White Oak Community College faculty and students trying to solve?

(b) EducText would allow students and faculty to text about assignments, course work, and anything school related for free. Why is this important to the faculty and/or students?

(c) Why is it important for the Committee to know whether the students send or receive at least 20 text messages each day?

Now, let's collect data about how often students in this class send or receive text messages.

(2) (a) What is the typical number of texts that students in this class send and receive each day? Discuss with your group the best way to approach this question. Try to find a number. Think about how you got that number.

(b) Based on the results of your class, would it be worth buying EducText at WOCC? Should the college purchase EducText?

Figure 1 below shows national data about the number of texts adults send and receive. These results are organized by age group.

Figure 1: Number of Texts Sent/Received Per Day, by Age Group in the U.S. during 2011[2]

Line graph showing the Number of Texts Sent/Received Per Day, by Age Group in the U.S. during 2011. Y-axis represents the number of texts sent/received per day, ranging from 0 to 120. X-axis represents the age group, ranging from 18-24 to 65 and over. 18-24: Median texts per day = 50; Mean texts per day = 109.5. 25-34: Median texts per day = 20; Mean texts per day = 41.8. 35-44: Median texts per day = 10; Mean texts per day = 25.9. 45-54: Median texts per day = 6; Mean texts per day = 14. 55-64: Median texts per day = 2; Mean texts per day = 9.8. 65 and over: Median texts per day = 2; Mean texts per day = 4.7.

(3) (a) Examine Figure 1. Be sure to read the labels in the figure carefully. Write at least two factually precise statements about the data. That is, write two statements that are based on information that is in the data chart above. Write your answer in 1–2 complete sentences. (It is important to write complete sentences because it helps your instructor better understand your mathematical thinking.) Be sure your answer is precise and complete.

Writing Tip! Writing precise and complete answers in complete sentences can be challenging. Here is an example of a possible answer to this question: *Twenty-five to thirty-four year-old adults in the U.S. send or receive an average of 41.8 texts per day and a median of 20 texts per day.*

(b) Now, use Figure 1 to draw one conclusion about texting and age. Write your answer in 1–2 **complete sentences**.

(c) If the texting data shown in Figure 1 for 18–24 year olds is representative of the texting habits of WOCC's students, would the EducText system be worth the expense?

(4) Why do you think the mean number of texts sent and received is significantly greater than the median? Again, write your answer in 1–2 complete sentences.

(5) Figure 1 shows that, for 45–54 year olds, the mean number of texts is 14 and the median is 6. Construct a sample of six individuals to illustrate what a data set might look like that has a mean of 14 and median of 6. As you are working, think about the process you use to determine this set of numbers.

(6) What will generally be true for a data set with a significantly greater mean than median?

(7) Write an example set of five data points for which the mean is significantly less than the median.

(8) In your research, you decided to learn more about different uses of information technology. You wanted to understand the possible options that might be available for staff and students to communicate at White Oak Community College. You found the following statistics. In each case. decide whether the mean, median, mode is being used. If the statistic is not the mean, median, or mode, select "none." Circle your answer.

i. Approximately half of all Americans between 18 and 24 send and receive at least 50 texts a day.

mean median mode none

ii. On average, American 18–29 year-olds send fifty more texts per day than 30–49 year-olds.

mean median mode none

iii. The best hour of the day to post on Twitter (measured by re-tweets) is at 5 p.m.

mean median mode none

iv. The average amount of time spent per Facebook visit is 20 minutes.

mean median mode none

v. 49% of 18–24-year-olds check Facebook when they wake up.

mean median mode none

vi. Among all social media sites, Facebook has the most users.

mean median mode none

vii. The average Facebook user is 40.5 years old.

mean median mode none

viii. Half of all Facebook users ages 18–29 have at least 300 Facebook friends.

mean median mode none

FURTHER APPLICATIONS

Broadband Speeds

Net neutrality is a complex issue now. It refers to the idea that information should be processed through the Internet in the order received, rather than allowing companies to pay more to allow their traffic to move through the Internet more quickly. This is an issue because the Internet is limited in how much information it can deliver, and how fast. Some people are frustrated by slow download speeds for media, such as Netflix streaming.

Imagine that, in studying this issue, you found some surprising information. You found following statistic:

The median internet usage in the U.S. is less than 2 gigabytes per month per person, while the mean usage is more than 9 gigabytes per month per person.

(9) Does this suggest that people are using approximately equal amounts of the Internet, or are some people using much greater amounts than others? Explain your answer in one or two complete sentences.

(10) Construct a scenario of five internet usage amounts in which the median of the five usage amounts is 2 gigabytes and the mean is 9 gigabytes.

Website Analytics

Your company maintains a website to sell its products. You wish to analyze the activity on your website. You measure the activity in terms of “visits” to your site. You want to report to the president of the company about the activity of the website. You find that on most days, your site received between 8,000 and 12,000 visits. However, there were eight days during the year in which the website was offline due to construction. Also, during the first month, the website had not yet been advertised on Google searches, so the number of visits was very small.

(11) Which will be greater, the mean or the median number of visits per day? What would offer the better representation of how active the website has been over the last year—the mean or the median number of visits per day? Explain your answer in one or two complete sentences.

(4) Construct a scenario of 5 values in which the median is 10,000 visits but the mean is 8,000 visits.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[2] http://www.pewinternet.org/2011/09/19/how-americans-use-text-messaging/

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8.6.1: Preparation 2.7-W How Do Students Text?

(1) The graph below is based on U.S. Census data.[1]

Line graph showing the median and average sales prices of new homes sold in the United States annual data from 1963 to 2014. Y-axis represents the prices, ranging from $0 to $350,000. X-axis represents the years, ranging from 1963 to 2014. Median data line is shown starting in between $0 and $50,000 in 1963, increasing steadily to finish just in between $300,000 and $350,000 in 2014. Average data line is shown starting in between $0 and $50,000 in 1963, increasing steadily to finish just in between $250,000 and $300,000 in 2014.

(a) What was the average or mean price of a new home in 2005?

(b) What was the median price of a new home in 1984?

(c) Which of the following intervals had the largest increase in the median price?

- (i) 1966–1969
- (ii) 1981–1984
- (iii) 1996–1999
- (iv) 2002–2005

(d) In what two-year period was the largest drop in the average price?

- (i) 1990–1992
- (ii) 2002–2004
- (iii) 2007–2009
- (iv) 2009–2011

After Preparation 2.7 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 2.7, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
perform basic operations using quantities such as integers, fractions, or decimals with the aid of technology.	
find the mean, median, and mode of a set of numeric data.	
read a line graph.	

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[1] http://www.census.gov/const/uspriceann.pdf

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8.6.2: Exercise 2.7-W How Do Students Text?

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) The mean number of texts is more than double the median for 18–24 year-olds.

(ii) The mean and median of a data set are always very close together, but the mode might be very different.

(iii) When using averages or measures of central tendency, it is always important to ask questions about what the different types of measurements do and do not tell you about the data set.

(iv) The median is the middle number when a data set is listed in order. If there is an even number of points in the data set, the median is found by finding the mean of the middle two numbers.

DEVELOPING SKILLS AND UNDERSTANDING

Line graph showing the Number of Texts Sent/Received Per Day, by Age Group in the U.S. during 2011. Y-axis represents the number of texts sent/received per day, ranging from 0 to 120. X-axis represents the age group, ranging from 18-24 to 65 and over. 18-24: Median texts per day = 50; Mean texts per day = 109.5. 25-34: Median texts per day = 20; Mean texts per day = 41.8. 35-44: Median texts per day = 10; Mean texts per day = 25.9. 45-54: Median texts per day = 6; Mean texts per day = 14. 55-64: Median texts per day = 2; Mean texts per day = 9.8. 65 and over: Median texts per day = 2; Mean texts per day = 4.7.

(2) According to the graph from the collaboration, the mean number of texts sent and received per day by 45–54 year-olds is 14. Suppose there were eight such individuals with the following numbers of texts sent and received: 0, 0, 2, 4, 6, 8, 9, and 13. What would the ninth person's number of texts need to be in order for the mean of the set to be the same as for this age group overall?

(3) The median number of texts sent and received by a group of people aged 35–44 is 10. Seven individuals from this group sent the following number of texts per day: 0,0,4,8,13,22,44. How many texts would an eighth individual need to send to create a median value of 10 for this group?

(4) Which statistic (mean, median, or mode) is most appropriate in each of the following situations?

(a) Tables in the dining hall are numbered 1 through 12 for students who eat there. The principal calls out a number for the table that will go through the buffet line first. The other tables follow in order of the table numbers. One student is sure the principal calls certain tables more often. She keeps track of which numbers are called over a 21-day period.

(i) Mean

(ii) Median

(iii) Mode

(b) The offensive line of a football team is larger than in previous years. The program will list a statistic to show this fact.

(i) Mean

(ii) Median

(iii) Mode

(c) A reporter is doing a story on the falling prices of homes in a large neighborhood. The reporter wants to demonstrate how the prices have fallen for all homes, not just the most expensive houses.

(i) Mean

(ii) Median

(iii) Mode

(5) Lines at the Department of Motor Vehicles are so long! A supervisor decided to conduct a study on the number of people standing in line. At the beginning of each hour for an entire week, the supervisor counted the number of people in line and recorded the number. At the end of the week, the supervisor made the frequency table below. Note that the first column shows the number of people in line at the beginning of the hour. The second column shows the number of times that length of line occurred in the 40 observations.

Number of People in Line	Frequency
1	1

2	0
3	2
4	2
5	2
6	4
7	4
8	4
9	8
10	4
11	9

(a) How many times did the supervisor observe six people in line?

(b) Which calculation could be used to determine how many people the supervisor observed standing in line all together?

(i) $1 \times 1 + 2 \times 0 + 3 \times 2 + 4 \times 2 + 5 \times 2 + 6 \times 4 + 7 \times 4 + 8 \times 4 + 9 \times 8 + 10 \times 4 + 11 \times 9$

(ii) $1 \text{ x } 2 \text{ x } 3 \text{ x } 4 \text{ x } 5 \text{ x } 6 \text{ x } 7 \text{ x } 8 \text{ x } 9 \text{ x } 10 \text{ x } 11$

(iii) $1 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11$

(iv) $1 + 2 + 2 + 2 + 4 + 4 + 4 + 8 + 4 + 9$

(c) How many observations did the supervisor make?

(d) Determine the mode of the data in the frequency table.

(e) Determine the median of the data in the frequency table.

(f) Determine the mean of the data in the frequency table.

(g) The supervisor made observations again the following week. The table below shows the observations.

Number of People in Line	Frequency
1	3
2	4
3	5
4	4
5	5
6	1
7	2
8	1
9	2
10	6
11	8

The mean for the second week was 6.366, the median was 5, and the mode was 11. The supervisor wanted to make the argument that additional personnel were needed. Which of these arguments is correct?

(i) Most of the time, there are five or more people in line.

(ii) Most of the time, there are five people in line.

(iii) Most of the time, there are six people in line.

(iv) Most of the time, there are 10 or more people in line.

(6) Three descriptions of measures of central tendency are given below. They are labeled “A,” “B,” and “C.” Descriptions of data sets are listed below that. Match each data set with a description of measures of central tendency by writing the letter in the blank. Choices may be used more than once.

A The mean and median are close together.

B The mean is much higher than the median.

C The median is much higher than the mean.

(a) _____ The data set has a large range with a few very high numbers and many small numbers.

(b) _____ The data set has a large range with the numbers evenly spaced.

(c) _____ The data set has a small range with most of the numbers grouped in the middle.

(d) _____ The data set has a large range with a few very low numbers.

(7) If you lived in Canada in 2008, you might have seen the following headline:

Canada Below G7 Average for Productivity!

Here is some information to help you understand this headline.

Productivity is a way to measure the economy of a nation. One way to measure productivity is by Gross Domestic Product (GDP) per worker. You may recall from Lesson 2.4 that GDP is the value of all the goods and services produced in a country.

The **G7** is a coalition of the major industrial democracies in the world. It includes the United States, United Kingdom, France, Germany, Italy, Canada, and Japan.

(a) Which of the following is most likely what the author of the headline wanted the reader to think?

(i) Canada’s economy is weak and is falling behind other countries in the G7.

(ii) Canada’s economy is strong and is leading other countries in the G7.

(iii) Canada’s economy is very similar to other countries in the G7.

(iv) Canada’s economy should not be compared to other countries.

(b) Which of the following can you conclude from the headline?

(i) Canada is less productive than half of the G7 nations.

(ii) There is at least one G7 nation that is more productive than Canada.

(iii) There is at least one G7 nation that is less productive than Canada.

(iv) None of the above.

(c) A graph of the GDP per worker of the G7 nations is shown below.[3] Find the mean of the GDP per worker for the G7 nations. Round to the nearest hundred dollars.

Vertical bar graph showing the GDP per worker of the G7 nations in US dollars for 2008. Y-axis represents the US dollar amount in thousands, ranging from 0.0 to 120.0. X-axis represents the countries. Japan = 68.0 Italy = 71.9 Germany = 72.6 United Kingdom = 74.8 Canada = 76.2 France = 82.8 United States = 97.1

(d) Is the headline correct?

(e) Which of the seven G7 nations have “above average” productivity? There may be more than one correct answer.

(f) Which of the following are correct conclusions based on the data in the graph? There may be more than one correct answer.

(i) Canada is in the top half of the G7 in productivity.

(ii) Canada’s productivity is within \$7,000 of all the G7 nations except for the United States and Japan.

(iii) Canada is far behind the G7 nations in productivity.

(iv) Canada cannot be compared to other G7 nations.

MAKING CONNECTIONS ACROSS THE COURSE

In Collaboration 2.8, you will explore themes about the *purchasing power of money*, the role of the *Urban Consumer Price Index*, and the concept of index numbers.

The Purchasing Power of Money

The **purchasing power of money** is the value of a currency, such as the U.S. dollar expressed in terms of the amount of goods and services that one unit of that currency can buy. The following questions will help you learn about it.

Gasoline costs have varied significantly in recent years. The American Petroleum Institute posted an update on gasoline prices for June 15, 2011.[4]

> **U.S. Pump Price Update—June 15, 2011**
>
> The average U.S. retail price for *all grades* of gasoline fell this week for the fifth week in a row by 6.6 cents from the prior week to \$3.767 per gallon, according to the Energy Information Administration (EIA). This was at the highest level since August 2008 with the exception of the recent highs in the prior two months. Compared with the December 29, 2008 low of \$1.670, the all-grade average was higher by \$2.097 per gallon, or 125.6 percent. The average has been above \$3.50 per gallon since the beginning of March 2011. Nominal prices have been above the year-ago average for 66 weeks—and were up by 101.1 cents or 36.7 percent, from the year-ago average of \$2.756 per gallon.

(8) (a) How much was the average retail price for one gallon of gasoline a week before this article was published?

(i) \$3.518

(ii) \$3.701

(iii) \$3.767

(iv) \$3.833

(b) What is the relative change in the retail price for one gallon of gasoline from December 29, 2008, to June 15, 2011?

(i) 25.6%

(ii) 55.67%

(iii)125.6%

(iv) 225.6%

(c) If the retail price for one gallon of gasoline was \$2.756 a year before, then what is the absolute change in the retail price on June 15, 2011?

(i) \$1.011

(ii) \$1.086

(iii) \$1.670

(iv) \$2.097

(9) A key idea from this lesson is that the purchasing power of a dollar is not constant. For example, the price of gasoline varies greatly, so the amount of gas you can buy with \$1 varies over time. The **Urban Consumer Price Index (CPI-U)** is a tool designed to compare the price of goods and services. Comparisons made with this tool are made in terms of **base-year** dollars. The base year is the first year included in a comparison. You will be using the CPI-U in your next lesson. The following questions will help you learn about it.

Refer to the “Consumer Price Index” page of the Bureau of Labor Statistics website, www.bls.gov/cpi, to answer the following questions. You can also use a search engine like Google to look up “Bureau of Labor Statistics CPI”. **Hint**: If you are on the homepage of the Bureau of Labor Statistics, find “Consumer Price Index” in the subjects tab at the top of the page (under the heading “Inflation & Prices”).

(a) Go to the CPI Overview and read the “Coverage” section. Are user fees for services like water and sewer included in the CPI?

(b) Go to the CPI Overview and read the “Coverage” section. Are income taxes included in the CPI?

Read the “Uses” section. The website states that the CPI affects the income of almost 80 million people because their incomes are tied to changes in the CPI.

(c) Which option is the best estimate for the percentage of these people who receive **Social Security benefits**?

(i) A little less than 48%

(ii) More than 50%

(iii) Around 80%

(iv) 100%

(d) Which option is the best estimate for the fraction of these people who receive **food stamps**?

(i) About a third

(ii) About half

(iii) About three-fourths

(iv) 100%

(g) State one use for the CPI-U, based on information from the website.

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[3] http://www.bls.gov/fls/flsgdp.pdf

[4] http://www.api.org/aboutoilgas/gasoline/upload/PumpPriceUpdate.pdf

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8.7: Quantway Core 3.1 Workforce (Health Care) - Student Lesson

INTRODUCTION

In 1998, “NASA lost its $125-million Mars Climate Orbiter because spacecraft engineers failed to convert from English to metric measurements when exchanging vital data before the craft was launched. A navigation team at the Jet Propulsion Laboratory used the metric system of millimeters and meters in its calculations, while Lockheed Martin Astronautics in Denver, which designed and built the spacecraft, provided crucial acceleration data in the English system of inches, feet, and pounds. As a result, JPL engineers mistook acceleration readings measured in English units of pound-seconds for a metric measure of force called newton-seconds.”

"People make mistakes all the time," said Carl Pilcher, the agency's science director for solar system exploration. "I think the problem was that our systems designed to recognize and correct human error failed us.”

Even though this was an expensive mistake ($125 million!), the mistake allowed the engineers to recognize that their system for catching errors (like conversion mistakes) was not good enough and had to be improved. Not all mistakes cost $125 million, but mistakes often provide an opportunity to make something better!

In this collaboration, we will be discussing how to convert units both between and within the metric and the US measurement systems. The procedure is called dimensional analysis, or the Unit Factor Method. Dimensional analysis is commonly used by healthcare professionals, computer scientists, or NASA engineers, just to mention a few professions.

Quick Check-in:

Please discuss the following questions with your group:

- What specific concept from Unit 2.9 do you feel fairly confident about?
- Are there any specific concepts from Unit 2.9 that you find challenging?

Completing the CaS Chart

You may want to use the Comprehension and Synthesis (CaS) Chart in this collaboration. You first saw the CaS Chart when thinking about selecting credit cards to purchase a laptop. Remember, using the CaS Chart will help you have a deeper understanding of the problem situation. CaS Charts will help you understand the main issue(s) that need(s) to be resolved and will help you to recognize the quantitative information that is available in the problem situation that can help you to solve the problem. You will use CaS Charts in some of the Quantway lessons to “unpack” problem situations and support problem-solving. First, read through the steps for completing the CaS Chart below. Then, as you read the Problem Situation: Acetaminophen Confusion, complete the CaS Chart. You may wish to return to these steps as you complete the CaS Chart.

Read through the problem situation, **Problem Situation: Acetaminophen Confusion**. As you are reading:

- Complete Column A. Hint: What issues in the problem situation do you need to understand in order to solve the problem? Is there contextual information that you need to know in order to understand the problem situation?
- Complete Column B. Hint: What quantitative information is provided in the problem situation that will help you solve the problem? Hint: Quantitative information is often a number, but also could be a number word, like “two”.
- Complete Column C. Hint: It is not necessary to solve the problem or use calculations right now. In this column, brainstorm ways you might address the issues presented in the problem situation (Column A) using the quantitative information in Column B. There are no wrong answers.

Column A	Column B	Column C
What is/are the main issue(s) in this problem situation?	**What is the key quantitative information you need to solve the issue(s) in the problem situation?**	**Describe in writing how the information in Column B will help you address the issue(s) in Column A later in the lesson.**

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- the units found in a solution may be used as a guide to the operations required in the problem—that is, factors are positioned so that the appropriate units cancel.
- units provide meaning to the numbers that result from calculations.

By the end of this collaboration, you should be able to

- write a rate as a fraction.
- use a unit factor to simplify a rate.
- use dimensional analysis to help determine the factors in a series of operations to obtain an equivalent measure.

SPECIFIC LANGUAGE AND LITERACY OBJECTIVES

By the end of this collaboration, you should be able to

- read and comprehend the problem situation and dosage chart.
- complete CaS Chart with quantitative and health care information from the problem situation about acetaminophen overdoses.
- demonstrate an understanding of mathematics by writing complete and correct responses to questions.
- demonstrate the ability to describe, interpret, and synthesize information in lesson text.

PROBLEM SITUATION: ACETAMINOPHEN CONFUSION

Andy and Amanda are two new parents with a six-month-old little girl named Isabella. It is 2 a.m. Isabella just woke up crying. She has a fever of 102 degrees Fahrenheit. It is her first high fever. Andy and Amanda are worried. Amanda calls Isabella's doctor to see if they should bring the baby to the emergency room. The doctor tells her to give Isabella acetaminophen (Tylenol®) to try to lower the fever before bringing the baby to the hospital.

The doctor prescribes a teaspoon of acetaminophen. (Acetaminophen is the active ingredient in Tylenol®.) Andy rushes to his local grocery store to buy medicine for Isabella. While at the store, Andy sees different types of acetaminophen. He is confused (see Figures 1–3[1]).

Isabella's doctor did not specify the type of acetaminophen to use. Andy assumes the package would give the dose. But, the package only says to "consult a doctor" for children under 2 years. Andy notices two different types of acetaminophen for young children: children's or infants' acetaminophen. Andy thinks it might be a good idea to buy the infants' type, because Isabella is a six-month-old infant.

Andy looks closely at the formula on the back of each box. The concentration amount of acetaminophen is different in each formula.

Image showing different types of acetaminophen for children and infants, indicating the following: Active ingredient: Acetaminophen; purpose: pain reliever/fever reducer Image showing infants' acetaminophen drug facts, indicating the following: Active ingredient (in each 1.0 mL): Acetaminophen 80 mg; Purpose: pain reliever/fever reducer Image showing children acetaminophen drug facts, indicating the following: Active ingredient (in each 5 mL): Acetaminophen 160 mg; Purpose: pain reliever/fever reducer

The problem is that Andy is not sure what is best to give her. He does not know what a teaspoon is in milliliters (mL). He is worried he might give her too much or too little medicine.

(1) Which type of acetaminophen should Andy buy for Isabella? Is there any information missing that you need to be able to answer the question? Brainstorm possible answers with your group. Write your answer in **1–2 complete sentences**. (It is important to write complete sentences because it helps your instructor better understand your mathematical thinking.)

Acetaminophen Overdoses

In Preparation 3.1-C, you watched the video "'It Just Goes Silent': Brianna Hutto and Tylenol." The video shows the tragic effects of confusing infants' and children's acetaminophen. Baby Brianna's parents were faced with the same dilemma as Andy and Amanda above. In both cases, doctors were not clear about the type of acetaminophen to give.

In this collaboration, you will learn two ways to figure out dosages for patients. The first way is using a chart. The second way is to use the method called dimensional analysis.

Using a Dosage Chart

Let's practice using a dosage chart for acetaminophen.

Figure 4: Acetaminophen Dosage Chart[2]

Child's Weight	Child's Age	Dose (mg)
6 to 11 lbs	0-3 months	40
12 to 17 lbs	4-11 months	80
18 to 23 lbs	12-23 months	120
24 to 35 lbs	2-3 years old	160
36 to 47 lbs	4-5 years old	240
48 to 59 lbs	6-8 years old	320
60 to 71 lbs	9-10 years old	400
72 to 95 lbs	11 years	480
96+ lbs	12+	640

(2) Answer the questions below. Show your calculations.

(a) What is the concentration of the infants' formula? What is the concentration of the children's formula?

(i) Infants' formula:

(ii) Children's formula:

(b) How many mL of **infants' acetaminophen** should the doctor prescribe for a 5-year-old patient?

(c) How many mL of **children's acetaminophen** should the doctor prescribe for a 5-year-old patient?

(d) What would happen if too much acetaminophen is given for a particular concentration?

Using Dimensional Analysis

Dimensional analysis is a method of setting up problems that involves converting between different units of measurement. It is also called *unit analysis* or *unit conversion*. Many healthcare professionals—including pharmacists, dieticians, lab technicians, and nurses—use unit analysis. It is also useful for everyday conversions in cooking, finances, and currency exchanges.

Many people can do simple conversions without dimensional analysis. However, they are more likely to make mistakes on more complex problems. In healthcare situations, like calculating dosages of medicine, mistakes can be very serious, even fatal. Doctors and nurses rely on dimensional analysis to check their calculations.

The advantage of using dimensional analysis is that it is a way to check your calculations. It is always important that you develop your own methods to solve problems. But, this is a time when you are encouraged to learn and use a specific method. Once you have learned dimensional analysis, you can decide when to use it and when to use other methods.

Let's return to Andy and Amanda's dilemma.

At the grocery store, Andy calls the doctor but cannot reach him. Instead, Andy speaks to the on-call Advice Nurse.

Imagine you are this Advice Nurse. You want to know how much medicine should be administered to Isabella. Isabella is a 6-month-old child who weighs 15 pounds (lb). The medicine comes in a liquid form. You will measure in teaspoons (tsp). The children's concentration is 160 mg per 5 mL.

Step 1: Start with what you need to know. You need to calculate how many teaspoons (tsp) should be given to a 15 pound (lb) child. The target unit for this calculation is tsp.

$$\rightarrow \text{tsp}$$

Step 2: We start with a rate that has the target unit in the numerator. Since 1 teaspoon equals 5 milliliters (mL) of liquid, we have:

1 tsp over 5 ml goes to tsp

Step 3: We want the mL to cancel out in the denominator, because this is not the unit we are looking for. In children's acetaminophen, every 5 mL of this medicine has 160 mg of the drug. We write it as a rate and multiply by it. Milliliters cancel in this step:

1 tsp over 5 ml times 5 ml over 160 mg goes to tsp

Step 4: Now we need to cancel milligrams (mg). We know Isabelle weighs 15 pounds and the recommended dosage for her weight is 80 mg (see the chart presented in Figure 4 above). We can use this weight to complete our calculation.

1 tsp over 5 ml times 5 ml over 160 mg times 80 mg over 1goes to tsp

The only unit left is tsp, which is our goal. Now we carry out the numerical calculation and obtain $\frac{1}{2}$ teaspoon of children's acetaminophen.

(3) Use dimensional analysis to calculate how many teaspoons (tsp) Andy should give Baby Isabella if he buys *infants' acetaminophen* instead.

(4) In this question you will explore the differences in the concentrations of children's acetaminophen and infants' acetaminophen. We consider little Isabella, who is a 6-month-old child and weighs 15 pounds, as an example.

(a) How many mg of acetaminophen should be given to Isabella?

(b) Using the dosages calculated for Isabella, compare the portion of a teaspoon dosage of infants' acetaminophen to the portion of a teaspoon dosage of children's acetaminophen. Calculate the ratio of children's acetaminophen concentration to the infants' acetaminophen concentration. Write a sentence that interprets this ratio in the given context.

(c) Recall that children's acetaminophen has a concentration of 160 mg per 5 mL and infants' acetaminophen has a concentration of 80 mg per 1 mL. Calculate the ratio of infants' acetaminophen concentration to the children's acetaminophen concentration. Write a sentence that interprets this ratio in the given context.

Write **1–2 complete sentences** to describe and explain the differences between the infants' and children's formulas. In your answer, use the Acetaminophen Overdoses problem situation and the calculations you have made in this lesson to help you discuss the differences between the two formulas.

(5) As parents of an infant, Andy and Amanda must consider what to feed Isabella. For newborns, there are two different types of formula: a "Ready-to-Feed" liquid formula, and a powder formula. Formula is very expensive. Andy and Amanda must calculate the cost for each formula to determine which one fits their budget. They must feed little Isabella every three hours, and at each feeding she consumes 140 mL. Remember, 1 ounce= 30 mL.

(a) The "Ready-to-Feed" liquid formula costs $8 for 32 ounces of formula. What is the cost of using the "Ready-to-Feed" formula for 1 month?

(b) The powdered formula costs $32 for 1 container of powder. This makes 169 ounces of formula when mixed with water. What is the cost of using the powdered formula?

FURTHER APPLICATIONS

(6) You are the on-call Advice Nurse on the night shift. A parent calls from home about his five-year-old son who is suffering from pain. Earlier that day, the son was seen in the emergency room for a broken arm. Although his arm is in a cast, he is in a lot of pain. You contact the on-call pediatric doctor to ask for what advice to give the parents.

The on-call pediatric doctor prescribes 7.5 mL of children's acetaminophen formula every four hours to lower the child's fever. The children's formula has 160 mg of acetaminophen per 5 mL.

The parent only has the infants' formula available at home. You must calculate the amount of infants' formula the parents can give their child.

(a) Determine the amount, in milliliters, of infants' formula that parents should administer.

(b) Provide recommendations to the parents about the use of children's and infants' formulas of acetaminophen. The parents have both an 8-month-old and a 5-year-old son at home. Explain any concerns you have about mixing up acetaminophen formulas (Write 2–3 complete sentences.)

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[1] http://www.fda.gov/ForConsumers/ConsumerUpdates/ucm284563.htm

[2] http://www.knowyourdose.org/sites/default/files/CHPA_Pediatrics_DosingChart_FINAL%5B1%5D.pdf

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8.7.1: Preparation 3.1-W Acetaminophen Overdoses

There was no Exercise for Unit 2.9. However, the content presented here will prepare you for Collaboration 3.1.

Equivalent Fractions

Four circles showing the following: 1st circle: Shown as one whole 2nd circle: Shown as two halves (one half is highlighted) 3rd circle: Shown as four quarters (two quarters are highlighted) 4th circle: Shown as eight eighths (four eighths are highlighted) The same part of an object: 1/2 = 2/4 = 4/8

The same part of an object: 1/2 = 2/4 = 4/8

Two fractions are equivalent if they have the same value or represent the same part of an object.

For example, the figure shows that $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ all represent the same part of a whole. They are equivalent fractions.

Recall that the **denominator** of a fraction represents the number of parts into which the whole has been divided. The numerator represents a count of the number of parts.
So, $\frac{4}{8}$ means that the whole is divided into eight equal parts, and four of these parts are counted.

(1) Write other fractions that are equivalent to $\frac{1}{2}$, other than $\frac{2}{4}$ or $\frac{4}{8}$.

Simplest Form of a Fraction

The fraction $\frac{50}{100}$ is equivalent to $\frac{1}{2}$. Note that you can write

$$\frac{50}{100} = \frac{1 \cdot 50}{2 \cdot 50} = \frac{1}{2} \cdot 1$$

The above calculation shows that both 50 and 100 can be written as a number times 50. You say that 50 and 100 have a “common factor of 50.” Another way to think of this is that the number “1” (written as $\frac{50}{50}$) is embedded in the fraction $\frac{50}{100}$.

The number “1” is a special number in mathematics because if you multiply any number by 1, you get a result that is equivalent to the original.

Simplify and Simplest Form

The word, **simplify** means to rewrite a fraction in an equivalent form with smaller numbers. The **simplest form** means that the fraction is written using the smallest possible whole numbers. In general, answers should always be given in simplest form unless the question specifically calls for a different form. For example:

$$\text{By dividing } \frac{50}{50} \text{ to get 1, you simplify } \frac{50}{100} \text{ to } \frac{1}{2}.$$

Caution! It is common to say that you “cancel” the 50 from the numerator (top number) and denominator (bottom number) and write the fraction in “reduced form.” This language is misleading for two reasons. While numbers can be “canceled” by dividing by 1, it is better to think about the operation (add, subtract, multiply, divide) so you can understand why it works.

Second, the value of the simpler fraction is the same as the original fraction, but the word “reduced” implies that the “reduced fraction” represents a smaller quantity. It can be confusing, so the terminology *simplest form* makes more sense.

(2) Write each fraction in its simplest form.

(a) $\frac{24}{36}$

(b) $\frac{9}{12}$

(c) $\frac{20}{30}$

(d) $\frac{35}{28}$

(3) Compare the two fractions to determine if they are equivalent.

(a) $\frac{3}{6}, \frac{6}{16}$

(b) $\frac{3}{4}, \frac{6}{9}$

(c) $\frac{10}{8}, \frac{5}{4}$

(d) $\frac{10}{24}, \frac{5}{8}$

Multiplying Fractions

The fact that common factors in the denominator and numerator of a number can be divided to make 1 can be used to make multiplying fractions easier. Consider the following multiplication problem.

Image showing two-thirds times seven-eighths.

If you see that there is a common factor of 2 in the numerator and denominator before multiplying, you can divide the common factors first. This makes the multiplication easier because you have smaller numbers to work with and the simplification is complete.

Image showing multiplication procedure for two-thirds times seven-eighths.

This is an important concept when working with ratios with units. You will learn more about this below.

Dividing Fractions

The division of fractions is not the most intuitive of procedures. So, instead of asking you to memorize the division rule, let's start by looking at a few examples.

Suppose you have $48 to spend on going to the movies during a month. How many tickets can you buy in a month? A movie ticket costs $8. One way to think about this is that you want to know how many groups of $8 there are in $48, or 48 ÷ 8.

In the same way, suppose you had $10 to spend on downloading songs for a 1/2 dollar. (For the sake of the mathematics, you are going to express "a half of a dollar" as a fraction instead of as a decimal.) This means you want to know how many 1/2 dollars there are in $10. Your common sense probably tells you that the answer is 20 because every one dollar has two halves. So you multiplied 102. Look at this written as a calculation:

One over two and two over one are called reciprocals.

Division is the same as multiplying by the *reciprocal*. Here are more examples:

Four divided by a half is the same as four times 2 over one.

Perform the calculations indicated in Questions 4–7. Write your answer as a whole number or a fraction in reduced form. **Note**: If your answer is an improper fraction (the numerator is bigger than the denominator), leave it that way. Don't convert it to a decimal or a mixed number. Improper fractions are easier to work with and easier to type into a computer.

(4) $10 \cdot \frac{2}{5} =$

(5) $8 \div \frac{3}{4} =$

(6) $\frac{7}{8} \div \frac{5}{8} =$

(7) $\frac{7}{8} \cdot \frac{8}{5} =$

(8) Jarrod is helping with his daughter's school assembly. Every student will receive a decorative ribbon to wear. The ribbons have to be 2/3 of a foot long. A local store donates 30 feet of ribbon. How many decorative ribbons can Jarrod make?

(9) Lorinda has started to think about saving for retirement. She reads a recommendation that says she should save at least 3/10 of her income because she is over 40 years old. Lorinda makes $42,000 a year. How much should she save in one year according to this recommendation?

Conversion Factors

A fraction that is a ratio of quantities can be equivalent to 1, even when the numerator and denominator are not the same number. However, it is necessary that the numerator and denominator represent equivalent quantities. For example, the following fractions are all forms of 1:

16 ounces over 1 pound. 1 mile over 5280 feet. 60 minutes over 1 hour.

These types of ratios are sometimes called **conversion factors** because they can be used to convert between units.

(10) Complete the following fractions to make a conversion factor. Look up the conversion factor in a reference book or on the Internet if you do not know it.

(a) 1 yard over ? feet.

(b) ? fluid ounces over 1 gallon.

(c) ? inches over 1 yard.

Converting for Speed

A rate, like 35 miles per hour, can be expressed as a fraction: $\frac{35\ miles}{1\ hour}$. You can use the conversion factor of 60 minutes per 1 hour to convert the rate to the units of miles per 1 minute. The example below shows how to set up a multiplication problem with the rate and the conversion factor to convert miles per hour to miles per minute.

35 miles over 1 hour times 1 hour over 60 minutes.

Notice that the conversion factor is written so that the units of “hours” are in the numerator. This is because you want the “hours” to divide out in the same way that common factors divided out in the multiplication problems above. This leaves the units of miles/minute as shown below (on the right).

Multiplication of Fractions	Unit Conversion
$\frac{2}{5} \times \frac{15}{7} \rightarrow \frac{2}{3} \times \frac{5 \times 3}{7} \rightarrow \frac{6}{7}$	35 miles over 1 hour times 1 hour over 60 minutes equals 35 miles over 60 minutes which equals 0.58 miles over 1 minute.

Both $\frac{35\ miles}{hour}$ and $\frac{0.58\ mile}{minute}$ are **unit rates**. Unit rates are ratios that have 1 unit in the denominator. Here are some other examples of unit rates:

- $\frac{35,000\ people}{square\ mile}$
- $\frac{2,500\ calories}{day}$
- $\frac{56,661\ views}{YouTube\ Video}$

You will use this concept in Collaboration 3.1.

This assignment included a large number of vocabulary words that are important. Make sure you fully understand each term listed below:

- Equivalent fraction
- Simplify, simplest form
- Common factor
- Reciprocal
- Unit rate
- Conversion factor

How are Medication Dosages Calculated?

In Collaboration 3.1, you will learn how to calculate medicine dosages. Medication doses are often measured in grams or milligrams, which are very small units of mass. The base unit for this measurement is the kilogram.

- 1 kilogram ≈ 2.2 pounds. Example: If a person’s weight is 132 pounds (lb), this is approximately 60 kilograms (kg).

Medicine can come in capsules, tablets, chewable pills, or liquid form (like a syrup). Imagine a doctor prescribes a patient 300 mg of X medicine a day.

- Capsules: Suppose a medicine X that comes in capsules has 300mg of the active ingredient in each capsule. This information is listed on a label. If the dose is 300mg of X per day, the person would take 1 capsule each day.
- Tablets: The medicine could be in tablets instead of capsules. If the label says that each tablet has 300mg of the active ingredient, the person would take one tablet per day.

- Chewable pills: The medicine could come in chewable pills. If the label says 150mg in each chewable, the person would take 2 of these pills per day to get 300mg.
- Liquid form: The calculations for liquid medicine are different. Liquids can be measured with a unit called the liter. This is a measure of volume. Liquid is measured in liters, not grams (g) or milligrams (mg).

The following conversions will be helpful for you in the next lesson:

- 1 kilogram ≈ 2.2 pounds.
- 1 kilogram (kg) = 1,000 grams (g)
- 1 gram (g) = 1,000 milligrams (mg)
- 1 liter (L) = 1,000 milliliters (mL)
- 1 teaspoon (tsp) ≈ 5mL
- 1 tablespoon (tbsp) = 3 teaspoons

Sometimes medications have recommended dosage by weight for pediatric patients. Children and infants are pediatric patients. Question 11 explores why precisely calculating dosages is important.

(11) In the next lesson, we will look at an important issue in the healthcare field: overdoses of medicine. Often when we think about "overdoses" we think about illegal drugs, like cocaine or heroin. But it is also possible to overdose on medicines like doctor-prescribed drugs, allergy medicines, or acetaminophen. The video at the link below is about one case involving an overdose of acetaminophen (the main ingredient in Tylenol®). It is entitled: "'It Just Goes Silent': Brianna Hutto and Tylenol." Please be aware that this video is highly emotional.

Watch this video: https://carnegiemathpathways.org/go/yttylenol.

Write down any questions or words you do not understand. Then, answer the questions below:

(a) Questions I have or words I did not understand:

(b) Imagine you were the prescribing doctor in this case. What would you have done differently?

After Preparation 3.1 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = confident and 5 = very confident).

Before beginning Collaboration 3.1, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
multiply two fractions.	
divide two fractions.	
understand that a fraction can be simplified by dividing common factors in the numerator and denominator (simplify fractions).	
understand that multiplying by 1 doesn't change a value.	
be familiar with basic units of measure of length (feet, miles) and time (seconds, hours, minutes).	

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8.7.2: Exercise 3.1-C Acetaminophen Overdoses

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Units can be used to set up conversion problems by using the fact that common factors in the numerator and denominator of a fraction divide out to one.

(ii) The confusion about different concentrations of acetaminophen can result in dangerous consequences.

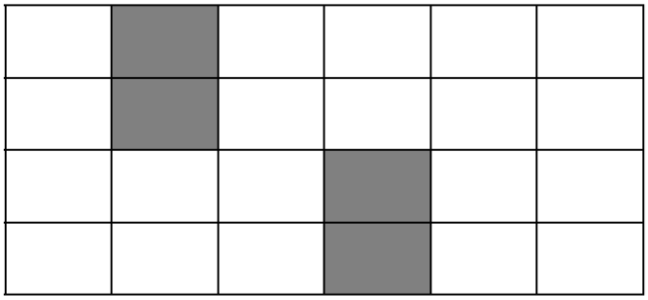
(iii) Units can be used to set up conversion problems by using the fact that common factors in the numerator and denominator of a fraction can be subtracted to equal zero.

(iv) It is possible to convert milligrams per milliliter into grams per liter.

DEVELOPING SKILLS AND UNDERSTANDING

(2) Use Figure 1 for the following questions.

Figure 1



(a) (i) What fraction of Figure 1 is shaded?

(ii) Shade the same fraction of the area in Figures 2 and 3.

Figure 2

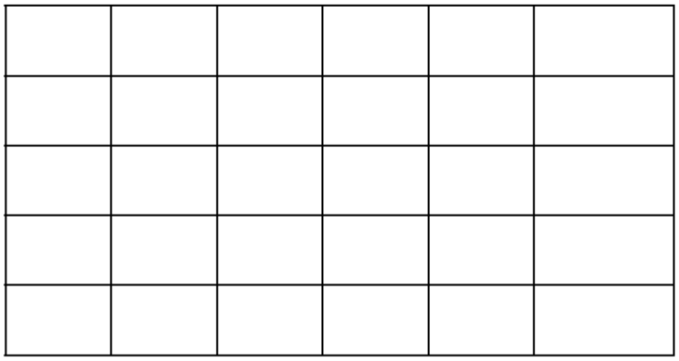
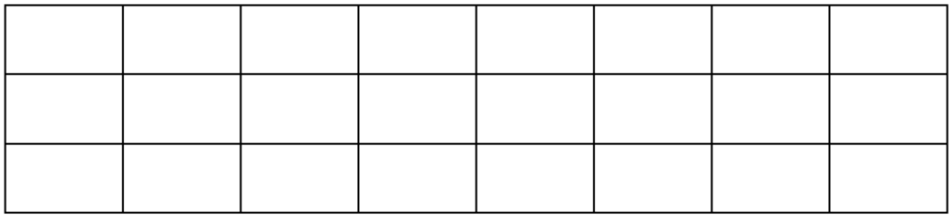


Figure 3



(b) How many boxes did you shade for Figure 2?

(c) How many boxes did you shade for Figure 3?

(3) You are a pediatric medical assistant. You have a patient who is six months old. Your patient has a fever of 102.5 degrees Fahrenheit. The pediatric doctor prescribes a dose of 2.5 mL of acetaminophen solution to help lower fever. The doctor does not specify which formula to use: children’s or infants’ formula.

Use dimensional analysis to determine which formula to use based on the doctor’s prescription.

(a) Children’s concentration is 160mg/5 mL. The doctor prescribes 2.5 mL.

This gives you __________ mg.

(b) Infants’ concentration is 80mg/1mL. The doctor prescribes 2.5 mL.

This gives you __________ mg.

(c) The doctor intends for you to administer the concentration for _______________.

(d) Explain to the patient’s parents what formula to use and why they need to be cautious about the type of formula they give their infant. (Write an explanation of 2–3 sentences.)

(4) You are working as a medical assistant in a clinic. You are in charge of restocking the office supply of acetaminophen. There are 160 mg of acetaminophen in 5 mL. You are ordering 2 L of acetaminophen. How many grams (g) of acetaminophen will you have available to give to patients? Use dimensional analysis to determine this. Remember 1,000 milligrams (mg) = 1 gram.

Other Applications of Dimensional Analysis

Dimensional analysis can be used in many different applications, not just calculating medicine dosages. In the following questions, you will be applying the method you learned in Collaboration 3.1 to other situations. Here are some helpful conversions: 60 seconds = 1 minute, 60 minutes = 1 hour, 1 meter = 3.28 feet, 5,280 feet = 1 mile.

(5) Nurses are often required to calculate dosages. That is, they must check the order that a doctor has given for the administration of a drug and decide whether the dosage is correct. To calculate correctly they must convert between different metric units. For example, 1,000 milligrams (mg) = 1 gram.

(a) Suppose a doctor has ordered a dose of 0.1 g of a medication. The drug comes in a solution concentration of 200 mg per 1 mL. How many milliliters of this solution is required?

(b) Suppose a doctor has ordered a dose of 3 g of a medication. The drug comes in a solution concentration of 300 mg per 1 ml. How many teaspoons of the solution is required?

(6) A 2014 Toyota Prius hybrid vehicle gets 48 mpg for highway driving. The tank holds 11.9 gallons of fuel.[3] Typically the low fuel warning light comes on when approximately two gallons of fuel remain in the tank. Which of the following calculations can be used to find the distance that can be traveled after the fuel light comes on and before the car runs out of gasoline?

(i) $\frac{1}{2\ gallons} \cdot \frac{48\ miles}{1\ gallon} = 24\ miles$

(ii) $\frac{2\ gallons}{1} \cdot \frac{48\ miles}{1\ gallon} = 96\ miles$

(iii) $\frac{11.9\ gallons}{1} \cdot \frac{48\ miles}{1\ gallon} = 571.2\ miles$

(iv) $\frac{2\ gallons}{1} \cdot \frac{1\ gallon}{48\ miles} = \frac{1}{24} miles$

MAKING CONNECTIONS ACROSS THE COURSE

(8) In Collaboration 1.6, you learned about a water footprint. Part of a person’s water footprint is the water used for cleaning. In this question, you will calculate the cost of water for laundry and bathing. You will use the City of New York 2014 rate of $9.27/100 cubic feet of water. Calculate the cost of each of the following based on this rate. Use the conversion factor of 7.48 gallons per cubic foot. [4]

(a) A standard washing machine uses approximately 50 gallons of water per load.[5] A household washes one load of laundry per week for 52 weeks. Find the total cost per year. Round to the nearest dollar per year.

(b) According to one study, the average American shower lasts for 8.2 minutes and uses 17.2 gallons of water.[6] A person showers once a day for a year. Find the total cost per year. Round to the nearest dollar per year.

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[3] http://www.toyota.com/prius/features.html#!/weights_capacities/1223/1225/1227/1229

[4] http://www.nyc.gov/html/dep/html/residents/wateruse.shtml

[5] ibid.

[6] ibid.

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8.8: Quantway Core 3.2 Workforce (Environment) - Student Lesson

INTRODUCTION

Greg owns a rental property in Beaufort, SC. His mortgage, insurance, and HOA fees total up to $1,100 per month. In addition, he's expecting the following maintenance expenses over the next year:

- Furnace replacement: $5,000
- Upgrading solar panels: $2,000
- Landscaping: $1,500

What's the minimum monthly rent Greg should be charging his tenants for the next year, just to break even? As a group, discuss your strategies for solving this problem.

Completing the CaS Chart

You may want to use the Comprehension and Synthesis (CaS) Chart in this collaboration. You may have used a CaS Chart in a previous collaboration. Remember, using the CaS Chart helps you have a deeper understanding of the problem situation. CaS Charts will help you to identify the main issue(s) that need(s) to be resolved and will help you to recognize the quantitative information that is available in the problem situation that can help you to solve the problem. You will use CaS Charts in some of the Quantway collaborations to "unpack" problem situations and support problem-solving.

As you read the problem situation, complete the CaS Chart. You may wish to return to these steps as you complete the CaS Chart. Read through the problem situation, **Problem Situation: Solar Energy in Homes**. As you are reading:

- Complete Column A. Hint: What issues in the problem situation do you need to understand in order to solve the problem? Is there contextual information that you need to know in order to understand the problem situation?
- Complete Column B. Hint: What quantitative information is provided in the problem situation that will help you solve the problem? Hint: Quantitative information is often a number, but also could be a number word, like "two".
- Complete Column C. Hint: It is not necessary to solve the problem or use calculations right now. In this column, brainstorm ways you might address the issues presented in the problem situation (Column A) using the quantitative information in Column B. There are no wrong answers.

Column A	Column B	Column C
What is/are the main issue(s) in this problem situation?	**What is the key quantitative information you need to solve the issue(s) in the problem situation?**	**Describe in writing how the information in Column B will help you address the issue(s) in Column A later in the lesson.**

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- units can be used in dimensional analysis to set up calculations.
- precision should be based on several factors, including the size of the numbers used and the precision of the original values. Rounding can produce large differences in results.

By the end of this collaboration, you should be able to

- solve a complex problem with multiple pieces of information and steps.
- use dimensional analysis.
- investigate how changing certain values affects the result of a calculation.

SPECIFIC LANGUAGE AND LITERACY OBJECTIVES

By the end of this collaboration, you should be able to

- read and comprehend the problem situation about solar energy in homes.
- complete a Comprehension and Synthesis (CaS) Chart with contextual and quantitative information from the problem situation about solar energy in homes.
- demonstrate an understanding of mathematics by writing complete and correct responses to questions.
- demonstrate the ability to interpret, analyze, and synthesize answers about solar energy efficiency in homes.
- use appropriate quantitative and environmental science vocabulary to discuss mathematics in this collaboration.

PROBLEM SITUATION: SOLAR ENERGY IN HOMES

You are a solar energy specialist at QW Solar Now. QW Solar Now is a company that specializes in helping homeowners plan for and install solar panels in their homes. The Pressman family is interested in installing a solar energy system in their New Jersey home. You are at their home to provide an estimate of potential costs and benefits of installing solar panels.

You first provide a brief background to Tali and David Pressman about the environmental benefits of solar energy. You explain that solar panels collect, store, and convert energy from the sun into electricity. Since solar is renewable energy, they do not have to worry about running out of solar energy. Solar energy is also good for the environment. Carbon dioxide (CO2) and other greenhouse gases are not released in the air when using solar energy. Burning fossil fuels such as gas and coal releases CO2 and greenhouse gases. CO2 and greenhouse gases contribute to global warming. The Pressmans are excited about helping the environment. They also care about costs.

You know that installing solar panels can be expensive. But, over time, solar panels save customers money. You want to help the Pressmans understand that solar panels will save them money in the long run. The most common system for homes is called a 5 kilowatt (5 kW) solar energy system. It costs about $20,000 to install. However, eventually the homeowners will save money on electricity bills because collecting and using solar energy is free.

Tali and David could even make a profit from installing solar panels. Sometimes solar panels collect more solar energy than a family needs to power their home. If homeowners collect more solar energy than they need, they can sell the excess energy back to the electric company.

Your main task today is to help the Pressman family learn the financial benefits of installing solar panels. To understand potential benefits for the Pressmans, you need to know how much they pay for electricity and the average amount of electricity they use per month.

You analyze the Pressmans’ electric bill from the previous year. You discover that they are paying 16¢ per kWh of electricity. On average, they use 903 kWh per month.

(1) Think about this problem situation and what you learned in Collaboration 2.3 about alternative energy. Do you think solar energy will be a popular source of energy in homes in the future? Why or why not? Write your answer in 1-2 complete sentences. Don’t forget to use the quantitative information you wrote in Column B of the CaS Chart to support your thinking.

(2) In order for Tali and David Pressman to better understand their own energy use, they must first understand how energy is measured.

First, you explain to the Pressmans that a watt (W) is a unit of power. Power is the rate at which energy is produced or consumed. For example, a 60W light bulb uses energy at a higher rate than a 25W light bulb.

A watt-hour (Wh) is a unit of energy. For example, if you leave a 25W (watt) light bulb on for one hour, it will use 25 Wh (watt-hours of energy). If you leave the same 25W light bulb on for four hours, it will use 100 Wh of energy. Power companies, because they are often measuring large amounts of energy, measure the amount of energy produced or consumed in kilowatt-hours (kWh). 1 kilowatt-hour equals 1,000 watt-hours.

Suppose the Pressmans want to power the six lights in their kitchen for a full 24-hour day. Each light uses a 60W bulb. How many kilowatt-hours of energy would be required?

(3) Approximately how much do the Pressmans pay for electricity annually? (Hint: Use information from the CaS Chart and problem situation. You need to know the cost of electricity and how much the Pressmans use.) Use dimensional analysis to solve this problem. Show your work. Round your answer to the nearest cent.

(4) Tali and David Pressman are considering installing a 5 kW solar energy system for their home. This is the typical solar energy system installed in a single-family residential home. A 5 kW solar energy system generates about 5000 kWh of energy per year. How much would the Pressmans save in electricity costs in one year using a 5 kW solar energy system?

(5) How many 5 kW solar energy systems would the Pressmans need to install to get more energy than they need (an energy surplus)?

(6) New Jersey pays 7.5¢ per kilowatt-hour to buy back surplus electricity from home solar energy systems. Using your answer to Question 4, calculate the total annual revenue the Pressmans would earn from installing enough solar energy systems to generate surplus energy.

(7) A federal tax credit reduces the amount of income tax an individual or family pays the government each year. A federal tax credit of 30% is offered on all home-installed solar energy systems.

(a) If the Pressmans purchased one 5 kW solar energy system, how many years will pass, assuming the cost of energy remains the same, until the money saved on electricity is greater than the initial cost of the solar energy system? Recall from the problem situation, the most common system for homes is called a 5 kilowatt (5 kW) solar energy system. It costs about $20,000 to install.

(b) If the Pressmans purchased three 5 kW solar energy systems, how long until the money saved on electricity combined with the profit from the energy generated is greater than the initial cost of the solar energy system?

(8) Think back to Question 1: "Do you think solar energy will be a popular source of energy in homes in the future? Why or why not?"

Has your opinion about the future of solar energy changed? Why or why not? Discuss in your group. Use the quantitative information from your answers to Questions 3–7 to support your thinking in 1–2 sentences.

FURTHER APPLICATIONS

(9) Write an email message to David and Tali Pressman explaining which you think is the better investment. Make a recommendation to the Pressmans, and explain your reasoning. Be sure to use quantitative information from the previous questions.

Writing Tip!

It can be a challenge to use numbers to back up your statement. However, doing this makes your statement much stronger. When writing this recommendation email, you should be sure to include costs associated with initial investment, the cost of electricity, and profit generation.

(10) Most current solar energy systems are only guaranteed to last for 25 years. How does this affect the benefits of buying a solar energy system?

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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8.8.1: Preparation 3.2-W Solar Energy

Ratios and Unit Rates

Unit rates are ratios with a denominator of “1,” although they are not always written as fractions. For example, 60 mph written as a unit rate is $\frac{60 \ miles}{1 \ hour}$.

The language “miles per hour” implies that the operation is miles divided by hours.

(1) Write each expression as a unit rate in fractional form.

(a) 23 mpg (miles per gallon)

(b) 12 ft/sec (feet per second)

(c) 5 gal/min (gallons per minute)

(d) $7.15/hr (dollars per hour)

(2) Convert each rate into a unit rate.

(a) $\frac{250 \ miles}{12.5 \ gallons}$

(b) $\frac{450 \ miles}{8 \ hours}$

(c) $\frac{\$382}{40 \ hours}$

In the next collaboration, you will learn about solar energy. You will learn about potential costs and benefits of using solar energy in residential homes. Please watch this short video, “Renewable Energy 101: Solar Power” prior to the next class. It will help you better understand how solar panels work.

Video: https://carnegiemathpathways.org/go/ytsolar

After Preparation 3.2 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 3.2, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	Rating from 1 to 5
use dimensional analysis in a contextual problem.	

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8.8.2: Exercise 3.2-W Solar Energy

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Solar energy is the most efficient form of renewable energy.

(ii) A small difference in rounding does not make much difference in results.

(iii) A small difference in rounding can significantly change results when working with large numbers.

(iv) You can calculate the cost of gas per mile by dividing the price of gas by the gas mileage of the car.

DEVELOPING SKILLS AND UNDERSTANDING

(2) In Arizona, there is bright direct sunshine throughout much of the year. A 5 Kw solar energy system can collect 9000 kWh of energy per year if installed on the south-facing roof of a building. However, in Arizona, electricity is eleven cents per kilowatt hour. The average Arizona home uses 66 million BTU per year.

(a) Given that 1 BTU = 0.000293 kWh, find the number of kilowatt hours used by the average home in Arizona in a given year.

(b) How much does a typical Arizonan homeowner spend on electricity per year?

(c) If a typical Arizona household were to install a 5 kW solar energy system, how much would it save on electricity per year?

(d) Given that the 5 kW system would cost $20,000 to install (minus a 30% federal tax credit), how many years would it take to recoup the initial investment?

MAKING CONNECTIONS ACROSS THE COURSE

(3) One way to compare the solar power capabilities between New Jersey and Arizona is by using the Sun Index. California has a Sun Index value of 1, Arizona's index value is 1.18, and New Jersey's index value is 0.81. How much sunlight does Arizona get compared to New Jersey? Write your answer as a percent.

(4) The National Center for Children in Poverty (NCCP) posted the following information in 2011:

Nearly 15 million children in the United States—21% of all children—live in families with incomes below the federal poverty level ($22,050 a year for a family of four).[1]

(a) The graph below can be used to illustrate the statement from the NCCP. Identify what each section and the full circle represent in terms of the context and the percentage. Section 1 is the darker section.

Circle graph showing two sections. Section 1 = 21% Section 2 = 79%

The full circle represents ____and is _______%.

Section 1 represents ____and is _______%.

Section 2 represents ____and is _______%.

(b) Can the statement from the NCCP be restated to say that 21% of people below the federal poverty level in the United States are children? Select the best answer.

(i) Yes, because the reference value in both statements is the children in the United States.

(ii) Yes, because both statements are about children living in poverty so the percentages are the same. The reference values do not matter.

(iii) No, because the percentage of children is really much higher because families really need much more money than the federal poverty level.

(iv) No, because the reference value in the first statement is the total number of children. The reference value in the second statement is the total number of people below the poverty level.

(c) Based on the information in the paragraph, which of the following is the best estimate for the number of children in the United States in 2011?

(i) 30 million

(ii) 75 million

(iii) 100 million

(iv) There is not enough information to make an estimate.

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[1] http://www.nccp.org/topics/childpoverty.html

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8.9: Quantway Core 3.7 Workforce (Health Care) - Student Lesson

INTRODUCTION

Lucas Fuel Treatment is formulated for both gasoline and diesel engines. It gives your fuel system what it really needs – a blend of super slick oils and additives with a high detergent action that allows the engine to operate at maximum efficiency. Also, it cleans and lubricates the carburetor and injectors and causes the fuel to burn more thoroughly for increased power and less fuel consumption.

Image of Lucas Oil website. Key Benefits: A great tune-up in a bottle Cleans and lubricates fuel system Neutralizes low sulfur fuel problems Increases power and miles per gallon by burning excess exhaust emissions Directions: Pour Lucas Oil directly into fuel tank Recommended dosage is 2-3 ounces of fuel treatment per 10 gallons of fuel

In your group, use the information from the image to answer the following two questions:

- What is the recommended dosage of Fuel Treatment per 50 gallons of fuel?
- What is the recommended dosage of Fuel Treatment per 37 gallons of fuel?

Completing the CaS Chart

You can use the Comprehension and Synthesis (CaS) Chart in this collaboration. Remember, using the CaS Chart will help you have a deeper understanding of the problem situation. CaS Charts help you understand the main issue(s) that need(s) to be resolved, and will help you to recognize the quantitative information that is available in the problem situation that can help you to solve the problem. You will use CaS Charts in some of the Quantway lessons to “unpack” problem situations and support problem-solving.

Read through the steps for completing the CaS Chart below. As you read the problem situation, complete the CaS Chart. You may wish to return to these steps as you complete the CaS Chart.

Read through the problem situation below, **Problem Situation: Medical Dosages in Hospitals**. As you are reading:

- Complete Column A. Hint: What issues in the problem situation do you need to understand in order to solve the problem? Is there contextual information that you need to know in order to understand the problem situation?
- Complete Column B. Hint: What quantitative information is provided in the problem situation that will help you solve the problem? Hint: Quantitative information is often a number, but also could be a number word, like “two”.
- Complete Column C. Hint: It is not necessary to solve the problem or use calculations right now. In this column, brainstorm ways you might address the issues presented in the problem situation (Column A) using the quantitative information in Column B. There are no wrong answers.

Column A	Column B	Column C
What is/are the main issue(s) in this problem situation?	**What is the key quantitative information you need to solve the issue(s) in the problem situation?**	**Describe in writing how the information in Column B will help you address the issue(s) in Column A later in the lesson.**

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- proportional relationships are based on a constant ratio.
- rules for solving equations can be applied in unfamiliar situations.

By the end of this collaboration, you should be able to

- set up a proportion based on a contextual situation.
- solve a proportion with algebraic methods.

SPECIFIC LANGUAGE AND LITERACY OBJECTIVES

By the end of this collaboration, you should be able to

- read and comprehend the problem situation about medical dosages.
- complete the Comprehension and Synthesis (CaS) Chart with contextual and quantitative information from the problem situation about medical dosages.
- demonstrate an understanding of mathematics by writing complete and correct responses to questions.
- demonstrate the ability to interpret, predict, analyze, and revise answers about the medical dosages.
- use appropriate quantitative and health care vocabulary to discuss mathematics in this collaboration.

PROBLEM SITUATION 1: MEDICAL DOSAGES IN HOSPITALS

Michael is a night nurse at Central County Hospital. He is responsible for administering medicines to three patients. This is his first night on the job. He wants to be very careful about the medicine he administers, because of a new study.

This study has shown that medical errors in hospitals are the third leading cause of the death in the U.S. As many as 440,000 people in the U.S. die from medical errors each year.[1] If there is a medication error, it is most commonly the result of calculation errors by the nurse or doctor.[2]

Nurses must work with many different types of medicines. Drugs are generally measured according to:

- weight of the drug (for example, grams or milligrams),
- volume of the drug (for example, liters or milliliters), or
- strength of the drug when dissolved in liquid, for example, milligrams per milliliter (also called concentration).

Often the doctor prescribes a medicine using one type of measurement, but the medicine is available to nurses in a different measurement.

Michael’s first patient, Viola, has been diagnosed with pneumonia and prescribed amoxicillin. Aiyana, his second patient, has tonsillitis and was prescribed penicillin. His third patient, Thomas, was prescribed colchicine to treat the gout in his hands.

Below are the prescriptions and availability of medicine for Michael’s first three patients. How can Michael calculate the correct dosage for each patient?

	Patient Name: Viola Shore **Prescription**: 80 mg Amoxicillin		The vial of medicine at Central County Hospital is labeled 100 mg/mL.
	Patient Name: Aiyana Jones **Prescription**: 100,000 units penicillin.		The label for the penicillin on hand is 250,000 units/mL.
	Patient Name: Thomas Grady **Prescription**: 0.6 mg colchicine		The colchicine in the hospital medicine room is labeled 250 mcg/mL.
	* 1 mg= 1,000 mcg		

(1) What is the correct dosage for each patient?

(a) Patient One (Viola):

(b) Patient Two (Aiyana):

(c) Patient Three (Thomas):

(2) These three examples show how proportions are useful in assigning correct dosages to patients. In each case, the problem required knowing the *concentration* of the drug. Why is it necessary to know the concentration of a drug before you can prescribe the drug’s dosage? Explain your reasoning **in two or three complete sentences**.

PROBLEM SITUATION 2: YOLANDA, THE PEDIATRIC NURSE

In the same hospital, one floor above, Yolanda is a pediatric nurse. Pediatric nurses work with infants and children. It is essential that the pediatric nurses calculate the correct dosage of medicine, because children and infants weigh much less than adults. If children or infants are given the wrong dose of medicine, it could result in severe illness or even death. Medications for infants and children are commonly prescribed by body weight to ensure that patients are receiving the exact dose for their size.

Medicines are often prescribed in milligrams (mg) or micrograms (mcg) per kilogram (kg) of body weight. This is different from the U.S. common measurement of weight (i.e., pounds). At Central County hospital, patients are weighed using scales that measure in pounds. So, Yolanda must first convert the patient's weight in pounds to kilograms before calculating patient dosage. Here is the conversion rate for 1 kg:

1 kg = 2.2 lbs

(3) A 20-pound child is given the following prescription: Solu-Medrol 1.5 mg/kg. Solu-Medrol is one drug prescribed as a treatment for leukemia. How much Solu-Medrol should the child receive? (Hint: Remember, you must first convert kg to lbs.)

(4) Anthony is a 5-year-old boy who weighs 40 lbs. He is in the hospital with meningitis. The doctor prescribed 100 mg/kg/day of ceftriaxone administered through an IV once daily. The drug is already pre-diluted in a concentration of 40 mg/mL. How many mL of ceftriaxone should Anthony receive each day?

(5) In this collaboration, you have learned about the importance of proportions in helping nurses make decisions about how much medicine an infant, child or adult patient should receive.

(a) In your own words, first explain why there is often a mismatch between the amount of medicine that doctors prescribe and the amount of medicine that nurses give to patients.

(b) Now explain why knowing how to do dimensional analysis calculations helps nurses give patients the correct dosages of medicine.

FURTHER APPLICATIONS

Nurses are required to determine whether children's prescribed dosages are in the safe range. This "safe range" change ensures that the doctor prescribes the appropriate dosage, and that the nurse makes the correct calculations to administer the correct dose. This requirement is important because administering the wrong dose to an infant or small child could result in long-term damage or death.

(6) Determine whether a dosage of 400 mg of Phenytoin is safe for a 55 lb child. The "safe range" is 15-20 mg/kg.

(7) Sometimes there is a 24-hour safe range for a medicine. In this case, the nurse must also determine how many doses of the medicine it is safe to give a patient within a 24-hour period.

For an infant who weighs 11 lbs, a doctor ordered 18 mg/kg every 6 hours. The maximum daily dose for this medicine is 40 mg/kg/day. Does the doctor's prescription exceed the maximum daily dose? Why or why not?

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[1] http://www.npr.org/blogs/health/2013/09/20/224507654/how-many-die-from-medical-mistakes-in-u-s-hospitals

[2] http://www.nursingtimes.net/how-to-ensure-patient-safety-in-drug-dose-calculation/5050561.article

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8.9.1: Preparation 3.7-W Medical Dosages

(1) Ben has $75 in his savings account. He plans to deposit $35 per week to build his account balance.

(a) Complete the following equation to represent the amount of money (A) Ben will have in his account after any number of weeks. Use x as your input variable.

A =

(b) What does your variable represent in this problem?

(c) Which of the following values could be the value of the variable in this context? There may be more than one correct answer.

(i) -5

(ii) 3

(iii) 4.2

(iv) 18

(d) Ben wants to use his savings to buy a computer for $740. Use your algebraic expression to determine the number of weeks it will take him to save enough money to buy the computer.

Proportionality

Length-to-width and width-to-length ratios

The dimensions of a figure can be written as a ratio. Imagine a rectangle with a length of ten inches and a width of three inches.

You can say that the ratio of the length to width is 10:3 or 10/3. It is also correct to say that the ratio of the width to length is 3:10 or 3/10. In Collaboration 3.7, when we compare multiple figures using their respective ratios, it will be important to be consistent.

Equivalent Fractions (review)

As you recall from Unit 3.1, equivalent fractions (ratios) can be written in many equivalent forms. For example, all of the following fractions simplify to $\frac{10}{3}$ in lowest terms:

$$\frac{10}{3}=\frac{20}{6}=\frac{30}{9}=\frac{100}{30}$$

Proportional Figures

Now, we can define what it means for two figures to be proportional. Consider the following figures and compare them using their length-to-width ratios.

The length-to-width ratio of the figure on the left is $\frac{10}{3}$.

The length-to-width ratio of the figure on the right is $\frac{20}{6}=\frac{10}{3}$ in lowest terms. Note that $\frac{10}{3}$ and $\frac{20}{6}$ are equivalent fractions.

While these figures are not equivalent, they are *proportional* to each other because their dimensions have the same ratio.

(2) Use the figure below to answer the following questions.

Diagram showing the cylinder, indicating the following: Height = 9 feet Width = 5 feet

(a) Write the ratio of the dimensions of the cylinder shown above in the form of diameter to height.

(b) Give the dimensions of a cylinder that would be proportional to the one shown.

Diameter:

Height:

(3) Which of the following fractions has a ratio of 4:3? There may be more than one correct answer.

(i) $\frac{24}{18}$

(ii) $\frac{16}{9}$

(iii) $\frac{9}{12}$

(iv) $\frac{20}{15}$

(v) $\frac{8.8}{6.6}$

After Preparation 3.7 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 3.7, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	**Rating from 1 to 5**
interpret the meaning of ratios, including when written as fractions.	
understand the use of a variable to represent an unknown.	
solve a two-step equation such as 2x + 9 = 13.	

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8.9.2: Exercise 3.7-W Medical Dosages

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Nurses have to be very aware of proportionality and know how to solve proportions.

(ii) The rules for solving equations are the same for all types of equations.

(iii) The rules for solving equations depend on the type of equation.

(iv) To solve for a variable in the denominator of a fraction, multiply both sides of the equation by the variable.

(2) In Module 1, you learned that a statement such as “30% of voters support Candidate A” can be interpreted as 30 out of 100.

(a) How many voters out of 1,000 support Candidate A?

(b) How many voters out of 1,500 support Candidate A?

(c) Is this a proportional relationship? Explain your answer.

DEVELOPING SKILLS AND UNDERSTANDING

(3) The tables below each give the dimensions of four different rectangles. Write in the blank if the four rectangles in each set are “Proportional” or “Not Proportional”.

(a) _____________

	Width	**Length**
Rectangle 1	17	110.5
Rectangle 2	23.4	152.1
Rectangle 3	33	214.5
Rectangle 4	52.2	339.3

(b) _____________

	Width	**Length**
Rectangle 1	7.4	15.5
Rectangle 2	17	40.8
Rectangle 3	23.4	68.5
Rectangle 4	33	72.6

(4) A marine biologist would like to feed some dolphins a mix of fish that consists of 9 parts cod to 4 parts mackerel. List a combination that would be an acceptable mixture of these fish.

(5) Identify the proportions that have the same solution as the one below. There may be one more correct answer.

$$\frac{3}{x}=\frac{24}{56}$$

(i) $\frac{x}{3}=\frac{24}{56}$

(ii) $\frac{3}{24}=\frac{x}{56}$

(iii) $\frac{24}{3}=\frac{56}{x}$

(iv) $\frac{x}{24}=\frac{3}{56}$

(v) $\frac{x}{3} = \frac{56}{24}$

(6) Solve the following proportions (round to one decimal place):

(a) $\frac{x}{5} = \frac{10}{13}$

(b) $\frac{3}{4} = \frac{15}{2x}$

(7) Erica would like to bake an 8-pound roast for a family gathering. The cookbook tells her to bake a 5-pound roast for 135 minutes. Create and solve a proportion that would allow Erica to cook her 8-pound roast.

(8) Cefaclor is a medication used for infections. It is often given in liquid form by mixing the powdered medication with a fluid. A pharmacist is mixing a dosage for a child. The instructions indicate that 125 mg of the medication should be mixed with 5 mL of fluid. If the child only requires a dosage of 100 mg of Cefaclor, how much fluid should the pharmacist use?

MAKING CONNECTIONS ACROSS THE COURSE

(9) A company is making pennants or flags for a sports team. The team wants small versions for fans and large versions that will fly over the stadium. The dimensions of the small version are shown below.

Diagram showing the small version indicating the following: Height = 8 inches Width = 20 inches

(a) The large version needs to be 12.5 feet across the base (the short side of the triangle). How long should it be?

(b) How much material will be used for the large version of the flag? Round up to the nearest tenth of a square foot to ensure the company has enough material.

(10) A staircase is made up of individual steps that should be consistent in height and width. The height of each step is called the *rise*, and the width of the step is called the *run*.

(a) The staircase below is made up of four steps each with a rise of 6.5" and a run of 8.25". Find the height (H) and depth (D) of the entire staircase.

Image of a staircase made up of four steps each with a rise of 6.5" and a run of 8.25".

(b) Builders have to follow guidelines on the rise and run of stairs when building a staircase. One acceptable ratio is a rise of 7-3/4 inches for a run of 9-3/4 inches. If a builder is using this ratio to build a staircase that is 15 feet high, how deep will the staircase need to be (d in the drawing below)? Note that the drawing does not show the correct number of steps. Round to the nearest tenth of a foot.

Image of a staircase that is 15 feet high with a run of d

GRAPHING ON A COORDINATE PLANE

Now, you will once again return to graphing on a coordinate plane. You may have noticed that the two axes split the coordinate plane into four sections. These are called **quadrants** and are numbered using Roman numerals as shown below.

For practical reasons, only a small part of the coordinate plane can be shown, but understand that the axes can go on infinitely in all four directions. The **scale** of the grid tells you which numbers are included in the portion of the plane that is shown. You can change the scale to make graphs with very large or very small numbers. The scale on a single axis must be consistent. In other words, if the distance between the gridlines represents five units on one part of the horizontal axis, then that same distance must always represent five units on that axis. However, the vertical and horizontal axes can have different scales as in the example below. As you have seen with other types of graphs, it is important to pay close attention to the scale.

Image showing a blank graph indicating the following: Top-right corner = Quadrant I Top-left corner = Quadrant II Bottom-left corner = Quadrant III Bottom-right corner = Quadrant IV

Blank coordinate plane graph with both x and y axes ranged from -14 to 14.

(11) Place the following points on the graph above. Label each with its ordered pair.

(a) (–90, 7)

(b) (0, 19)

(c) (63, –16)

(12) Indicate if each statement is true or false.

(a) The point (–7, –5) is in Quadrant II.

i) True ii) False

(b) The point (0, 5) lies on the vertical axis.

i) True ii) False

(c) All the points in Quadrant IV have a positive horizontal coordinate and a negative vertical coordinate.

i) True ii) False

(d) The points (20, 12) and (20, 200) lie on the same horizontal line.

i) True ii) False

Many applications use only positive numbers. In these cases, only Quadrant I of the graph is usually shown because that is the only quadrant that is used. An example of this is given below.

(13) You learned about the *golden ratio* in Exercise 3.5. A rectangle whose dimensions match the golden ratio is called a *golden* rectangle. The graph below shows the widths and lengths of golden rectangles.

Line graph showing the Golden Rectangles. Y-axis represents the length, ranging from 0 to 35. X-axis represents the width, ranging from 0 to 25. 0 = 0 5 = in between 5 and 10 10 = just over 15 15 = just under 25 20 = in between 30 and 35 25 = over 35

(a) Based on the graph, is a rectangle with a width of 17 inches and a length of 30 inches a golden rectangle?

i) Yes ii) No

(b) Use the graph to complete the table of values below. Estimate to the nearest whole number.

Width	Length
5	(i)
8	(ii)
(iii)	20
(iv)	31

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8.10: Quantway Core 4.2 Workforce (IT)- Student Lesson

INTRODUCTION

Quick Check-in:

Please discuss the following questions with your group:

- What specific concept from Unit 4.1 do you feel fairly confident about?
- Are there any concepts from Unit 4.1 that you find challenging?

Completing the CaS Chart

You may want to use the Comprehension and Synthesis (CaS) Chart in this collaboration. You may have used a CaS Chart in a previous collaboration. Remember, using the CaS Chart helps you have a deeper understanding of the problem situation. CaS Charts will help you to identify the main issue(s) that need(s) to be resolved and will help you to recognize the quantitative information that is available in the problem situation that can help you to solve the problem. You will use CaS Charts in some of the Quantway collaborations to “unpack” problem situations and support problem-solving.

As you read the problem situation, complete the CaS Chart. You may wish to return to these steps as you complete the CaS Chart.

Read through the problem situation, **Problem Situation: Analyzing Instagram and Twitter Trends**. As you are reading:

- Complete Column A. Hint: What issues in the problem situation do you need to understand in order to solve the problem? Is there contextual information that you need to know in order to understand the problem situation?
- Complete Column B. Hint: What quantitative information is provided in the problem situation that will help you solve the problem? Hint: Quantitative information is often a number, but also could be a number word, like “two”.
- Complete Column C. Hint: It is not necessary to solve the problem or use calculations right now. In this column, brainstorm ways you might address the issues presented in the problem situation (Column A) using the quantitative information in Column B. There are no wrong answers.

Column A	Column B	Column C
What is/are the main issue(s) in this problem situation?	**What is the key quantitative information you need to solve the issue(s) in the problem situation?**	**Describe in writing how the information in Column B will help you address the issue(s) in Column A later in the lesson.**

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand

- that linear models are appropriate when the situation has a constant rate of increase/decrease or can be approximated by a constant rate.
- that the rate of change (slope) has units in context.
- the difference between a positive slope and a negative slope.
- that the linear models for authentic situations have limitations in using them to make predictions.

By the end of this collaboration, you should be able to

- make a linear model when given data or information in context.
- calculate a slope given data or information in context.
- estimate the value that makes two linear models equivalent.

SPECIFIC LANGUAGE AND LITERACY OBJECTIVES

By the end of this collaboration, you should be able to

- read and comprehend the problem situation.
- complete the CaS Chart with quantitative and IT information from the problem situation.
- demonstrate an understanding of mathematics by writing complete and correct responses to questions.
- demonstrate the ability to describe, analyze, and synthesize information using linear models and slope.
- use appropriate quantitative and information technology vocabulary to discuss mathematics in this collaboration.

PROBLEM SITUATION: ANALYZING INSTAGRAM AND TWITTER TRENDS

Ben recently graduated from college with a degree in Computer Science. He is very interested in working in social media. Social media are websites and apps where users create, share, and participate in an online community. Facebook, Twitter, and Instagram are some examples of social media sites.

Ben wants to work at a popular social media company. But, he is concerned about getting a job at a company that will continue to be successful. For example, Ben knows that Myspace was once the most visited website in the United States, and hired over 1,600 employees. Today, Myspace only employs 150 people.

Ben is particularly interested in working at Twitter or Instagram. He analyzes user statistics to help make his decision about where to apply. Ben wants to use these data to predict which company will be more successful in the future so he can decide which company to join.

Ben decides to analyze the number of Twitter and Instagram followers. Companies report this information or data by Quarter. Businesses often refer to “quarters”, which are three-month periods of time. For example, Quarter 1 (Q1) is the first three months of the year (January, February, March). There are four quarters in the year, which we will refer to as Q1, Q2, Q3, Q4.

The data show the number of Twitter and Instagram users by Quarter. The data indicate that in the first quarter (Q1) of 2012, Twitter had 138 million users. In the first quarter (Q1) of 2014, the number of Twitter users had increased to 255 million. In the first quarter (Q1) of 2012, Instagram had only 27 million users. However, in the first quarter (Q1) of 2014, Instagram had 200 million users.

(1) Without doing any calculations, to which company do you think Ben should apply? Explain your answer. **Write your answer in 1–2 complete sentences**. Be sure to include both numbers and facts from the problem situation to support your answer.

Will Instagram ever catch up to Twitter in terms of its number of users? If so, when would we expect that to happen? Let’s find out!

For this problem, we will assume that the change in membership for both sites is linear. We will build linear models for the membership of both sites. To develop a linear model, you will need to calculate the rate of change. Linear models have a constant rate of change. The rate of change is also referred to as the slope, because it describes how steeply the graph is increasing or decreasing.

The rate of change is the amount of change in one quantity (in this case the number of users) per change in the amount of another quantity (in this case, time). It can be calculated as

$$rate\ of\ change = \frac{absolute\ change\ in\ number\ of\ users}{absolute\ change\ in\ time}$$

Here, we can define two variables:

U = number of users (in millions)

t = number of years

(2) Find the rates of change for both Twitter and Instagram users. Round to the nearest tenth of a million users. Be sure to specify the units of the rates.

Twitter =

Instagram =

(3) Based on the rates of change, predict the number of Twitter and Instagram users in Quarter 1 (Q1) of 2019.

(4) To predict when Instagram will overtake Twitter, create linear models for the membership of these websites. Use the subscripts (shown below) to distinguish between the number of users of Twitter and Instagram:

UT = Number of Twitter users

UI = Number of Instagram users

These variables will represent the output of the models. The input variable is time. However, in mathematical models, time is not represented as it is on a clock or calendar. Instead, time is represented in **elapsed time**. **Elapsed time** is the amount of time that has passed since some starting point. In this case, the starting point will be 2012. Therefore,

t = the number of years since 2012

Using these variables, develop linear models for the number of users of Instagram and Twitter.

(5) Use the models developed above to fill out the following table:

Year	Years Since Q1 2012 (t)	Twitter Users (millions)	Instagram Users (millions)
2012	0		
2014	2		
2015	3		
2016	4		
2017	5		
2018	6		
2019	7		

(6) Plot both sets of data on one graph. **Note**: If completing this problem online, follow the instructions given online to create your graph.

(7) Based on the graph, when would we predict that Instagram will overtake Twitter in membership?

(8) Now, using the models you created for Twitter and Instagram users, find an algebraic solution for the approximate year and quarter (Q1, Q2, Q3, or Q4) when Instagram will surpass (that is, overtake) Twitter in membership.

FURTHER APPLICATIONS

(9) In 2000, China had approximately 18 million internet users. However, by 2012 China had about 580 million internet users. In 2000, the United States had 116 million internet users, and by 2012 that figure had risen to 260 million internet users.

Create linear models to estimate the year in which the number of internet users in China surpassed the number of internet users in the United States.

(a) China

(b) US

(c) In which year will the number of internet users in China surpass the number of internet users in the United States?

(10) The table of values below shows the relationship between the velocity of a car in miles per hour (mph) and the braking distance in feet.

(a) Calculate the slope between each of the two points in the table. The first answer is shown as an example.

Velocity (mph)	Braking Distance (ft)	Slope Between Two Points
5	0.98	
15	8.84	The slope between (5, 0.98) and (15, 8.84) is 0.786.
20	15.72	Find the slope between (15, 8.84) and (20, 15.72).
30	35.37	

50	98.24	

(b) Is this a linear relationship? Explain your answer.

(c) Which of the following is the best explanation for the meaning of the first slope in the table?

- (i) At speeds between 5 and 15 miles per hour, the braking distance increases exactly 0.786 feet for every increase of 1 mile per hour in speed.
- (ii) On average, at speeds between 5 and 15 miles per hour, the braking distance increases 0.786 feet for every increase of 1 mile per hour in speed.
- (iii) At speeds between 5 and 15 miles per hour, the braking distance decreases exactly 0.786 feet for every increase of 1 mile per hour in speed.
- (iv) On average, at speeds between 5 and 15 miles per hour, the braking distance decreases 0.786 feet for every increase of 1 mile per hour in speed.

(d) Use the trend of the data in the table to make predictions about the braking distance for speeds between 50 and 70 mph. Which of the following is a correct statement?

- (i) The braking distance between 50 and 70 mph will increase by about 3.144 feet per mile.
- (ii) The braking distance between 50 and 70 mph will increase by exactly 3.144 feet per mile.
- (iii) The braking distance between 50 and 70 mph will increase by more than 3.144 feet per mile.
- (iv) The braking distance between 50 and 70 mph will increase by less than 3.144 feet per mile.

(e) Explain your answer to Question 2(d) in **1–2 complete sentences**.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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8.10.1: Preparation 4.2-W Trends in Social media

(1) In Exercise 3.4, you used a model to determine the amount owed on a loan of $5,000 with simple interest of 4.25%. The model was A = 5000 + 5000(0.0425t), where A is the amount (total principal plus interest) required to repay a loan, and t is the time, in years.

(a) What is the slope of this model? Include units in your answer. (**Hint**: Think about simplifying the equation or use the equation to find a few values and see how much the amount increases each year.)

(b) In Exercise 3.4, you estimated how long it would take for the loan amount to reach $7,000. Use the model to solve for this answer. Round to the nearest tenth. Be sure to show your work.

(2) Ali wants to save money to buy a new video game. Ali's mother agrees to help him by paying him for each "A" he receives on a test in school. Ali's mother creates a spreadsheet to show Ali how much money he will have based on the number of tests on which he earns an "A." The spreadsheet with the formula is shown below.

Screen capture of data in an excel sheet, indicating the following in cells: Column B formula = 5 plus 1.5 multiplied by A2 A1 = Number of Tests A2 = 0 A3 = 1 A4 = 2 A5 = 3 A6 = 4 A7 = 5 A8 = 6 A9 = 7 A10 = 8 A11 = 9 A12 = 10 B1 = Amount Saved B2 = $5.00 B3 = $6.50 B4 = $8.00 B5 = $9.50 B6 = $11.00 B7 = $12.50 B8 = $14.00 B9 = $15.50 B10 = $17.00 B11 = $18.50 B12 = $20.00

Indicate if each statement is true or false.

(a) Ali did not have any money saved to begin with.

i) True

ii) False

(b) Ali will receive $1.50 for each "A" he earns.

i) True

ii) False

(c) The slope of this model is $1.50/test with a grade of "A."

i) True

ii) False

(d) The vertical intercept of this model is $1.50.

i) True

ii) False

(e) The horizontal intercept of this model is $1.50.

i) True

ii) False

(f) If Ali's grandfather offers to add $0.50 to the amount Ali gets for each test, the slope of the model would increase.

i) True

ii) False

After Preparation 4.2 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 4.2, you should understand the concepts and demonstrate the skills listed below:

Skill or Concept: I can …	**Rating from 1 to 5**
understand a rate of change as a ratio or proportion.	
solve for an unknown variable in a linear relationship.	
create a linear model from an authentic context with units.	

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8.10.2: Exercise 4.2-W Trends in Social Media

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Instagram usership is increasing faster than Twitter usership.

(ii) A slope is a ratio with units that describes how two variables change in relationship to each other.

(iii) A slope tells you how much one variable has changed.

(iv) A slope can be written as a fraction or a decimal.

(2) You have learned about absolute change, relative change, and average rate of change. Identify which type of change best describes each of the following values and explain your answer.

(i) 30%

(ii) $30/year

(iii) $30

DEVELOPING SKILLS AND UNDERSTANDING

(3) The three graphs below model the relationship between time spent in class and time spent studying for three students. Use the graphs to answer the following questions.

Line graph showing the relationship between time spent in class and time spent studying for three students. Y-axis represents the number of hours spent studying, ranging from 0 to 24. X-axis represents the number of hours spent in class, ranging from 0 to 12. 0 = 0 2 = 4 4 = 8 6 = 12 8 = 16 10 = 20

Line graph showing the relationship between time spent in class and time spent studying for three students. Y-axis represents the number of hours spent studying, ranging from 0 to 6. X-axis represents the number of hours spent in class, ranging from 0 to 12. 0 = 0 2 = 1 4 = 2 6 = 3 8 = 4 10 = 5

Line graph showing the relationship between time spent in class and time spent studying for three students. Y-axis represents the number of hours spent studying, ranging from 0 to 12. X-axis represents the number of hours spent in class, ranging from 0 to 12. 0 = 0 2 = 2 4 = 4 6 = 6 8 = 8 10 = 10

(a) Which of the graphs has a slope of 2?

(b) Which graph shows a relationship of one hour studying for each hour in class?

(c) Which graph matches the following table?

Hours in class	1	4	7	9
Hours studying	2	8	14	18

(d) What is the equation for Graph B? Use C for hours in class and s for hours studying.

(e) Which graph would contain the point (20, 10) if it were extended?

(4) A parents' group is writing a grant to support an afterschool program for their children. They want to make the point that government funding for afterschool programs in their school has decreased over the last decade while the cost of offering programs has increased. Their argument is based on the data below.

	2000	**2010**
Government funding for afterschool programs ($/child)	$3,000	$1,800
Cost of services ($/child)	$3,800	$5,100

(a) Which of the following is the best interpretation of the average rate of change in the government funding?

(i) Government funding has changed by an average of $120 each year.

(ii) Government funding has decreased by an average of $120 per child each year.

(iii) Government funding has decreased by an average of $1,200 per child each year.

(iv) Government funding has decreased by an average of $1,800 per child each year.

(b) Write an interpretation of the average rate of change in the cost of services.

(c) Sketch a graph showing linear models of the two sets of data: government funding and cost of services. Be sure to include a title, scales, and labels on the axes. Label each of the models on the graph.

(5) In a study by PricewaterhouseCoopers (PwC), it was found that DVD sales totaled $12.2 billion in 2013.[1] According to PwC, it's expected that DVD sales will be $11.5 billion in 2014 and $8.7 billion in 2018.[2]

The study also projected an increase in sales from streaming videos. It's expected that revenue will double by 2018, from $8.5 billion in 2014 to $17 billion in 2018.

(a) What is the slope, including units, of the model for DVD sales?

(b) Write a statement interpreting the meaning of the slope in the context of the problem.

(c) Write an equation that represents the linear model for DVD sales. Let D = the sales of DVDs in billions of dollars, and t = the number of years after 2014.

(d) What is the slope, including units, of the model for streaming videos?

(e) Write a statement interpreting the meaning of the slope in the context of the problem.

(f) Write an equation that represents the linear model for streaming videos. Let S = the total sales of streaming videos in billions of dollars, and t = the number of years after 2014.

(g) In what year do you expect streaming video sales to exceed DVD sales?

(6) Suppose you work for a shuttle service. You see the following spreadsheet that gives information about what your company will charge for rides of different lengths. The formula shown is for cell B2. Write a statement that tells how the fare is calculated based on the number of miles.

Screen capture of data in an excel sheet, indicating the following in cells: Column B formula = A2 multiplied by 0.25 plus 3 A1 = Number of Miles A2 = 10 A3 = 15 A4 = 20 B1 = Total Cost of Ride B2 = $5.50 B3 = $6.75 B4 = $8.00

(7) Delilah wants to join a gym, so she shops around to find the one with the lowest overall price (she is not sure how long she will be a member). She finds the Harbor Square Athletic Club is running a special, and only charges a $25 initiation fee plus $87 a month to be a member. The local YMCA charges $100 to join, and has a monthly fee of $72.

(a) Find the equations for the linear models for the costs to join the Harbor Square Athletic Club and the YMCA. Use C = cost in dollars and m = months.

(b) Calculate how many months Delilah would have to be a gym member for it to be less expensive to join the YMCA. Then fill in the blank.

The YMCA will be less expensive if she remains a member for more than ________ months.

MAKING CONNECTIONS ACROSS THE COURSE

(8) In Unit 2.3, you used the following data about population changes in the United States. Suppose state planners in Indiana and Michigan think that the average rate of change will continue through 2015.

State	2000 Population	2010 Population
Indiana	6,080,485	6,483,802
Michigan	9,938,444	9,883,640

(a) Write equations to model the populations for each state. Use M for the population of Michigan; I for the population of Indiana and t for time in years after 2000.

(b) Use your equations to find the projected populations for Michigan and Indiana in 2015.

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[1] http://variety.com/2014/digital/news/digital-home-entertainment-to-exceed-physical-by-2016-study-finds-1201207708/

[2] http://www.pwc.com/gx/en/global-entertainment-media-outlook/assets/2014/images/filmed-large3.jpg

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8.11: Quantway Core 4.3 Workforce (Healthcare) - Student Lesson

INTRODUCTION

Completing the CaS Chart

You may want to use the Comprehension and Synthesis (CaS) Chart in this collaboration. You may have used a CaS Chart in a previous collaboration. Remember, using the CaS Chart helps you have a deeper understanding of the problem situation. CaS Charts will help you to identify the main issue(s) that need(s) to be resolved and will help you to recognize the quantitative information that is available in the problem situation that can help you to solve the problem. You will use CaS Charts in some of the Quantway lessons to “unpack” problem situations and support problem-solving.

As you read the problem situation, complete the CaS Chart. You may wish to return to these steps as you complete the CaS Chart. Read through the problem situation, **Problem Situation: Understanding Heart Rate**. As you are reading:

- Complete Column A. Hint: What issues in the problem situation do you need to understand in order to solve the problem? Is there contextual information that you need to know in order to understand the problem situation?
- Complete Column B. Hint: What quantitative information is provided in the problem situation that will help you solve the problem? Hint: Quantitative information is often a number, but also could be a number word, like “two”.
- Complete Column C. Hint: It is not necessary to solve the problem or use calculations right now. In this column, brainstorm ways you might address the issues presented in the problem situation (Column A) using the quantitative information in Column B. There are no wrong answers.

Column A	Column B	Column C
What is/are the main issue(s) in this problem situation?	**What is the key quantitative information you need to solve the issue(s) in the problem situation?**	**Describe in writing how the information in Column B will help you address the issue(s) in Column A later in the lesson.**

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- linear models can approximate nearly linear data.

By the end of this collaboration, you should be able to

- find the equation of a line that estimates nearly linear data by calculating the rate of change and estimating the vertical intercept of the line.
- use approximate linear models to interpolate and extrapolate.

SPECIFIC LANGUAGE AND LITERACY OBJECTIVES

By the end of this collaboration, you should be able to

- read and comprehend the problem situation about heart rate.
- read, interpret, and explain the data in scatterplots.
- complete the CaS Chart with quantitative and health care information about heart rates from problem situation.
- demonstrate an understanding of mathematics by writing complete and correct responses to questions.
- demonstrate the ability to describe, interpret, analyze, and synthesize information in lesson text.
- use appropriate quantitative and health care vocabulary to discuss mathematics in this collaboration.

PROBLEM SITUATION: UNDERSTANDING HEART RATE

You are a nurse working at a physical rehabilitation center. The physical rehabilitation center helps patients recover from injury or serious illness.

Miles is your new patient. He is a 53-year-old man who recently had a minor heart attack. Miles is determined to get healthy so he does not have another heart attack. You have to teach Miles to monitor his heart rate. First, you must explain what heart rate is. Second, you explain why it is important in monitoring his cardiovascular, or heart, health during his rehabilitation.

Heart rate is important in determining a person’s fitness level and health. To monitor heart rate, a patient’s *resting heart rate* and *maximum heart rate* must be calculated. Resting heart rate is the number of times your heart beats in one minute at rest (usually sitting). Most people have a resting heart rate of 60-80 beats per minute (BPM). A stronger heart can pump more blood per beat and has a lower BPM. A weaker heart pumps less blood per beat and has a higher BPM. A higher resting heart rate means that the heart has to work harder to do the same amount of work. This can lead to cardiovascular disease and heart attacks.

Miles’ resting heart rate is usually around 95 BPM. You must help Miles lower his resting heart rate. One way that Miles can lower his resting heart rate is through aerobic exercise, often referred to as “cardio”

It is very important that Miles gets enough exercise to strengthen his heart without exceeding his maximum heart rate.

“Maximum Heart Rate” is the highest heart rate that a person can achieve without causing health problems. This is important to understand in order to determine how much exercise is safe to do. When exercising, individuals should aim to be within the Target Heart Rate Zone (which is 50-85% of the Maximum Heart Rate[1]).

The problem is that Miles must improve his heart health. What information would you need to help Miles better understand heart rate? In this lesson you will learn how a person’s age plays an important role in determining maximum heart rate. Understanding this allows patients like Miles to exercise safely and prevent heart disease.

(1) Why is it important for Miles to know his maximum heart rate? Explain your answer in **1–2 complete sentences**.

The chart below shows a random sample of individuals of different ages and their corresponding heart rates. This chart is called a scatter-plot. A scatter-plot is a common tool that researchers use to help discover relationships between variables.

On the chart, the horizontal axis represents the age of an individual. This is called an *independent variable*. The vertical axis represents the maximum heart rate. This is called a *dependent variable*. In the next few questions, you will investigate how the heart rate depends on the age of a person.

Figure 1: Maximum Heart Rate by Age[2]

Graph showing the maximum heart rate by age. Y-axis represents the maximum heart rates, ranging from 120 to 220. X-axis represents the ages, ranging from 0 to 100. 20 = 190 30 = in between 180 and 190 40 = in between 170 and 180 50 = in between 170 and 180 60 = in between 160 and 170

(2) Use the scatterplot in Figure 1 to answer the following questions.

(a) Describe the relationship between age and maximum heart rate. Try to find at least two characteristics.

(b) Miles is 53 years old. Is it possible to use this scatterplot to estimate what his maximum heart rate should be when exercising? If it is possible, how would we go about doing this?

(3) Find a linear model to describe this data. Let H be the maximum heart rate, and a be the age of a person. Describe the meaning of a slope and a vertical-intercept in the context of this data. Use the points (20,200) and (60,160) to get the rate and vertical intercept.

(4) Using the obtained model:

(a) Determine the maximum heart rate for Miles, who is a 53-year-old man.

(b) Is this answer reasonable? Why?

(c) Using Miles’ maximum heart rate, calculate his target heart rate zone (50–85%).

(d) Determine the maximum heart rate for a 70-year-old person.

(6) Adam, Miles’ neighbor, was advised that he should aim for a target heart rate of 90 BPM when exercising at 50 percent intensity. Using the obtained model, determine how old Adam is.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[1] http://www.heart.org/HEARTORG/GettingHealthy/PhysicalActivity/Target-Heart-Rates_UCM_434341_Article.jsp

[2] http://www.ncbi.nlm.nih.gov/pubmed/17468581

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8.11.1: Preparation 4.3-W Understanding Heart Rate

Your next collaboration will focus on understanding heart rate. “Resting Heart Rate” is the number of times your heart beats in one minute at rest. According to the American heart Association, your resting heart rate should be approximately 60-80 beats per minute (BPM).

(1) Find your heartbeat. Put your index finger and middle finger on either side of your wrist or on your neck. Using a stopwatch, count how many beats occur in 10 seconds and multiply by 6. This is your resting heart rate. Now, do some jumping jacks. Find your heartbeat again. What do you notice?

(2) Calculate the relative change from BPM at rest at BPM after exercise. (Note: this is a percent).

After Preparation 4.3 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 4.3, you should understand the concepts and demonstrate the skills listed below:

Skill or Concept: I can …	**Rating from 1 to 5**
understand the meaning and use of variables.	
solve for an unknown variable in a linear relationship.	
calculate a rate of change (slope) from data.	
make a linear model given linear data in context.	

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8.11.2: Exercise 4.3-W Understanding Heart Rate

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Understanding heart rate is important to prevent heart attacks.

(ii) A linear model has a vertical intercept.

(iii) If a model based on data is created correctly, it will accurately predict values beyond the data.

(iv) Models created from data are based on assumptions. It is important to ask if the assumptions are correct or will remain correct over time.

DEVELOPING SKILLS AND UNDERSTANDING

The flight times and distances from Memphis, Tennessee to various cities are given in the table and graph below. Use this information to answer Questions 3–5.

Time to Fly from Memphis to Various Cities[3]

City	Distance (miles)	Time (minutes)
Atlanta	332	82
Baltimore	787	162
Boston	1139	191
Cincinnati	403	100
Detroit	611	121

Line graph showing the flight times and distances from Memphis, Tennessee to various cities. The y-axis represents the number of minutes, ranging from 0 to 250, and the x-axis represents the number of miles, ranging from 0 to 1,200, as described in the table on the previous page.

(2) Use what you know about the relationship of the equation to the graph to select the model from the list below that is most representative of the data. Do not actually find a linear model to make your choice. In each option, d represents the distance in miles and t represents the time in minutes.

(i) $t = –0.13d + 40$

(ii) $t = 0.13d + 40$

(iii) $t = 0.13d + 110$

(iv) $t = –0.13d + 110$

(3) Using the model, calculate the flight time from Memphis to Philadelphia if the distance between the two cities is 874 miles. Round your answer to the nearest tenth of an hour.

(4) Based on the model you chose, select the response that best tells how long it will take a Delta plane flying at full speed to travel 1 mile.

(i) The plane goes 0.13 miles in one minute so it takes between 7 and 8 minutes to go 1 mile.

(ii) It takes 40 minutes to go 1 mile.

(iii) It takes 0.13 minutes or around 8 seconds to go 1 mile.

(iv) It takes 40.13 minutes to go 1 mile.

WORLD RECORD SPEEDS OVER TIME

The table below shows the world record times in the men’s 100-meter dash from 1912 to 2009. Use this to answer Questions 5–8.

Men’s 100-meter dash[4]

Year	World Record Time (nearest tenth of a second)
1912	10.6
1921	10.4
1930	10.3
1936	10.2
1956	10.1
1960	10
1968	9.9
1999	9.8
2008	9.7
2009	9.6

(5) (a) Create a scatterplot using the data relating “years since 1900” and “World Record Time” for the Men’s 100-meter dash.

(b) Write an equation for a linear model representing the data. Define the variables you use in your equation.

(6) Use the model you created to predict in what year the world record will fall below 9.0 seconds.

(7) Identify which of the following statements are correct based on your model. Note that the values given are approximations. There may be more than one correct answer.

(i) On average, the men’s world record in the 100-meter dash has decreased about one-hundredth of a minute each year.

(ii) On average, the men’s world record in the 100-meter dash has decreased about one-tenth of a minute every 10 years.

(iii) On average, the men’s world record in the 100-meter dash has decreased about one-hundredth of a second each year.

(iv) On average, the men’s world record in the 100-meter dash has decreased about one-tenth of a second every 10 years.

(8) Models often only work well for a limited range of input values. Outside that range, the model is said to break down. Which of these explains why your world record model might break down? There may be more than one correct answer.

(i) The data do not form a perfect line.

(ii) Eventually the model will predict negative times for the race.

(iii) Changes in equipment or training might make for sudden improvements in times and change the trend in the current data.

(iv) The times are not really accurate.

(9) The following chart gives sunrise times for New York City in hours after midnight as measured in Eastern Standard Time (EST) on the fifteenth day of each month. The first month is August 2010, the second month is September 2010, and so on. Would it be appropriate to use a linear model to represent these data? Explain.

Line graph showing the sunrise times for New York City in hours after midnight as measured in Eastern Standard Time (EST) on the fifteenth day of each month. Y-axis represents the hours after midnight, ranging from 0 to 9. X-axis represents the months. Aug = 6 Sept = in between 6 and 7 Oct = in between 7 and 8 Nov = in between 7 and 8 Dec = in between 8 and 9 Jan = in between 8 and 9 Feb = in between 7 and 8 Mar = in between 7 and 8 Apr = in between 6 and 7 May = in between 5 and 6 Jun = in between 5 and 6 Jul = in between 5 and 6

MAKING CONNECTIONS ACROSS THE COURSE

In baseball, a team must score more runs than its opponent to win the game. Players have to reach base for a team to score runs. How often a player reaches base is measured by the *on-base percentage*. However, a player who reaches a base may or may not score. The scatterplot gives data relating the number of runs and the on-base percentage. Use the graph to answer Questions 10 and 11.

Line graph showing the relationship between the number of runs and the on-base percentage. Y-axis represents the number of runs, ranging from 0 to 1,000. X-axis represents the on-base percentage, ranging from 29% to 36% 30% = in between 500 to 600 runs (1 dot) 31% = in between 600 to 700 runs (1 dot) 31% = in between 700 to 800 runs (1 dot) 32% = in between 600 to 700 runs (3 dots) 33% = in between 600 to 700 runs (1 dot) 33% = in between 700 to 800 runs (2 dots) 34% = in between 700 to 800 runs (2 dots) 34% = in between 800 to 900 runs (1 dot) 35% = in between 800 to 900 runs (1 dot)

(11) Katy used the graph to create a linear model that she could use to predict the number of runs scored using the on-base percentage. Her model was $r = 30b + 450$, where r = number of runs and b = on-base percentage (i.e., for 32%, $b = 32$). What is wrong with Katy’s model?

(i) The data are not nearly linear so a linear model should not be used.

(ii) Katy put the rate and the vertical intercepts in the wrong places in the equation.

(iii) Katy used the vertical intercept in her equation where she should have used the horizontal intercept.

(iv) The vertical intercept of 450 runs is incorrect because the scale on the horizontal axis does not start at zero.

(12) Based on the slope in Katy’s model, if on-base percentage for a team goes up by 1% then by about how much does the number of runs scored go up?

(i) 480 runs

(ii) 450 runs

(iii) 30 runs

(iv) 0.3 runs

(12) The graph below shows the cost of basic cable service from 2002 to 2010.[5]

Vertical bar graph showing the cost of basic cable service from 2002 to 2010. Y-axis represents the cost in dollars, ranging from 0 to 70. X-axis represents the years, ranging from 2002 to 2010. 2002 = $34.99 2003 = $37.03 2004 = $40.07 2005 = $43.35 2006 = $47.00 2007 = $49.95 2008 = $53.65 2009 = $57.95 2010 = $61.99

(a) An employee of the cable company made a linear equation of the data using the 2002 cost as a starting value and the 2005 cost as a second data point. What is the **company’s equation**? Use C for the cost per month and t for time in years since 2002. Round the slope to two decimal places.

(b) A consumer advocate made a linear equation of the data using the 2002 cost as a starting value and the 2009 cost as a second data point. What is the **advocate’s equation**? Use C for the cost per month and t for time in years since 2002. Round the slope to two decimal places.

(c) What will the cost per month be in 2015 based on the company’s model found in (a)? Based on the consumer advocate’s model found in (b)? (**Hint**: Remember that t is the number of years since 2002.)

(i) Company's projection:

(ii) Consumer advocate’s projection:

(d) Which of the following is the most accurate estimate of the relative increase in the monthly cost of cable services from 2002 to 2010?

(i) 90–100%

(ii) 75–85%

(iii) 50–60%

(iv) 35–45%

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[3] Flight times retrieved for flights leaving July 28, 2011 from Delta.com.

[4] http://en.Wikipedia.org/wiki/Men%27s_100_metres_world_record_progression

[5] http://stopthecap.com/2010/07/20/happy-summer-rate-increase-comcast-customers-rates-up-for-a-second-time-in-10-months-for-many

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8.12: Quantway Core 4.7 Workforce (Environment) - Student Lesson

INTRODUCTION

Let’s start this collaboration by reviewing linear and exponential functions. As a group, try to come up with at least one example of each type:

- Exponential growth
- Exponential decay
- Linear growth
- Linear decay

To review, the following table summarizes exponential models. In this table, we assume that $r > 0$.

	Exponential Growth	**Exponential Decay**
Formula	$y = A(1 + r)^x$	$y = A(1 - r)^x$
Exponential growth/decay rate	r (percent of the previous value)	r (percent of the previous value)
Vertical intercept (starting value)	A	A
Applications	Any quantity (y) that increases by the same percent per unit increase in x	Any quantity (y) that decreases by the same percent per unit increase in x

To review, the following table summarizes linear models. In this table, we assume that $m > 0$.

	Linear Growth	**Linear Decay**
Formula	$y = A + mx$	$y = A - mx$
Linear growth/decay rate	m (constant)	m (constant)
Vertical intercept (starting value)	A	A
Applications	Any quantity (y) that increases by the same amount (m) per unit increase in x	Any quantity (y) that decreases by the same amount (m) per unit increase in x

Double-Entry Journal for Problem Situation

In this collaboration, you can use a Double-Entry Journal (DEJ) to help you better understand the problem situation.

The DEJ is designed to give you a better understanding of the reading. Sometimes a problem situation has additional information about a situation. A DEJ helps you focus on the key ideas in a problem situation. As you read through **Problem Situation: The Rising Sea Level**, you may want to complete the Double-Entry Journal using the steps below:

1. In the Left Column, list the **key vocabulary** in the problem situation.
2. In the Right Column, **write the definition** from the problem situation, or your own ideas and thoughts about what you think the key vocabulary word means.

Key Vocabulary or Concept in the Problem Situation	**My Definition**
1.	1.

SPECIFIC OBJECTIVES

By the end of this collaboration, you should understand that

- A process of doubling over time can be understood in terms of percent increase.
- Both linear and exponential models for growth may be applied to the same set of data, but result in significantly different long-term projections.

By the end of this collaboration, you should be able to

- build both exponential and linear models.
- compare an exponential and linear model, and make a conclusion based on the comparison.

SPECIFIC LANGUAGE AND LITERACY OBJECTIVES

By the end of this collaboration, you should be able to

- read and comprehend the problem situation.
- complete a Double-Entry Journal (DEJ) with vocabulary from the problem situation about the rising sea level in Cape Carmel.
- demonstrate an understanding of mathematics by writing complete and correct responses to questions.
- demonstrate the ability to interpret, predict, analyze, and compare information about the rising sea level in Cape Carmel.
- use appropriate quantitative and environmental science vocabulary to discuss mathematics in this collaboration.

PROBLEM SITUATION: RISING SEA LEVEL IN CAPE CARMEL

You work at the Cape Carmel Coastal Commission as an expert on Marine Policy. Cape Carmel Coastal Commission is an organization devoted to protecting the coastal habitat. In the past year, many citizens, particularly homeowners, have come to you with concerns about the rising sea level. They are especially worried about the rising sea level in Cape Carmel. A major reason for their worry is that there are many homes built at, or nearly at, sea level along the coast of Cape Carmel. The total value of these homes is more than 100 million dollars.

Global warming is a climate change concern. It is the primary cause for the rise in sea level. Human activities are largely responsible for this climate change, in particular the burning of fossil fuels. When fossil fuels are burned, they release carbon dioxide (CO2) and other gases into the air. These are called greenhouse gases. Greenhouse gases create a blanket around the atmosphere of the Earth. They trap heat and cause the Earth to warm.

Global warming causes ice in the ocean to melt, especially at or near the North and South Poles of the Earth. This ice melting increases the sea level. Global sea level is measured relative to the sea level in the year 1870. If the sea level in 1870 is represented by 0 inches, then Cape Carmel sea level has risen from 6.8 inches to 8.4 inches in the years from 1992 to 2012.[1] If the sea level rises to 24 inches, it will have devastating effects on the homes along the coast of Cape Carmel.

Many of the Cape Carmel residents want to know if, and when, they should sell their property before it is washed away into the sea. You must help residents understand more about the rising sea level in Cape Carmel to help them predict when they should sell their homes.

We can predict the rise of the sea level in Cape Carmel by creating a linear model. First we must find the slope.

(1) If we assume the trend of rising sea level in Cape Carmel is linear, what is the annual rate of increase in the sea level? (**Hint**: Refer to DEJ for quick access to this quantitative information.)

(2) Using the slope calculated above, develop a linear model to describe sea level, starting in the year 1992. Let t be the years since 1992 and R be the rise in sea level.

(3) Based on this model, how long will it take for houses to be washed away into the sea because of the rising sea level in Cape Carmel?

The rise in sea level is only one of the many consequences of global warming. Some of the consequences result in an even larger increase in global warming. For example, as temperatures increase, forests will be drier and more likely to burn. These burning forests will release more CO2 into the atmosphere. This additional CO2 in the air will lead to more global warming.

Another example is the melting ice. As the ice melts and the sea level rises, oceans will become darker. This will cause them to absorb more heat from the sun. As the ocean warms, it will release stored CO2 into the air.

These changes are called “climate change feedback”. Many climate scientists believe that climate change feedback will play a significant role in future global warming.

(4) What kind of mathematical increase could result from the climate change feedback effect? Have you seen any examples of such a mathematical process in this course?

(5) One of the most well-known climate scientists in the world, James Hansen, has argued that climate change feedback has already begun to have a significant impact. Hansen believes that the rate of ice melt in Greenland will double every five to ten years.[2] Consider how a model showing the rate of ice melt might appear.

(a) What sort of mathematical increase results from doubling over a fixed unit of time?

(b) Now, use your answer to Question 2 to answer the following question: What was the percent increase in sea level from 1992 to 1993? Round your answer to the nearest tenth of a percent.

(6) (a) Think back to previous work where you calculated a relative increase. Write down or discuss with your group the method you used.

(b) Use your answer to Question 5 to devise an exponential model for the rise in sea level over time. Let the variables be the same as in Question 2.

(7) Use the model you developed in Question 6(b) to predict the sea level in Cape Carmel in 2012.

(8) In Question 3 you predicted when the houses in Cape Carmel would be washed into the sea. Does the model you developed in Question 6(b) predict a similar result for the sea level in that year? Explain.

(9) On the same axes, graph both the linear model and the exponential model for the following values of time: t=0, t=20, t=40, t=60, t=80, t=100, and t=120. For the exponential model, use the graph to predict when the homes in Cape Carmel would be destroyed due to the rising sea level. **Note**: If completing this problem online, follow the instructions given online to create your graph.

Blank graph

(10) Imagine you are writing to the owners of a home in Cape Carmel. Use one of the models you developed in this lesson to tell the homeowners whether they should sell their home, and if so, when. Explain your reasons. **Write your answer in 1-2 complete sentences. Be sure to use quantitative information from the model in your response**.

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

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[1] http://www.epa.gov/climatechange/science/future.html

[2] http://news.nationalgeographic.com/2015/07/150721-james-hansen-sea-level-rise-climate-change-global-warming-science/

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8.12.1: Preparation 4.7-W The Rising Seas

(1) Kim is trying to do a better job of managing her money. A book on personal finance recommends that she track how much she spends on different items. Kim knows she spends a lot on “nonessentials” such as buying a cup of coffee or buying small items from street vendors. She finds it hard to track these purchases so she decides to set aside a certain amount of money in cash that she will use for these purchases. Every Sunday, she records how much she has left. She keeps a record for 10 weeks. Her data are approximately linear so she made the following graph in which she extends the model past 10 weeks.

Line graph showing the non-essential expenses. Y-axis represents the amount left to spend in dollars, ranging from 0 to 600. X-axis represents the number of weeks, ranging from 0 to 25. 0 = 500 5 = in between 300 and 400 10 = in between 200 and 300 15 = in between 100 and 200 20 = 0

(a) Complete the table below by matching the following mathematical terms to the appropriate contextual meaning and estimating the values from the graph.

- Slope
- Vertical intercept
- Horizontal intercept

Contextual Meaning	Mathematical Term	Value (including units)
Amount Kim sets aside for non-essential items		
Time it takes Kim to spend the entire amount		
Amount Kim spends per week		

(b) Write an equation for the model. Let A = amount left to spend ($), and w = the number of weeks.

(2) In many computer applications such as spreadsheets, a power is indicated by a *caret* symbol (^). For example, 3^4 would be written as 3^4. As a part of her budget management, Kim has put money into an Individual Retirement Account. She creates a spreadsheet to estimate her earnings over time.

She enters the formula: $=1500*\left(1+\frac{0.06}{4}\right)\wedge(4*A2)$, where A2 = the time in years.

(a) How much did Kim invest?

(b) What annual rate of interest does she expect to earn over time? Write your answer as a percentage.

(c) What is the compounding period?

(i) Annual

(ii) Quarterly

(iii) Monthly

(iv) Daily

After Preparation 4.7 (survey)

You should be able to do the following things for the next collaboration. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Collaboration 4.7, you should understand the concepts and demonstrate the skills listed below.

Skill or Concept: I can …	Rating from 1 to 5
create and use linear equations, including understanding of vertical and horizontal intercepts and slope in the context of a problem.	

create and use exponential equations, including understanding of vertical intercept and percentage change in the context of an interest-bearing account.	

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8.12.2: Exercise 4.7-C The Rising Seas

MAKING CONNECTIONS TO THE COLLABORATION

(1) Which of the following was one of the main *mathematical* ideas of the collaboration?

(i) Exponential growth starts slowly, but then speeds up. Linear growth is constant.

(ii) Global temperatures are most likely increasing at an exponential rate.

(iii) Water levels will increase at a steady constant rate for the foreseeable future.

(iv) It would be advisable to procure a houseboat.

(2) The graph below is an exponential model of the levels of toxic waste in a lake. The amount of waste is decreasing at a rate of 12% each year.

Line graph showing the levels of toxic waste in Lake Meadows. Y-axis represents the amount of toxic waste in pounds, ranging from 0 to 1,200. X-axis represents the number of years, ranging from 0 to 14. 0 = 1,000 2 = in between 600 and 800 4 = 600 6 = in between 400 and 600 8 = in between 200 and 400 10 = in between 200 and 400 12 = 200

(a) If the trend continues, which of the following statements would be correct?

(i) The decrease in the level of waste from Year 12 to Year 14 will be the same as the decrease from Year 10 to 12.

(ii) The decrease in the level of waste from Year 12 to Year 14 will be less than the decrease from Year 10 to 12.

(iii) The decrease in the level of waste from Year 12 to Year 14 will be more than the decrease from Year 10 to 12.

(b) Would your answer in (a) change if this had been a linear model? Explain your answer.

DEVELOPING SKILLS AND UNDERSTANDING

(3) Match the following descriptions with the graphs (on the next page). Each description may match more than one graph. Some graphs may match more than one situation and some graphs may not match any situation.

(a) This graph represents the monthly cost of a gym membership plan. Its monthly fee is \$20 with an additional charge of \$5 per class, where Y = monthly cost of gym membership and t = number of classes.

(b) This graph represents the monthly cost for a gym membership that has a fixed monthly charge of \$20 with free unlimited classes where Y = monthly cost of gym membership and t = number of classes.

(c) This graph represents the amount in an account that decreases by \$5 each month where Y = balance in the account and t = months.

(d) This graph represents the amount in an account that decreases by 25% each month where Y = balance in the account and t = months.

(e) This is a graph of $Y = -5t + 40$

(f) This is a graph of $Y = 30(1.25)t$

(g) This is a graph of an exponential function.

(h) This is a graph of a linear function.

I. Graph showing an increasing curb, indicating the following: Start: 0 = 30 End: 5 = just above 90	II. Graph showing a decreasing straight-line, indicating the following: Start: 0 = 40 End: 5 = in between 10 and 20
III. Graph showing a straight-line, indicating the following: Start: 0 = 20 End: 5 = 20	IV. Graph showing a decreasing curb, indicating the following: Start: 0 = 50 End: 5 = in between 10 and 20
V. Graph showing an increasing squiggly line, indicating the following: Start: 0 = 30 End: 5 = in between 40 and 50	VI. Graph showing an increasing straight-line, indicating the following: Start: 0 = 20 End: 5 = 45

(4) In a previous collaboration, you saw that a person's Blood Alcohol Content (BAC) after t hours could be approximated by Widmark's equation. Paula is part of a research study to verify Widmark's equation. She weighs 130 pounds. The researchers ask her to drink three beers. After one hour, her BAC is at 0.10. After three hours her BAC is at 0.08.

(a) Find an equation of the line using her BAC measurements where B is her BAC after t hours.

Hint: The time and BAC can be written as ordered pairs: (1, 0.10) and (3, 0.08). It might help to graph the two points and draw the line between them. Round to the nearest tenth.

(b) What is the slope of your equation and how would you interpret this value in the context of the situation?

(c) What are the horizontal and vertical intercepts of your equation and how would you interpret these values in this scenario?

(d) Recall Widmark's equation:

$$B = -0.015 \cdot t + \left(\frac{2.84 \cdot N}{W \cdot g} \right)$$

where N is the number of "standard drinks," W is his/her weight in pounds, and g = gender constant (0.68 for men and 0.55 for women). One standard drink is a 12-ounce beer, 5-ounce glass of wine, or 1.5-ounce shot of liquor. Simplify this equation for Paula's case, leaving t as a variable. What is the slope and vertical intercept for this simplified equation? How would you interpret these values?

(e) Looking at the models you created in (a) and (d), compare the slopes of the two equations. Which model has the steepest slope? What does that mean in this context?

(5) Caffeine is another chemical in food and beverages. However, caffeine is processed in the body differently than alcohol. For most people, five hours after eating or drinking something with caffeine only about half of the caffeine is left in the bloodstream. In another five hours (10 hours total) half of that is left, and so on. This situation is exponential because the amount of caffeine decreases by 50% every five hours.[3]

(a) Suppose Fred drinks a tall Café Americano from Starbucks, which has about 150 mg of caffeine.[4] Complete the table below for this situation. Round to the nearest tenth. The first two entries are done for you.

Time in Hours	Amount of Caffeine Left in His Body (mg)
0	150
5	75
10	
15	
20	
25	
30	

(b) Plot the data from (a) on a graph.

(c) Suppose his wife, Wilma, drinks a 12-oz Diet Coke, which has 45 mg of caffeine. However, Wilma is using an oral contraceptive. When someone is on "the pill," it takes about 10 hours (twice as long) for half of the caffeine to leave the bloodstream, and another 10 hours for half of the remaining to leave, etc. Complete the table below for this situation. Round to the nearest tenth. Then plot the amount of caffeine left in Wilma's body on the same graph as in (b).

Time in Hours	Amount of Caffeine Left in Her Body (mg)
0	
10	
20	
30	

(d) Looking at your graph of Fred and Wilma's data, who has:

(i) more caffeine in their body in the first few hours?

(ii) more caffeine in their body in the long term?

(e) Based on your graphical models, is there a time that either Fred or Wilma will have no caffeine left in their body? Explain your reasoning.

MAKING CONNECTIONS ACROSS THE COURSE

(6) In OCE 4.5, you modeled the world population based on the growth rate in 2014. To plan for future needs, it is necessary to project how many people there will be in a certain place, but it is also necessary to know information about the ages of the people. This type of information is often displayed in a type of graph called a *population pyramid*. The population pyramid shown below is from the CIA World Factbook. Look at it carefully and think about what information is shown before continuing.[5]

Horizontal bar graph showing the population in millions by age group in the United States in 2014. Y-axis represents the age groups, ranging from 0-4 to over 100. X-axis represents the population in millions, ranging from 0 to 15. 0-4: Male = in between 9 and 12; Female = in between 9 and 12. 5-9: Male = in between 9 and 12; Female = in between 9 and 12. 10-14: Male = in between 9 and 12; Female = in between 9 and 12. 15-19: Male = in between 9 and 12; Female = in between 9 and 12. 20-24: Male = in between 9 and 12; Female = in between 9 and 12. 25-29: Male = in between 9 and 12; Female = in between 9 and 12. 30-34: Male = in between 9 and 12; Female = in between 9 and 12. 35-39: Male = in between 9 and 12; Female = in between 9 and 12. 40-44: Male = in between 9 and 12; Female = in between 9 and 12. 45-49: Male = in between 9 and 12; Female = in between 9 and 12. 50-54: Male = in between 9 and 12; Female = in between 9 and 12. 55-59: Male = in between 9 and 12; Female = in between 9 and 12. 60-64: Male = in between 9 and 12; Female = in between 9 and 12. 65-69: Male = in between 6 and 9; Female = in between 6 and 9. 70-74: Male = in between 3 and 6; Female = in between 6 and 9. 75-79: Male = in between 3 and 6; Female = in between 3 and 6. 80-84: Male = in between 0 and 3; Female = in between 3 and 6. 85-89: Male = in between 0 and 3; Female = in between 0 and 3. 90-94: Male = in between 0 and 3; Female = in between 0 and 3. 95-99: Male = in between 0 and 3; Female = in between 0 and 3. Over 100: Male = in between 0 and 3; Female = in between 0 and 3.

This population pyramid is for the U.S. in 2014. The left side of the pyramid represents males; the right represents females. Each bar represents the age group listed in the center column. This pyramid measures the population in millions along the horizontal axis. Sometimes these types of graphs measure the population in percentages instead of numbers. Use this graph to answer the following questions.

(a) Based on the graph, which of the following statements is the most accurate?

(i) In the population under 30 years old, the number of men and the number of women are similar.

(ii) In the population over 79 years old, the number of men and the number of women are equal.

(iii) There are more people in the age range of 35–44 than in the range of 45–55.

(b) The graph below illustrates Mexico's population in 2014.[6]

Horizontal bar graph showing the population in millions by age group in Mexico in 2014. Y-axis represents the age groups, ranging from 0-4 to over 100. X-axis represents the population in millions, ranging from 0 to 6. 0-4: Male = in between 4.8 and 6; Female = in between 4.8 and 6. 5-9: Male = in between 4.8 and 6; Female = in between 4.8 and 6. 10-14: Male = in between 4.8 and 6; Female = in between 4.8 and 6. 15-19: Male = in between 4.8 and 6; Female = in between 4.8 and 6. 20-24: Male = in between 4.8 and 6; Female = in between 4.8 and 6. 25-29: Male = in between 4.8 and 6; Female = in between 4.8 and 6. 30-34: Male = in between 3.6 and 4.8; Female = in between 3.6 and 4.8. 35-39: Male = in between 3.6 and 4.8; Female = in between 3.6 and 4.8. 40-44: Male = in between 3.6 and 4.8; Female = in between 3.6 and 4.8. 45-49: Male = in between 2.4 and 3.6; Female = in between 3.6 and 4.8. 50-54: Male = in between 2.4 and 3.6; Female = in between 2.4 and 3.6. 55-59: Male = in between 1.2 and 2.4; Female = in between 2.4 and 3.6. 60-64: Male = in between 1.2 and 2.4; Female = in between 1.2 and 2.4. 65-69: Male = in between 1.2 and 2.4; Female = in between 1.2 and 2.4. 70-74: Male = in between 0 and 1.2; Female = in between 1.2 and 2.4. 75-79: Male = in between 0 and 1.2; Female = in between 0 and 1.2. 80-84: Male = in between 0 and 1.2; Female = in between 0 and 1.2. 85-89: Male = in between 0 and 1.2; Female = in between 0 and 1.2. 90-94: Male = in between 0 and 1.2; Female = in between 0 and 1.2. 95-99: Male = in between 0 and 1.2; Female = in between 0 and 1.2. Over 100: Male = in between 0 and 1.2; Female = in between 0 and 1.2.

Indicate if each statement applies to the United States, Mexico, or both.

(i) The general trends in age categories are similar for men and women.

(ii) The number of people 0–19 years old is about the same as the number of people 20–39 years old.

(iii) The population could be described as being weighted towards younger ages.

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[3] http://en.Wikipedia.org/wiki/Caffeine

[4] http://www.starbucks.com/menu/drinks/espresso/caffe-americano?foodZone=9999

[5] https://www.cia.gov/library/publications/the-world-factbook/geos/us.html

[6] https://www.cia.gov/library/publications/the-world-factbook/geos/mx.html

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